SATYAM SIR

ADAPTIVE PROBLEMS BOOK IN PHYSICS

PHYSICS FACTOR
Fall in Love with Physics

Master this Chapter from Basic to Advance

MOTION IN 1D

4

SOLVED QUESTION BANK

ADAPTIVE

PROBLEMS BOOK

IN PHYSICS

Volume 04

Motion in a Straight Line

for

Board Exams

State Engineering Entrance

NEET Medical Entrance Exam

IIT JEE Mains & Advanced Exams

KVPY & International Physics Olympiad

2nd Edition (Feb 2024)

Disclaimer

In spite of many efforts taken to present this book without errors, still some errors might be possible. The author doesn't take any responsibility for such errors in any form. We request you to kindly give error information to author at satyamquestapps@gmail.com , so that he will rectify those mistakes in his next edition of his book.

Author’s Message

Physics is a subject to understand nature. How it works? There are certain laws that applies to nature that we study in physics in terms of conceptual theory. Then we apply these theories in day-to-day life or its applications in form of numerical. It is the toughest subject for those who tend to mug up this subject. A subject requires an IQ. However, an IQ is a subjective prospect. To measure, it has numerous parameters. It grows, as we tend to understand the subject. The intension is to create this book to present physics subject as in a most systematic approach to learn in depth knowledge and develop a good problem solving skill. All the chapters presented in this book starts with an adaptive topic wise question bank, that personally I have made especially for other teacher to use in their class. However, it is not limited to do so. These problems are given in order of difficulty level. Further, the chapter gives a plenty of unsolved questions exercise questions to practice, plus last year asked JEE Mains and Advanced Questions along with NCERT questions. This book also contains selected BOARD level problems in every chapter, which gives a good boost in your school and board exams. I hope you would enjoy this journey of learning. Though an enormous of hard work and efforts have been to make this book as error free as possible still I expect feedbacks and content mistakes (if any), so to remove in the next upcoming editions.

Enjoy learning Physics!

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Ex- FIITJEE, AAKASH, ALLEN TEACHER

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Motion in a Straight Line

Chapter Summary

Motion and Rest

Motion is relative. The same body can be in motion and at rest simultaneously at the same time with respect to two different observers.

Frame of Reference

This is a point in 3-D space about which the position or motion of a body is defined.

Distance: It is defined as the actual length of path covered by an object.

It is a +ve scalar quantity.

Its SI unit is m (meter).

Displacement: Displacement is the change in position vector i.e., vector joining initial and final position, or we can say it is a minimum possible distance between two positions.

It is a vector quantity.

Position vector of $A = \overrightarrow{OA} = \vec{r}_1$

Position vector of $B = \overrightarrow{OB} = \vec{r}_2$

Displacement from position A to $B = \overrightarrow{AB}$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \vec{r}_2 - \vec{r}_1$

Speed: It is defined as the rate of change of distance, with respect to time.

$$(v) = \frac{ds}{dt}$$

It's the derivative of distance with respect to time.

If speed is uniform, then

$$\text{Speed} = \frac{\text{Distance}}{\text{time}} = \frac{S}{T}$$

It is a scalar quantity.

Its S.I. unit is m/s

Average Speed

The average speed of a particle is defined as ratio of total distance travelled to the total time taken

$$\text{Average Speed} = \frac{\text{Total Distance travelled}}{\text{Total time taken}}$$

Velocity

It is defined as the rate of change in displacement with respect to time. $v = \frac{d\vec{r}}{dt}$

It is a vector quantity.

Its S.I. unit is m/s

Average Velocity

The average velocity of a particle for a given interval of time is defined as the ratio of its displacement to the time taken.

Average velocity $= \vec{v}_{av} = \frac{\text{Displacement}}{\text{Time}} = \frac{\Delta\vec{S}}{\Delta t}$

Acceleration

It is defined as rate of change of velocity with respect to time.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

Also, $v = \frac{dx}{dt}$, $a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{vdv}{dx}$

It is a vector quantity

Its S.I. unit is m/s^2

Average acceleration

It is defined as ratio of change in velocity to the time interval in which change takes place.

Suppose a particle moving along *x*-axis has velocity v_1 at time t_1 and velocity v_2 at time t_2 average acceleration is given by

$$a_{av} = \frac{\text{change in velocity}}{\text{Time}}$$

Equations of Motion

If the acceleration of the particle is constant, then we can write these three useful equations of motion as:

$$v = u + at$$

$$S = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Where, v=final velocity, u=initial velocity

S=displacement, t=time, a=constant acceleration

Motion Under Gravity

Case 1:

When an object is thrown in upward direction (taking positive) space with initial velocity v_0.

Acceleration $= -g$ (in downward direction)

So, equation of motion will be

$$v = v_0 - gt$$

$$h = v_0 t - \frac{1}{2}gt^2$$

$$v^2 = v_0^2 - 2gh$$

Case 2:

When an object is thrown in downward direction (taking positive) in space with initial velocity v_0.

Acceleration $= +g$ (in downward direction)

So, equation of motion will be

$$v = v_0 + gt$$

$$s = v_0 t + \frac{1}{2}gt^2$$

$$v^2 = v_0^2 + 2gh$$

Graphs in Motion

There are 3 commonly drawn graphs in motion:

Displacement–Time Graph

(i) Slope of positive vs. time graph gives velocity

(ii) Slope at a particular point of the graph gives instantaneous velocity

(iii) Slope of a line joining initial position to final position gives average velocity between two points.

(iv) The maximum slope at any point on the graphs gives the maximum velocity.

(v) When graph will be straight line parallel to x-axis, it means slope is zero so velocity will be zero.

$$v = \tan\theta = 0 = \tan 0°$$

(vi) Area of displacement-time graph has no significance.

Velocity–Time Graph

(i) Slope of velocity vs. time graph gives acceleration.

(ii) Area under curve gives displacement and area on negative side gives negative displacement.

Acceleration–Time Graph

1. The slope of acceleration vs. time graphs gives rate of change of acceleration with respect to time, which has no physical significance.

2. Area under curve of this graph gives change in velocity.

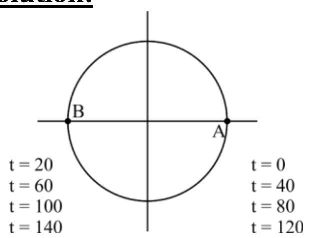
Beginner	**Distance and Displacement**

1. An athlete completes one round of a circular track of radius R in 40 s. What will be his displacement at the end of 2 min. 20 s?

(a) Zero **(b) 2R**

(c) 2πR **(d) 7πR**

Solution:



At t = 140 sec (2 min 20 sec)

He will be diametrically opp. end.

Hence, the displacement will be '2R'

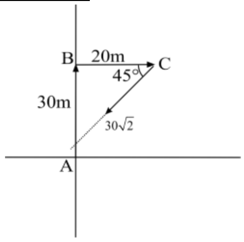
=> Option (B) is correct.

2. A person moves 30 m north and then 20 m towards east and finally 30√2 m in south-west direction. The displacement of the person from the origin will be

(a) 10 m along north (b) 10 m long south

(c) 10 m along west (d) Zero

Solution:



x = 20 - $30\sqrt{2}\cos 45°$

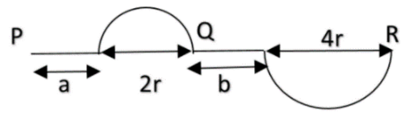
= 20 - $30\sqrt{2} \times \frac{1}{\sqrt{2}}$= -10

y = 30 - $30\sqrt{2}\cos 45°$ = 0

Final co-ordinate = (-10, 0)

=> Option C is correct.

3. A car starts from P and follows the path as shown in figure. Finally, car stops at R. Find the distance travelled and displacement of the car if



$a = 7m$, $b = 8\ m$ and $r = \frac{11}{\pi}m$?[Take = $\frac{22}{7}$]

Solution:

Distance is a scalar quantity, hence can be measured by the scalar sum (normal sum) of individual distances.

Hence, distance

= a + πr + b + π x (2r) = 3πr + a +b

Hence total distance = 3 x π x $\frac{11}{\pi}$ + 7 + 8 = 33 + 15 = 48 m

How do we measure displacement?

Displacement is a vector quantity, hence can be measured by the laws of vector addition. In general to get displacement, we just find the distance between initial and final point.

The distance between initial and final point

= a + 2r + b +4r

Displacement

= a + b + 6r = 7 + 8 + 6 x $\frac{11}{\pi}$

= 15 + $\frac{66}{\frac{22}{7}}$ = 15 + 3 x 7 = 15 + 21 = 36m

4. When a person leaves his home for sightseeing by his car, the meter reads 12352 km. When he returns home after two hours the reading is 12416 km. During the journey, he stays for 15 minutes at midway.
(a) What is the average speed of the car during this period?
(b) What is the average velocity?

Solution:

(a) Total distance covered is given as the difference in both the readings

Total distance traveled =12416 km -12352 km

So, Total distance traveled = 64 km

and

Total time taken is 2 hours

So, V avg = 64/2 = 32 km/hr

(b) As he returns to his house, the displacement is zero, And we know,

Average Velocity =0

Hence the average velocity is 0.

Expert **Distance and Displacement**

1. A wheel of radius 1 meter rolls forward half a revolution on a horizontal ground. The magnitude of the displacement of the point of the wheel initially in contact with the ground is

(a) 2π **(b) $\sqrt{2}\pi$**

(c) $\sqrt{\pi^2+4}$ **(d) π**

Solution:

Correct option is C.

We have,

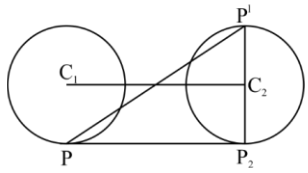
R = 1m

Horizontal distance covered by the wheel in the half revolution is: πR

So, the displacement of the point which was initially in contact with ground = AA'

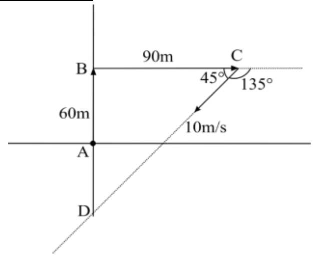
$\sqrt{(\pi R)^2 + 2R^2}$

= R $\sqrt{\pi^2+4}$



2. A body starts from origin and moving in north direction with the speed 20 m/s and moves for 3 seconds. After that it turns right and moves with speed 30 m/s for next 3 seconds. Thereafter, it turns 135° clockwise and starts moving with 10 m/s for next 20 seconds. Find
(a) Total distance travelled by the body
(b) Magnitude of displacement & its direction

Solution:



(a) AB = V_1t_1

= 20 x 3 = 60 m

BC = V_2t_2

= 30 x 3 = 90 m

CD = V_3t_3

= 10 x 20 = 200 m

∴ Total distance = 60 + 90 + 200

= 350 m

(b) Total x- displacement = 90 – (CD) cos45°

= 90 – (200) x $\frac{1}{\sqrt{2}}$

= 90 – 141.4

= -51.4 m

Total y- displacement = 60 – (CD) cos45°

= 60 – (200) x $\frac{1}{\sqrt{2}}$

= 60 – 141.4

= -81.4 m

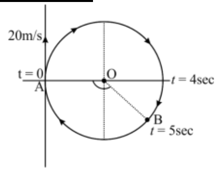
∴ Net Displacement = $|\vec{\Delta s}| = \sqrt{x^2+y^2}$

= $\sqrt{51.4^2+81.4^2}$

= 96.3 m

3. A body starts from origin and moving in north direction with the constant speed 20 m/s in a circular motion clockwise direction. The time period of its motion is 8 seconds. Find the distance & displacement in 5 seconds.

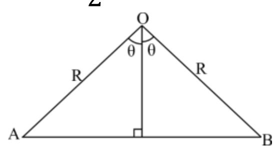
Solution:



(a) distance = vt = 20 x 5 = 100m

(b) displacement = $|AB|$

$\theta = \frac{135}{2}$



Radius = R = $\frac{20 \times 8}{2\pi}$

= $\left(\frac{80}{\pi}\right) m$

AB = 2Rsin θ

= $2\left(\frac{80}{\pi}\right)\left(sin\frac{135}{2}\right)$

= 47.05 m

Beginner **Speed and Velocity**

1. One car moving on a straight road covers one third of the distance with 20 km/hr and the rest with 60 km/hr. The average speed is

(a) 40 km/hr **(b) 80 km/hr**

(c) $46\frac{2}{3}$ km/hr **(d) 36 km/hr**

Solution:

Correct option is D.

Velocity for one third of distance v_1 = 20km/h

Velocity for two third distance v_2 = 60 km/h

Time taken to cover two third of distance

$t_1 = \frac{d/3}{v_1} = \frac{d/3}{20}$

Time taken to cover two third of distance

$t_2 = \frac{2d/3}{v_2} = \frac{2d/3}{60}$

Average velocity = $\frac{Total\ distance}{total\ time}$

$v_{av} = \frac{d}{t_1 + t_2}$

$= \frac{d}{\frac{d/3}{20}+\frac{2d/3}{60}}$ = 36 km/h

2. Boston Red Sox pitcher Roger Clemens could routinely throw a fastball at a horizontal speed of 160 km/hr. How long did the ball take to reach home plate 18.4 m away?

Solution:

We assume that the ball moves in a horizontal straight line with an average speed of 160 km/hr. Of course, in reality this is not quite true for a thrown baseball.

We are given the average velocity of the ball's motion and also a particular displacement, namely Δx = 18.4 m. Equation gives us:

$\bar{v} = \frac{\Delta x}{\Delta t} \quad \Rightarrow \quad \Delta t = \frac{x}{\bar{v}}$

But before using it, it might be convenient to change the units of $\bar{v}$. We have:

$$\bar{v} = 160\frac{km}{hr}\cdot\left(\frac{1000m}{1km}\right)\cdot\left(\frac{1hr}{3600s}\right) = 44.4\frac{m}{s}$$

Then we find:

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{18.4m}{44.4\frac{m}{s}} = 0.414s$$

The ball takes 0.414 seconds to reach home plate.

3. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km/h. The average speed of the man over the interval of time 0 to 40 min. is equal to

(a) 5 km/h **(b) $\frac{25}{4}$ km/h**

(c) $\frac{30}{4}$ km/h **(d) $\frac{45}{8}$ km/h**

Solution:

Correct option is D.

Average velocity = $\frac{Total\ distance}{total\ time}$

Time taken to go to the market = $\frac{distance}{speed}$

= $\frac{2.5}{5} hour = \frac{1}{2} hour$ = 30 minutes

Time taken to return back = $\frac{distance}{speed}$

= $\frac{2.5}{7.5} hour = \frac{1}{3} hour$ = 20 minutes

Distance travelled in 10 minutes while returning back = $\frac{10}{60} \times 7.5 = 1.25$ km

(Speed is constant at 7.5 km/hr)

Hence Distance traveled in 40 minutes (30 mins towards the market and 10 mins towards the home) = 2.5 + 1.25 = 3.75 km

So average speed in 0 to 40 minutes

= $\frac{3.75\ km}{\frac{40}{60}hr}$

= $\frac{45}{8} km/hr$

4. If a car covers 2/5th of the total distance with v_1 speed and 3/5th distance with v_2 then average speed is?

Solution:

Time taken to cover $\frac{2}{5}th$ distance: $t_1 = \frac{2}{5}\frac{s}{v_1} = \frac{distance}{velocity}$ where s is total distance

Time taken to cover $\frac{3}{5}th$ distance: $t_2 = \frac{3}{5}\frac{s}{v_2}$

Average speed = $\frac{total\ distance}{total\ time}$

$v_{mg} = \frac{s}{t_1 + t_2}$

$= \frac{s}{\frac{2s}{5v_1}+\frac{3s}{5v_2}}$

$= \frac{5v_1v_2s}{s(2v_2s+3v_1s)} = \frac{5v_1v_2}{3v_1+2v_2}$

Expert Speed and Velocity

1. The distance travelled by a particle in time t is given by $s = (2.5)t^2$. Find (a) the average speed of the particle during the time 0 to 5.0 s, and (b) the instantaneous speed at $t = 5.0\ s$.

Solution:

Expression for velocity:

v = $\frac{ds}{dt}$

v = $\frac{d}{dt}(2.5m/s^2)t^2$

v = $(5m/s^2)t$

(a) Average speed from 0 to 5 sec

Distance travelled during 0 to 5 sec = $(2.5m/s^2)t^2$

= $(2.5m/s^2)(5s)^2$ = 62.5 m

Average speed:

v = $\frac{Total\ distance\ travelled}{total\ time}$

v = $\frac{62.5}{5}$

v = 12.5 m/s

(b) Instantaneous Speed at t = 5sec

v = $(5m/s)t$

= $(5m/s^2)(5s)$

= 25 m/s

2. $x = t^2 - 4t$ Find out the average speed and velocity in 5 seconds of its motion.

Solution:

x = $t^2 - 4t$

v = $\frac{dx}{dt}$ = 2t - 4 = 0

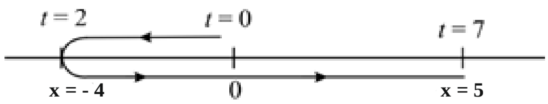
t = 2 sec

Velocity of particle is zero. This means, the particle will change its direction of motion at t = 2sec.

At t = 0, x = 0

t = 2, x = -4

t = 5, x = 5



Total distance travelled = 4 + 4 + 5 = 13 m

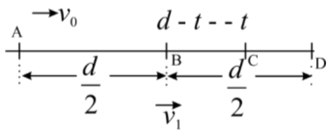
Total displacement = 5m

$v_{avg} = \frac{13}{5}$ = 2.6 m/s

$\overrightarrow{v_{avg}} = \frac{5}{5}$ = 1 m/s

3. A particle travels half of its journey with speed v_0 and remaining half it travels in two equal time intervals with speeds v_1 and v_2. Find out the average velocity of the particle.

Solution:



$v_{avg} = \frac{d}{t}$

$= \frac{d}{t_{AB} + t_{BD}}$

$= \frac{d}{\frac{d/2}{v_0} + 2t}$ ……………(1)

$v_1 t + v_2 t = \frac{d}{2}$

2t = $\frac{d}{v_1+v_2}$ ……(2)

$v_{avg} = \frac{d}{\frac{d/2}{v_0} + \frac{d}{v_1+v_2}}$

$= \frac{1}{\frac{1}{2v_0} + \frac{1}{v_1+v_2}}$

$= \frac{2v_0(v_1+v_2)}{v_1+v_2+2v_0}$

Beginner Acceleration & Calculus

1. A particle moves along a straight line such that its displacement at any time t is given by $S = t^3 - 6t^2 + 3t + 4$ meters. The velocity when the acceleration is zero is

(a) 3ms^{-1} **(b) -12ms^{-1}**

(c) 42 ms^{-1} **(d) -9ms^{-1}**

Solution:

Correct option is D.

S = $t^3 - 6t^2 + 3t + 4$

v = $\frac{ds}{dt}$

So v = $3t^2$-12t + 3

Again differentiate, a = $\frac{dv}{dt}$

6t – 12 = 0

t = 2s

v = $3(2)^2$-12 x 2 +3 = -9ms^{-1}

2. The displacement of a particle is given by $y = a + bt + ct^2 - dt^4$. The initial velocity and acceleration are respectively

(a) $b, -4d$ **(b)** $-b, 2c$

(c) $b, 2c$ **(d)** $2c, -4d$

Solution:

Correct option is C.

Initial velocity is given by

$v_{t=0} = \frac{dy}{dt}|_{t=0} = [b + 2ct - 4dt^3]_{t=0}$ = b

Initial acceleration,

$v_{t=0} = \frac{dy}{dt}|_{t=0} = [2c - 12dt^2]_{t=0}$ = 2c

3. A particle moves along x-axis as $x = 4(t-2) + a(t-2)^2$ Which of the following is true?

(a) The initial velocity of particle is 4

(b) The acceleration of particle is $2a$

(c) The particle is at origin at $t = 0$

(d) None of these

Solution:

Given

x = 4(t – 2) + a $(t-2)^2$ at x = 4

At t = 0

x = 4(0 – 2) + a $(0-2)^2$

= - 8 + 4a

=> v = $\frac{dx}{dt}$

= 4 + 2a(t-2)

At t = 0,

v = 4 – 4a

= 4 (1-a)

But acceleration a = $\frac{d^2x}{dt^2}$ = 2a

Hence, Option B is correct.

4. The velocity of a body depends on time according to the equation $v = 20 + 0.1t^2$. The body is undergoing

(a) Uniform acceleration

(b) Uniform retardation

(c) Non-uniform acceleration

(d) Zero acceleration

Solution:

Correct option is C.

$\vec{a} = \frac{dv}{dt} = \frac{d}{dt}(20 + 0.1t^2) = 0.2t$

Since, acceleration, a, does depend on time, t, it is non-uniform acceleration.

5. Starting from rest, acceleration of a particle is a = 2(t – 1) . The velocity of the particle at t = 5s is

(a) 15 m/sec **(b) 25 m/sec**

(c) 5 m/sec **(d) None of these**

Solution:

a = 2(t-1)

= 2t – 2

dv = (2t – 2) dt

Integrating both side we get,

$\int dv = \int (2t-2)dt$

v = $t^2 - 2t$

= $(5)^2 - 2x5$

= 25 – 10

= 15 m/s

6. The position of a particle as a function of time t, is given by x(t) = at + bt² – ct³ where, a, b and c are constants. When the particle attains zero acceleration, then its velocity will be:

(a) $a + \frac{b^2}{4c}$ **(b)** $a + \frac{b^2}{3c}$

(c) $a + \frac{b^2}{c}$ **(d)** $a + \frac{b^2}{2c}$

Solution:

(b) x = at + bt² – ct³

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}\left(at + bt^2 + ct^3\right)$

= a + 2bt – 3ct²

Acceleration, $\frac{dv}{dt} = \frac{d}{dt}\left(a + 2bt - 3ct^2\right)$

or 0 = 2b – 3c × 2t

$\therefore\ t = \left(\frac{b}{3c}\right)$

and $v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2 = \left(a + \frac{b^2}{3c}\right)$

7. If the velocity of a particle is v = At + Bt², where A and B are constants, then the distance travelled by it between 1s and 2s is

(a) 3A + 7B **(b)** $\frac{3}{2}A + \frac{7}{3}B$

(c) $\frac{A}{2} + \frac{B}{3}$ **(d)** $\frac{3}{2}A + 4B3$

Solution:

(b) Velocity of the particle is given as

v = At + Bt²

where A and B are constants.

$\frac{dx}{dt}$ = At + Bt² $\left[\because v = \frac{dx}{dt}\right]$

dx = (At + Bt²) dt

Integrating both sides, we get

$\int_{x_1}^{x_2} dx = \int_1^2 \left(At + Bt^2\right)dt$

$\Delta x = x_2 - x_1 = A\int_1^2 t dt + B\int_1^2 t^2 dt$

$$= A\left[\frac{t^2}{2}\right]_1^2 + B\left[\frac{t^3}{3}\right]_1^2$$

$$= \frac{A}{2}\left(2^2 - 1^2\right) + \frac{B}{3}\left(2^3 - 1^3\right)$$

∴ Distance travelled between 1s and 2s is

$$\Delta x = \frac{A}{2}\times(3) + \frac{B}{3}(7) = \frac{3A}{2} + \frac{7B}{3}$$

Expert **Acceleration & Calculus**

1. The displacement x of a particle varies with time t as $x = \alpha t^2 - \beta t^3$

(a) particle will return to its starting point after time $\frac{\alpha}{\beta}$

(b) the particle will come to rest after time $\frac{2\alpha}{3\beta}$

(c) the initial velocity of the particle was zero, but its initial acceleration was not zero.

(d) no net force act on the particle at time $\frac{\alpha}{3\beta}$

Solution:

Correct options are A, B, C and D

Given, $x = \alpha t^2 - \beta t^3$

Particle will return to its starting point when,

$x = \alpha t^2 - \beta t^3 = 0$

or,

$t = \frac{\alpha}{\beta}$

Velocity = v = $\frac{dx}{dt}$ = $2t\alpha - 3\beta t^2$ …..(1)

at t = 0, v = 0 so the initial velocity zero.

Acceleration = a = $\frac{dv}{dt}$ = $2\alpha - 6t\beta$ …..(2)

at t = 0, a = 2α so initial acceleration does not zero.

The particle will come to rest when,

$2t\alpha - 3\beta t^2 = 0$ from (1)

or t = $\frac{2\alpha}{3\beta}$

At, $t = \frac{\alpha}{3\beta}$, $a = 2\alpha - 6\beta\left(\frac{\alpha}{3\beta}\right) = 0$

So net force = ma = 0, thus no net force acct on the particle when t = $\frac{\alpha}{3\beta}$

2. A particle moves along a straight line such that the relation between time t and displacement s is s^2 = t, then

(a) Acceleration is positive and directly proportional to v^2

(b) Acceleration is positive and directly proportional to v^3

(c) Acceleration is negative and directly proportional to v^2

(d) Acceleration is negative and directly proportional to v^3

Solution:

$s^2 = t$

2s.$\frac{ds}{dt}$ = 1

2.$\frac{ds}{dt} = \frac{1}{s} = t^{-1/2}$

a = $\frac{d^2s}{d^2t} = \frac{1}{s} = t^{-\frac{1}{2}}$

a = $\frac{d^2s}{d^2t} = \frac{-1}{2}$ x $t^{-\frac{3}{2}}$

$\because v = \frac{dy}{ds} \propto \frac{1}{s}$

$a = -\frac{1}{4s^3} \propto v^3$

∴ Acceleration is negative and it is proportional to v^3

∴ Option (d)

3. A particle moves along x-axis and displacement varies with time t as $x = \left(t^3 - 3t^2 - 9t + 5\right)$. Then

(a) in the interval 3< t< 5, the particle is moving in +x direction

(b) the particle reverses its direction of motion twice in entire motion if it starts at t = 0

(c) the average acceleration from 1≤t ≤ 2 seconds is 6m/s².

(d) in the interval 5≤t ≤6 seconds, the distance travelled is equal to the displacement.

Solution:

Correct options are A and D.

$x = t^3 - 3t^2 - 9t + 5$

$v = 3t^2 - 6t - 9$

a = 6t – 6

the particle to move in positive x direction

v > 0

$3t^2 - 6t - 9 > 0$

$t^2 - 2t - 1 > 0$

(t – 3) (t + 1) > 0

So,

v < 0 for T < 3

v > 0 for T > 3

So, reverses direction only once.

$a = \frac{\Delta v}{\Delta t} = \frac{v_{t=2} - v_{t+1}}{\Delta t} = \frac{3}{2} = \frac{(3.2^2 - 6x2 - 9) - (3.1^2 - 6x1 - 9)}{2}$

$= \frac{3}{2} = 1.5\ m/s^2$

Since, in 5≤ $t \leq 6$, v does not change its direction, distance = displacement

Answer is A and D.

4. The relation between time and distance is $t=\alpha x^2+\beta x$, where α and β are constants. The retardation is

(a) $2\alpha v^3$ **(b)** $2\beta v^3$

(c) $2\alpha\beta v^3$ **(d)** $2\beta^2 v^3$

Solution:

Correct option is A.

$\vec{v} = \frac{\overrightarrow{dx}}{dt}$

and

$\vec{a} = \frac{\overrightarrow{dv}}{dt}$

$t = \alpha x^2 + \beta x$

Differentiating with respect to time on both sides, we get

$1 = 2\alpha \frac{dx}{dt} x + \beta \frac{dx}{dt}$

$\therefore v = \frac{1}{\beta + 2\alpha x}$

$\frac{dv}{dt} = \frac{-2\alpha v}{(\beta+2\alpha x)^2} = -2\alpha v^3$

Negative sign shows retardation.

5. Position of a particle moving along x – axis is given by x = 2 + 8t – 4t^2 where t is time in sec. The distance travelled by the particle in the first two seconds is:

(a) 2 units **(b) 8 units**

(c) 10 units **(d) 16 units**

Solution:

Correct option is B.

x = 2 + 8t - 4t^2

=> velocity = $\frac{dx}{dt}$ = 0 + 8 – 4(2)t

Or v = 8 – 8t

At t = 0 v = 8m/s & t = 1 => v = 0

t = 2 => v = 8 – 16 = -8

so plotting the graph we get there points

A ≡ (0,8)

B ≡ (1,0)

C ≡ 2, -8

Distance = Area OAB + Area BOC

$= \frac{1}{2} OB \ x \ OA + \frac{1}{2} O'B \ x \ O'C = \frac{1}{2} \ x \ 1 \ x \ 8 + \frac{1}{2} \ x \ (2-1)8 = 8m$

6. A particle is moving with speed v = $\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time t = τ (assume that the particle is at origin at t = 0).

(a) $\frac{b^2\tau}{4}$ **(b)** $\frac{b^2\tau}{2}$

(c) b^{2}τ **(d)** $\frac{b^2\tau}{\sqrt{2}}$

Solution:

(b) Given, $v = b\sqrt{x}$

or $\frac{dx}{dt} = bx^{1/2}$

or $\int_0^x x^{-1/2}dx = \int_0^t bdt$

or $\frac{x^{1/2}}{1/2} = bt$

or $x = \frac{b^2t^2}{4}$

Differentiating w.r.t. time, we get

$\frac{dx}{dt} = \frac{b^2 \times 2t}{4}$ (t = τ)

or $v = \frac{b^2\tau}{2}$

7. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by X_P (t) = at + bt^2 and X_Q (t) = ft – t^2. At what time do the cars have the same velocity?

(a) $\frac{a-f}{1+b}$ **(b)** $\frac{a+f}{2(b-1)}$

(c) $\frac{a+f}{2(1+b)}$ **(d)** $\frac{f-a}{2(1+b)}$

Solution:

(d) Velocity of each car is given by

$$V_P = \frac{dx_P(t)}{dt} = a + 2bt$$

and $V_Q = \frac{dx_Q(t)}{dt} = f - 2t$

It is given that $V_P = V_Q$

a + 2bt = f – 2t

$t = \frac{f-a}{2(b+1)}$

8. In an arcade video game a spot is programmed to move across the screen according to x = 9.00t – 0.750t^3, where x is distance in centimeters measured from the left edge of the screen and t is time in seconds.

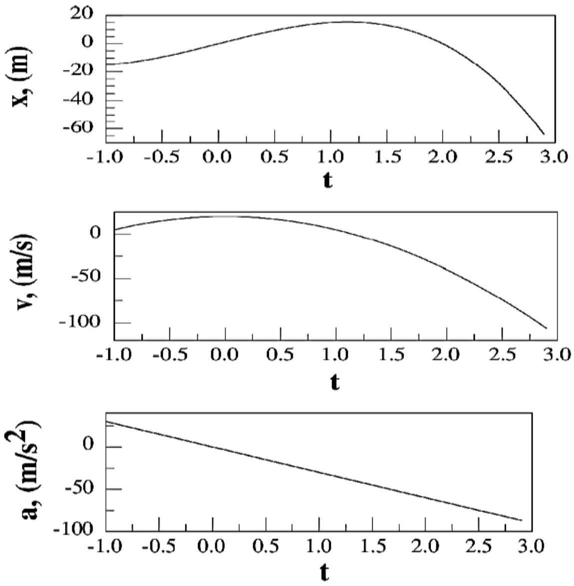
When the spot reaches a screen edge, at either x = 0 or x = 15.0 cm, t is reset to 0 and the spot starts moving again according to x(t).
(a) At what time after starting is the spot instantaneously at rest?
(b) Where does this occur?
(c) What is its acceleration when this occurs?
(d) In what direction is it moving just prior to coming to rest?
(e) Just after?
(f) When does it first reach an edge of the screen after t = 0?

Solution:

(a) This is a question about the instantaneous velocity of the spot. To find v(t) we calculate:

$$v(t)=\frac{dx}{dt}=\frac{d}{dt}\left(9.00t-0.750t^3\right)=9.00-2.25t^2$$

Where this expression will give the value of v in cm/s when t is given in seconds.



We want to know the value of t for which v is zero, i.e. the spot is instantaneously at rest. We solve:

$$9.00-2.25t^2=0 \implies t^2=\frac{9.00}{2.25}=4.00s^2$$

(Here we have filled in the proper units for t^2 since by laziness they were omitted from the first equations!) The solutions to this equation are

t = ±2.00s

but since we are only interested in times after the clock starts at t = 0, we choose t = 2.00s.

(b) In this part we are to find the value of x at which the instantaneous velocity is zero. In part (a) we found that this occurred at t = 3.00s so we calculate the value of x at t = 2.00s:

$x(2.00s) = 9.00\,(2.00) – 0.750\,(2.00)^3 = 12.0$ cm

(where we have filled in the units for x since centimeters are implied by the equation). The dot is located at x = 12.0 cm at this time. (And recall that the width of the screen is 15.0 cm.)

(c) To find the (instantaneous) acceleration at all times, we calculate:

$$a(t)=\frac{dv}{dt}=\frac{d}{dt}\left(9.00-2.25t^2\right)=-4.50t$$

Where we mean that if t is given in seconds, a will be given in m/s^2. At the time in question (t = 2.00s) the acceleration is

a(t = 2.00s) = –4.50 (2.00) = –9.00

That is, the acceleration at this time is –9.00 m/s^2.

(d) From part (c) we note that at the time that the velocity was instantaneously zero the acceleration was negative. This means that the velocity was decreasing at the time. If the velocity was decreasing yet instantaneously equal to zero then it had to be going from positive to negative values at t = 2.00 s. So just before this time its velocity was positive.

(e) Likewise, from our answer to part (d) just after t = 2.00 s the velocity of particle had to be negative.

(f) We have seen that the dot never gets to the right edge of the screen at x = 15.0 cm. It will not reverse its velocity again since t = 2.00 s is the only positive time at which v = 0. So it will keep moving to back to the left, and the coordinate x will zero when we have:

$x = 0 = 9.00t – 0.750t^3$

Factor out t to solve:

$t(9.00 – 0.750t^2) = 0$

$$\begin{cases} t=0 & \text{or} \\ \left(9.00-0.750t^2\right)=0 & \text{otherwise.} \end{cases}$$

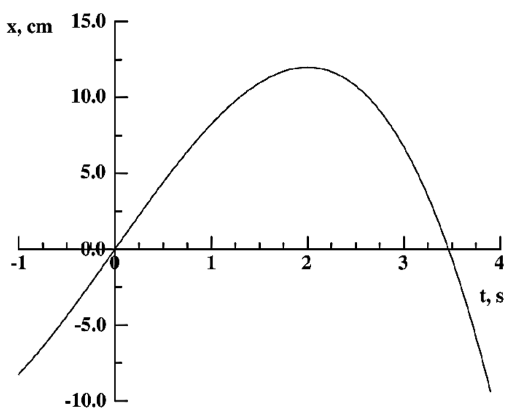


Fig. Plot of x vs t for moving spot. Ignore the parts where x is negative!

The first solution is the time that the dot started moving, so that is not the one we want.

The second case gives:

$(9.00 - 0.750t^2) = 0$

$t^2 = \frac{9.00}{0.750} = 12.0s^2$

Which gives

t = 3.46s

since we only want the positive solution. So the dot returns to x = 0 (the left side of the screen) at t = 3.46s.

If we plot the original function x(t) we get the curve given in Fig. which shows that the spot does not get to x = 15.0 cm before it turns around. (However as explained in the problem, the curve does not extend to negative values as the graph indicates.)

9. A particle is moving in a straight line. The velocity v of the particle varies with time t, as v = t^2 – 4t, then the distance travelled by the particle in t = 0 to t = 6s (where t is in seconds and v is in m/s).

(a) $\frac{64}{3}m$ **(b) zero**

(c) $\frac{32}{3}m$ **(d) None**

Solution:

Correct option is A.

$v = t^2 - 4t$=0

t(t-4) = 0

t = 4 sec

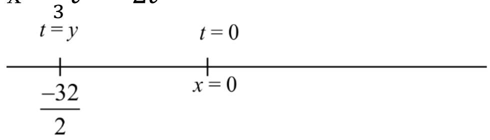
Particle will stop at t = 4sec

$\frac{dx}{dt} = t^2 - 4t$

$\int dx = \int (t^2 - 4t)dt$

x = $\frac{t^3}{3} - \frac{4t^2}{2}$

x = $\frac{1}{3}t^3 - 2t^2$



t = 4

x = $\frac{64}{3} - 32 = \left(\frac{-32}{3}\right)$

t = 6

x = $\frac{1}{3} x\ 6\ x\ 6\ x\ 6 - 2(36)$

= 72-72

= 0

Distance = $\frac{32}{3} + \frac{32}{3} = \frac{64}{3}$ m

10. A particle moves along a straight line such that its displacement x changes with time t as $x = \sqrt{at^2 + 2bt + c}$ where a, b and c are constants, then the acceleration varies as

(a) $\frac{1}{x}$ **(b)** $\frac{1}{x^2}$

(c) $\frac{1}{x^3}$ **(d)** $\frac{1}{x^4}$

Solution:

Correct option is C.

$x = \sqrt{at^2 + 2bt + c}$

$x^2 = at^2 + 2bt + c$

Differentiate w.r.t. time t

$2x\frac{dx}{dt} = 2at + 2b$

Or xv = at + b

Again differentiate

x$\frac{dv}{dt}$ + v$\frac{dx}{dt}$ = a

x(acceleration) + v^2 = a

acceleration = $\frac{a - v^2}{x}$

v ∝ $\frac{1}{x}$

so, $v^2 \propto \frac{1}{x^2}$

Acceleration ∝ $\frac{v^2}{x} \propto \frac{\frac{1}{x^2}}{x}$

Acceleration ∝ $\frac{1}{x^3}$

Pro	Acceleration & Calculus

1. Given a = 5 m/s², Initial position = 5m, Initial velocity = 2 m/s. Find the following functions: x=f(t), v=f(t), v=f(x)

Solution:

a = 5 m/s^2

x_0 = 5m

v_0 = 2m/s

v$\frac{dv}{dx}$ = a = 5

$\int_{v_0}^{v} v dv = \int_{x_0}^{x} 5dx$

$\frac{1}{2}[v^2 - 2^2] = 5[x - 5]$

v^2 – 4 = 10 (x-5)

$v = \sqrt{(10x - 46)}$

$5 = \frac{dv}{dt}$

$\int_{v_0}^{v} dv = \int_0^t 5dt$

v – 2 = 5t

v = 2 + 5t

$v = \frac{dv}{dx} = 2 + 5t$

$\int_{x_0}^{x} dx = \int_{x_0}^{t} (2 + 5t)dt$

$x - 5 = 2t + \frac{5}{2}t^2$

$x = \frac{5}{2}t^2 + 2t + 5$

$x = \frac{5}{2}t^2 + 2t + 5$

2. Given v = x^2, Initial position = 5 m
Find the following:
a=f(x), x=f(t), v=f(t), a=f(t)

Solution:

$x_0 = 5$

$v = x^2$

$a = v\frac{dv}{dx} = (x^2)(2x)$

$= 2x^3$

$a = 2x^3$

$\frac{dx}{dt} = x^2$

$\int \frac{dx}{x^2} = \int dt$

$-\left[\frac{1}{x}\right]_5^x = t$

$\frac{1}{x} - \frac{1}{5} = -t$

$\frac{1}{x} = \frac{1}{5} - t$

$\frac{1}{x} = \frac{1-5t}{5}$

$x = \frac{5}{1-5t}$

$v = \frac{dx}{dt} = \frac{25}{(1-5t)^2}$

$a = \frac{dv}{dt} = \frac{250}{(1-5t)^3}$

3. A radius vector of a particle varies with time t as r = at (1 - α t), where a is a constant vector and α is a positive factor. Find:
(a) The velocity v and the acceleration w of the particle as functions of time;
(b) The time interval Δt taken by the particle to return to the initial points, and the distance s covered during that time.

Solution:

(a) $\vec{r} = \vec{a}t(1-\alpha t)$

$\vec{v} = \frac{d\vec{r}}{dt} = \vec{a}(1-2\alpha t)$

$\vec{\omega} = \frac{d\vec{r}}{dt} = -2\alpha\vec{a}$

(b) It's motion in one dimension as position, velocity and acceleration are all parallel to each other.

So, we can write the equations as:

$r = at(1-\alpha t)$

$v = a(1-2\alpha t)$

Particle is at origin then r = 0

i.e., $at(1-\alpha t) = 0$

$t = 0, \frac{1}{\alpha} \Rightarrow \Delta t = \frac{1}{\alpha}$

At forward most point, v=0

$\Rightarrow a(1-2\alpha t) = 0, \qquad t = \frac{1}{2\alpha}$

Distance covered $= 2\times$ forward distance

$= 2\times r_t = \frac{1}{2\alpha}$

$= 2\times\left[a\frac{1}{2\alpha}\left(1-\alpha\frac{1}{2\alpha}\right)\right] = \frac{a}{2\alpha}$

4. At the moment t = 0, a particle leaves the origin and moves in the positive direction of the x-axis. Its velocity varies with time as $v = v_0\left(1-\frac{t}{\tau}\right)$, where v_0 is the initial velocity vector whose modulus equals v_0 = 10.0 cms^{-1}; τ = 5.0 s. Find:
(a) The x coordinate of the particle at the moments of time 6, 10, and 20 s;
(b) The moments of time when the particle is at the distance 10.0 cm from the origin;
(c) The distance s covered by the particle during the first 4.0 and 8.0 s; draw the approximate plot s(t).

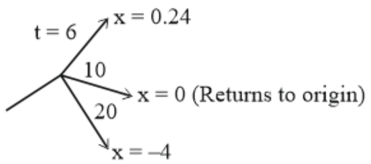
Solution:

$$v = \frac{dx}{dt} = v_0\left(1 - \frac{t}{2}\right)$$

$$\Rightarrow \int_0^x dx = \int_0^t v_0\left(1 - \frac{t}{\tau}\right)dt$$

$$\Rightarrow x = v_0\left(t - \frac{t^2}{2\tau}\right)$$

(a) $x = 10\left(t - \frac{t^2}{10}\right)$



(b) $\pm 10 = 10\left(t - \frac{t^2}{10}\right)$

$t^2 - 10t - 10 = 0$

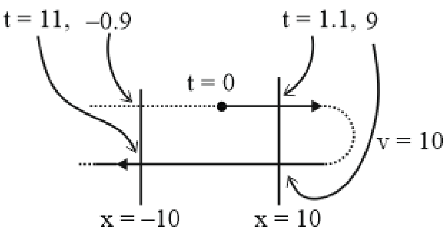
$\downarrow$

$t = 11, -0.9$

$t^2 - 10t + 10 = 0$

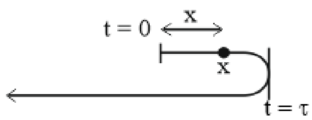
$\downarrow$

$t = 1.1, \quad 9$



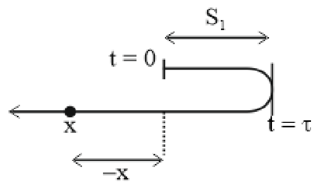
(c) $v = v_0\left(1 - \frac{t}{2}\right)$

$$x = v_0\left(t - \frac{t^2}{2\tau}\right)$$



$t < \tau$

$$\text{distance}: S = v_0\left(t - \frac{t^2}{2\tau}\right)$$



$t \geq \tau$

$$\text{distance}: S = 2s_1 + (-x)$$

$$2\left(\frac{v_0\tau}{2}\right) + -\left[v_0\left(t - \frac{t^2}{2\tau}\right)\right]$$

$$= v_0\tau - v_0 t\left(1 - \frac{t}{2\tau}\right)$$

$$S_1 = v_0\left(\tau - \frac{\tau^2}{2\tau}\right)$$

$$= \frac{v_0\tau}{2}$$

5. The velocity of a particle moving in the positive direction of the x axis varies as $v = \alpha\sqrt{x}$, where α is a positive constant. Assuming that at the moment t = 0 the particle was located as the point x = 0, Find :
(a) The time dependence of the velocity and the acceleration of the particle;
(b) The mean velocity of the particle averaged over the time that the particle takes to cover the first s meters of the path.

<u>Solution:</u>

$$v = \frac{dx}{dt} = \alpha\sqrt{x}$$

$$\Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha t$$

$$\Rightarrow 2\sqrt{x} = \alpha t$$

(a) $x = \frac{\alpha^2 t^2}{4} \xrightarrow[w.r.t.\ time]{differentiating}$

$$v = \frac{dx}{dt}$$

$$= \frac{\alpha^2 t}{2} \xrightarrow[w.r.t.\ time]{differentiating}$$

$$a = \frac{dv}{dt} = \frac{\alpha^2}{2}$$

(b) $v_{av} = \xrightarrow[time]{total\ distance}$

$S = \frac{\alpha^2 t^2}{4}$ (given)

$v_{av} = \frac{S}{t} = \frac{S}{\frac{2\sqrt{S}}{\alpha}} = \frac{\alpha\sqrt{S}}{2}$

6. A point moves rectilinearly with deceleration whose modulus depends on the velocity v of the particle as $w = \alpha\sqrt{v}$, where a is a positive constant. At the initial moment the velocity of the point is equal to v_0. What distance will it traverse before it stops? What time will it take to cover the distance?

Solution:

(a) $a_c = \frac{v\,dv}{dx} = -a\sqrt{v}$

- Indicates deceleration

$\Rightarrow \int_{v_0}^{v} \sqrt{v}\,dv = -a\int_0^x dx$

$\Rightarrow \frac{2}{3}\left(v^{\frac{3}{2}} - v_0^{\frac{3}{2}}\right) = -ax$

$\Rightarrow At \quad v = 0, \quad x = \frac{2}{3a}v_0^{\frac{3}{2}}$

(b) $\frac{dv}{dt} = -a\sqrt{v}$

$\Rightarrow \int_{v_0}^{v} \frac{dv}{\sqrt{v}} = -\int_0^t a\,dt$

$\Rightarrow 2\left(\sqrt{v} - \sqrt{v_0}\right) = -at$

$At\ v = 0,\ t = \frac{2\sqrt{v_0}}{a}$

7. A radius vector of a point A relative to the origin varies with time t as r = at i -bt^{2}j, where a and b are positive constants, and i and j are the unit vectors of the x and y axes. Find :
(a) the equation of the point's trajectory y(x); plot this function;
(b) the time dependence of the velocity v and acceleration w vectors, as well as of the moduli of these quantities;
(c) The time dependence of the angle α between the vectors w and v;
(d) The mean velocity vector averaged over the first t seconds of motion, and the modulus of this vector

Solution:

$r = at\hat{i} - b^2 j \xrightarrow[\text{w.r.t time}]{\text{diff.}} v = a\hat{i} - 2bt\vec{j} \xrightarrow{diff.}$ $w = -2b\hat{j}$

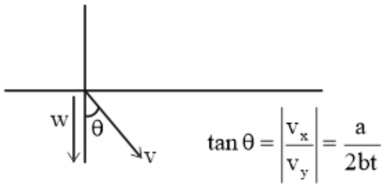
$\downarrow \qquad \downarrow$

$x = at \quad y = -bt^2$

Eliminating t

$y = -b\left(\frac{x^2}{a^2}\right)$

equation of trajectory



$\vec{v}_{av} = \frac{\vec{r}_f - \vec{r}_i}{t}$

$= \frac{at\hat{i} - bt^2\hat{j}}{t}$

$= a\hat{i} - bt\hat{j}$

8. A point moves in the plane xy according to the law $x = a\sin\omega t$, y = a (1 - cos ω t), where a and ω are positive constants, Find :
(a) The distance s traversed by the point during the time x;
(b) The angle between the point's velocity and acceleration vectors

Solution:

(a) $x = a\sin\omega t$

$y = a(1 - \cos\omega t) \xrightarrow{differentiate}$

$v_x = a\omega\cos\omega t$
$v_y = a\omega\sin\omega t$ $\xrightarrow{differentiate}$ $a_x = -a\omega^2\sin\omega t$, $a_y = a\omega^2\cos\omega t$

$\downarrow a_c a\omega^2$

$\Downarrow$

$v_z^2 + v_y^2 = a^2\omega^2$

$\Downarrow$

$|v|\,a\omega$ (a constant)

Distance $S = vt = a\omega t$

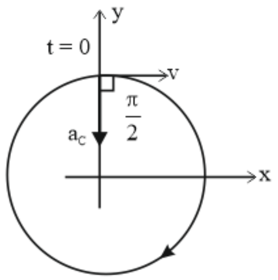
(b) Angle between velocity and acceleration:

$\cos\theta = \frac{\vec{v}.\vec{a}}{|v||a|}$

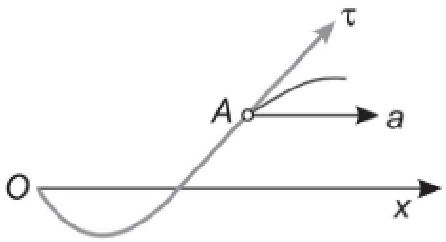
$= \frac{-a^2\omega^3\cos\omega t\sin\omega t + a^2\omega^3\sin\omega t\cos\omega t}{|v||a|}$

$= 0 \qquad \Rightarrow \theta = \frac{\pi}{2}$

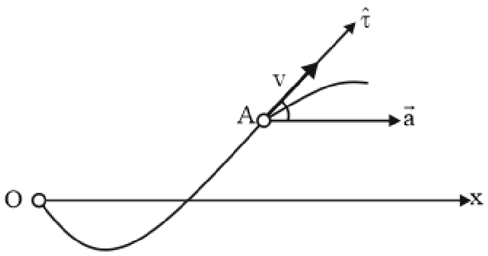
Uniform circular motion



9. A particle A moves in one direction along a given trajectory with a tangential acceleration $w_\tau = a\tau$, where a is a constant vector coinciding in direction with the x axis (Fig.), and τ is a unit vector coinciding in direction with the velocity vector at a given point. Find how the velocity of the particle depends on x provided that its velocity is negligible at the point x = 0.



Solution:



We know that,

Tangential acceleration:

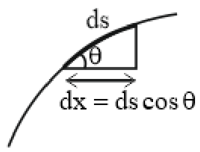
$$wt = v(speed)\frac{dv}{ds(\text{distance})}$$

Given:

$$w_t = \vec{a}.\vec{\tau} = a\tau \cos\theta$$

$$\Rightarrow \frac{vdv}{ds} = a\cos\theta$$

$$\Rightarrow vdv = a\ ds\cos\theta$$



$$\Rightarrow vdv = a\ dx$$

$$\Rightarrow \int_0^v vdv = a\int_0^x dx$$

$$\Rightarrow \frac{v^2}{2} = ax \qquad \Rightarrow v = \sqrt{2ax}$$

10. A particle moves along the plane trajectory y[x) with velocity v whose modulus is constant. Find the acceleration of the particle at the point x = 0 and the curvature radius of the trajectory at that point if the trajectory has the form.

(a) Of a parabola y = ax^2;

(b) Of an ellipse $(x/a)^2 + (y/b)^2 = 1$; a and b are constants here.

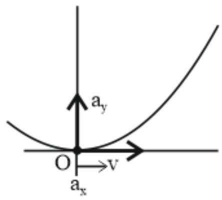
Solution:

∵ speed is constant:

$a_t = 0 \quad y = ax^2$

(a) At origin,

$at = a_x = 0$



$$\Rightarrow v_y = \frac{dy}{dt} = 2ax\frac{dx}{dt}$$

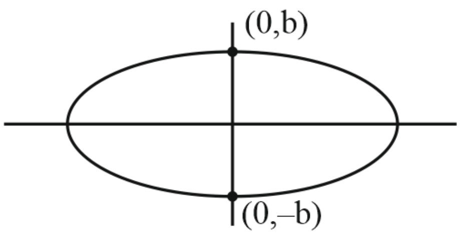
$$\Rightarrow a_y = \frac{d^2y}{dt^2}$$

$$= 2a\left(\frac{dx}{dt}\right)^2 + 2ax\frac{d^2x}{dt^2}$$

$$\therefore a_{net} = \sqrt{a_x^2 + a_y^2} = 2av^2$$

$$R_c = \frac{v^2}{a_1} = \frac{v^2}{a_y} = \frac{v^2}{2av^2} = \frac{1}{2a}$$

(b)



$a_t = a_x - 0$

$diff.\ \frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$diff.\ \frac{2x}{a^2}\frac{dx}{dt}+\frac{2y}{b^2}\frac{dy}{dt}=0$

$$\frac{1}{a^2}\left(\left(\frac{dx}{dt}\right)^2+x\frac{d^2x}{dt^2}\right)+\frac{1}{b^2}\left(\left(\frac{dy}{dt}\right)^2+y\frac{d^2y}{dt^2}\right)=0$$

$\therefore a_{net}=\sqrt{a_x^2+a_y^2}=\pm b\frac{v^2}{a^2}$

$R_c=\frac{v^2}{a_\perp}=\frac{v^2}{a_y}$

$=\frac{v^2a^2}{\pm bv^2}=\pm\frac{a^2}{b}$

11. A particle travels according to the equation a = A – Bv where a is the acceleration. A and B are constants, v is the velocity of the particle. Find its velocity as a function of time. Also find its terminal velocity.

Solution:

a = A – Bv

$\frac{dv}{dt}$ = A – Bv

$\int_0^v \frac{-dv}{(Bv-A)} = \int_0^t dt$

$[\ln(Bv-A)] = -Bt$

$\ln(Bv-A) - \ln(-A) = -Bt$

$\ln(\frac{A-Bv}{A})$ = -Bt

$(\frac{A-Bv}{A}) = e^{-Bt}$

$A - Bv = A(e^{-Bt})$

$V = \frac{A}{B}(1 - e^{-Bt})$

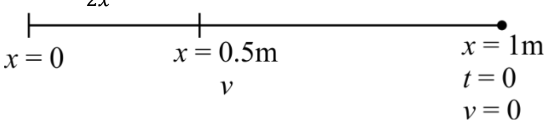
At t=infinity, terminal velocity = A/B

12. A particle of mass 10^{-2} kg is moving along the positive x-axis under the influence of a force $F(x) = -\frac{k}{2x^2}$, where k = 10^{-2} Nm^2. At time t = 0, it is at x = 1.0 m and its velocity is v = 0. Find its velocity when it reaches x = 0.5 m.

Solution:

m = $10^{-2}kg$

F(x) = $\frac{-k}{2x^2}$, where k = 10^{-2}Nm^2



$F(x) = \frac{-k}{2x^2}$

$\therefore a(x) = \frac{-k}{2mx^2}$

$v\frac{dv}{dx} = -\frac{k}{2mx^2}$

$\int_0^V Vdv = -\frac{k}{2m}\int_1^x \frac{1}{x^2}dx$

$\frac{V^2}{2} = \left(\frac{k}{2m}\right)\left[\frac{1}{x}\right]_1^x$

V(x) = $\pm\frac{1}{\sqrt{\left(\frac{k}{m}\right)\left(\frac{1}{x}-1\right)}}$

$= \pm\sqrt{\left(\frac{1}{x}-1\right)}$

(a) V (x = 0.5m) = $\pm\sqrt{\left(\frac{1}{0.5}-1\right)}$ = $\pm$1m/s

Therefore, velocity of particle at x = 0.5 m is V = -1 m/s

13. A particle starts from rest, with an acceleration $a=\frac{\lambda}{x^2}$, where λ > 0 and x is the distance of the particle from a fixed-point O. The particle is at a distance μ from O, when it is at rest. Its velocity when at a distance 2μ from O is

(a) $\sqrt{\frac{\lambda}{\mu}}$ **(b)** $\sqrt{\frac{\lambda}{2\mu}}$

(c) $\sqrt{\frac{2\lambda}{\mu}}$ **(d) None**

Solution:

$a=\frac{\lambda}{x^2}$

$v\frac{dv}{dx}=\frac{\lambda}{x^2}$

$vdv=\frac{\lambda}{x^2}dx$

$\int_0^v vdv=\int_\mu^{2\mu}\frac{\lambda}{x^2}dx$

The implies, v=$\sqrt{\frac{\lambda}{\mu}}$

Hence, Option (A) is correct.

14. A particle starts from a point x = 0 along the positive X – axis with a velocity v, varying with x as $v=\mu\sqrt{x}$, the average velocity of the particle over the first s meters of its path is

(a) $\mu\sqrt{s}$ (b) $\mu\sqrt{\frac{s}{2}}$

(c) $2\mu\sqrt{s}$ (d) $\frac{\mu}{2}\sqrt{s}$

Solution:

u = 0

v = $\mu\sqrt{x}$

distance covered = s

s = ut + $\frac{1}{2}at^2$

s = $\frac{1}{2}at^2$

a = $\frac{v-u}{t}$ => a = $\frac{\mu\sqrt{s}}{t}$

s = $\frac{1}{2}$ x $\frac{\mu\sqrt{s}}{t}t^2$

$\frac{s}{t}$ = average velocity = $\frac{\mu}{2}\sqrt{s}$

∴ Correct option is (d)

15. A particle moving in a straight line has velocity (v) and displacement(s) related as $v = 4\sqrt{1+s}$, where velocity (v) is in m/s and displacement (s) is in meters. Then (at t = 0, s = 0)

(a) Acceleration of the particle is 8 m/s^2

(b) Velocity of the particle at t = 2s is 20 m/s

(c) Displacement of the particle at t = 2s is 24

(d) Displacement of the particle at t = 1s is 8 m

Solution:

v = $4\sqrt{1+s}$

at t = 0, s = 0

v = 4m/s

a = v. $\frac{dv}{ds}$ = 4 x 4 x $\frac{d(1+s)^{1/2}}{ds}$

= 4 x 4 $\left[\frac{1}{2}x\frac{1}{\sqrt{1+s}}\right]x\sqrt{1+s}$

= $\frac{16}{2}$ = 8m/s^2

∴ Option (a) is u

For t = 2s,

u = 4m/s

a = 8m/s^2

v = u + at

= 4 + 8 x 2 = 16 + 4

v = 20 m/s

∴ Option (b) is correct

If v = 20m/s,

20 = $4\sqrt{1+s}$

25 = 1+s

∴ s = 24 m

∴ Option (c) is correct.

At t = 1s,

u = 4m/s

a = 8m/s^2

v = u + at

= 4 + 8 = 12m/s

$v^2 - u^2 = 2as$

s = $\frac{(4+12)(12-4)}{2\,x\,8} = \frac{16\,x\,8}{16}$

s = 8m

∴ Option (d) is correct.

Hence, all (a), (b), (c) & (d) are correct.

16. Velocity v of a moving particle, on x – axis varies with its x – coordinate as $V = \beta x^{\frac{1}{3}}$ when β is a positive constant. Assuming the particle to start from origin.

(a) Acceleration of particle is variable.

(b) Mean velocity of point averaged over time T from starting is $\left(\frac{2\beta}{3}\right)^{\frac{3}{2}}\sqrt{T}$.

(c) Mean velocity of point averaged over the time taken by the point to move from the origin to the point where its x – coordinate becomes x_0 is $\frac{2\beta x_0^{\frac{1}{3}}}{3}$.

(d) All above statements are false.

Solution:

$v = \beta x^{1/3}$

=> $\frac{dv}{dx} = \frac{1}{3}\beta x^{-2/3}$

$v.\frac{dv}{dx} = a = \frac{1}{3}\beta^2 x^{1/3}$

∴ Acceleration is variable

=> Option (a) is correct.

$v = \beta x^{1/3}$

$\frac{dx}{dt} = \beta x^{1/3}$

$\int x^{-1/3}dx = \beta\int dt$

$\frac{3x^{2/3}}{2} = \beta t$ ----------- (1)

Total displacement => x = $\left(\frac{2}{3}\beta\right)^{3/2}T^{3/2}$

Avg velocity in time T = $\frac{Total\ displacement}{T}$

= $\left(\frac{2}{3}\beta\right)^{3/2}\frac{T^{3/2}}{T}$

= $\left(\frac{2}{3}\beta\right)^{3/2}\sqrt{T}$

∴ Option (b) is correct.

From eq. (1),

$\beta t = \frac{3x^{2/3}}{2}$,where t is time taken to cover distance x

∴ time t to cover distance x_0 is given by

t = $\frac{x_0^{2/3}}{(2/3\beta)}$

Total displacement = x_0

∴ $v_{avg} = \frac{x_0}{x_0^{2/3}} x \frac{2}{3}\beta = \frac{2}{3}\beta x_0^{v_3}$

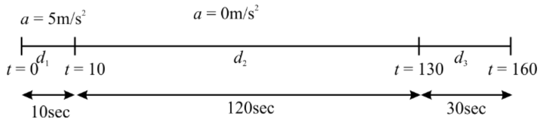
∴ Option (c) is correct.

∴ (a), (b) and (c) is correct.

Beginner **Equation of Motion**

1. A particle starts from rest, moves with constant acceleration of 5 m/s^2 for 10 seconds. After that it moves with constant speed for next 2 minutes. Then it retards and comes to rest in 30 seconds. Find the followings:
(a) Total time
(b) Maximum Velocity
(c) Total distance
(d) Average velocity in 1 minute
(e) Ratio of distance travelled in 3rd & 3 seconds

Solution:



$V_B = u + at$
= 0 + 5 x 10 = 50 m/s

a) Total time = 10 + 120 + 30 = 160 sec

b) $V_{max} = u + at$
= 5 x 10
= 50 m/s

c) $d_{Total} = d_1 + d_2 + d_3$

$d_1 = ut + \frac{1}{2}at^2$
= 0 + $\frac{1}{2} x\ 5\ x\ 10^2$
= 250 m

d_2 = vt
= 50 x 120
= 6000 m

To calculate d_3,
v = u +at
0 = 50 + a (30)

∴ $a = \frac{-5}{3} m/s^2$

$d_3 = ut + \frac{1}{2}at^2$
= 50 x 30 + $\frac{1}{2}\left(\frac{-5}{3}\right) 30^2$
= 750 m

Therefore, $d_{Total} = 250 + 6000 + 750$
= 7000 m
= 7km

(d) Avg velocity = $\frac{250+(50\ x\ 50)}{60}$
= $\frac{2750}{60}$
= $\frac{275}{6} m/s$

(e) $d_3 = \frac{1}{2}at^2$
= $\frac{1}{2} x\ 5\ x\ 3^2$
= $\frac{45}{2}\ m$

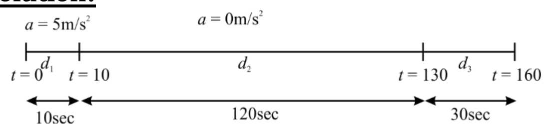
$d_{3rd} = d_3 - d_2$
= $\frac{1}{2}a(3^2 - 2^2)$
= $\frac{1}{2}\ x\ 5\ x\ (9-4)$
= $\frac{5}{2} x5$
= $\frac{25}{2} m$

$\frac{d_{3rd}}{d_3} = \frac{\frac{25}{2}}{\frac{45}{2}} = 5:9$

2. A particle starting from rest travels a distance x in first 2 seconds and a distance y in next two seconds, then

(a) y = 8x **(b) y = 2x**
(c) y = 3x **(d) y = 4x**

Solution:



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= 250 m

d_2 = vt
= 50 x 120
= 6000 m

To calculate d_3,
v = u +at
0 = 50 + a (30)
$\therefore a = \frac{-5}{3} m/s^2$
$d_3 = ut + \frac{1}{2}at^2$
$= 50 \times 30 + \frac{1}{2}\left(\frac{-5}{3}\right) 30^2$
= 750 m
Therefore, $d_{Total} = 250 + 6000 + 750$
= 7000 m
= 7km
(d) Avg velocity = $\frac{250+(50 \times 50)}{60}$
$= \frac{2750}{60}$
$= \frac{275}{6} m/s$
(e) $d_3 = \frac{1}{2}at^2$
$= \frac{1}{2} x\ 5\ x\ 3^2$
$= \frac{45}{2}\ m$
$d_{3rd} = d_3 - d_2$
$= \frac{1}{2}a(3^2 - 2^2)$
$= \frac{1}{2}\ x\ 5\ x\ (9-4)$
$= \frac{5}{2} x5$
$= \frac{25}{2} m$
$\frac{d_{3rd}}{d_3} = \frac{\frac{25}{2}}{\frac{45}{2}} = 5:9$

3. A particle experiences a constant acceleration for 20 sec after starting from rest. If it travels a distance S_1 in the first 10 sec and a distance S_2 in the next 10 sec, then
(a) $S_1 = S_2$ **(b)** $S_1 = S_2/3$
(c) $S_1 = S_2/2$ **(d)** $S_1 = S_2/4$

Solution:
Correct option is B.
Let a be the constant acceleration of the particle.
Then s = ut + $\frac{1}{2}at^2$
or $s_1 = 0 + \frac{1}{2} x\ a\ x\ (10)^2 = 50a$
and $s_2 = \left[0 + \frac{1}{2}\ a\ (20)^2\right] - 50a = 150a$
$\therefore s_2 = 3s_1$
Alternatively: Let a be constant acceleration and
s = ut + $\frac{1}{2}at^2$
or $s_1 = 0 + \frac{1}{2} x\ a\ x\ 100 = 50a$
Velocity after 10s, is v = 0 + 10a
So $s_2 = 10a\ x\ 10 + \frac{1}{2}\ a\ x\ 100 = 150a$
=> $s_2 = 3s_1$

4. How long does it take an object to travel a distance of 30m from rest at a constant acceleration of 2m/s^2 ?Possible Answers:
(a) 2.7s **(b) 12.6s**
(c) 10.9s **(d) 7.3s**
(e) 5.5s

Solution:
Option (e)
Using the equation d = v_ot + (1/2) at^2, we can solve for time.
Since the object started at rest, v_0 = 0. Now we are left with the equation d = (1/2) at^2.
$30m = \frac{1}{2}\left(2\frac{m}{s^2}\right)t^2$
30s^2 = t^2
t = 5.5s
Plugging in the remaining values we can find that t = 5.5s.

5. A race car accelerates uniformly from 18.5 m/s to 46.1 m/s in 2.47 seconds. Determine the acceleration of the car and the distance traveled.

Solution:
a = (Delta v)/t
a = (46.1 m/s – 18.5 m/s)/(2.47s)
a = 11.2 m/s^2
d = v_i × t + 0.5 × a × t^2
d = (18.5 m/s)(2.47s) + 0.5 (11.2 m/s^2)(2.47s)^2
d = 45.7 m + 34.1 m
d = 79.8 m

6. An engineer is designing the runway for an airport. Of the planes that will use the airport, the lowest acceleration rate is likely to be 3m/s^2. The takeoff speed for this plane will be 65m/s. Assuming this minimum acceleration, what is the minimum allowed length for the runway?

Solution:
$v_f^2 = v_i^2$ + 2ad
(65 m/s)^2 = (0 m/s)^2 + 2(3 m/s^2)d
4225 m^2/s^2 = (0 m/s)^2 + (6 m/s^2)d
(4225 m^2/s^2)/(6 m/s^2) = d
d = 704 m

7. A bullet leaves a rifle with a muzzle velocity of 521 m/s. While accelerating through the board of the rifle, the bullet moves a distance of 0.840 m. Determine the acceleration of the bullet (assume uniform acceleration).

Solution:

$v_f^2 = v_i^2 + 2ad$

$(521\text{ m/s})^2 = (0\text{ m/s})^2 + 2(a)(0.840\text{ m})$

$271441\text{ m}^2/\text{s}^2 = (0\text{ m/s})^2 + (1.68\text{ m})a$

$(271441\text{ m}^2/\text{s}^2)/(1.68\text{ m}) = a$

$a = 1.62 \times 10^5\text{ m/s}^2$

8. The head of a rattlesnake can accelerate $50\frac{m}{s^2}$ in striking a victim. If a car could do as well, how long would it take to reach a speed of $100\frac{km}{hr}$ from rest?

Solution:

First, convert the car's final speed to SI units to make it easier to work with:

$$100\frac{km}{hr} = \left(100\frac{km}{hr}\right)\cdot\left(\frac{1000m}{1km}\right)\cdot\left(\frac{1hr}{3600s}\right) = 27.8\frac{m}{s}$$

The acceleration of the car is $50\frac{m}{s^2}$ and it starts from rest which means that $v_0 = 0$. As we've found, the final velocity v of the car is $27.8\frac{m}{s}$. (The problem actually that this is final speed but if our coordinate system points in the same direction as the car's motion, these are the same thing.) Equation 2.6 lets us solve for the time t:

$$v = v_0 + at \quad \Rightarrow \quad t = \frac{v - v_0}{a}$$

Substituting, we find

$$t = \frac{27.8\frac{m}{s} - 0}{50\frac{m}{s^2}} = 0.55s$$

If a car had such a large acceleration, it would take 0.55s to attain the given speed.

9. A body moving with uniform acceleration has a velocity of $12.0\frac{cm}{s}$ when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is –5.00 cm, what is the magnitude of its acceleration?

Solution:

In this problem we are given the initial coordinate (x = 3.00 cm), the initial velocity $\left(v_0 = 12.0\frac{cm}{s}\right)$, the final x coordinate (x = –5.00 cm) and the elapsed time (2.00 s). Using Eq. 2.7 (since we are told that the acceleration is constant) we can solve for a. We find:

$$x = x_0 + v_0t + \frac{1}{2}at^2 \quad \Rightarrow \quad \frac{1}{2}at^2 = x - x_0 - v_0t$$

Substitute things:

$$\frac{1}{2}at^2 = -5.00cm - 3.00cm - \left(12.0\frac{cm}{s}\right)(2.00s)$$

$$= -32.0cm$$

Solve for a:

$$a = \frac{2(-32.0cm)}{t^2} = \frac{2(-32.0cm)}{(2.00s)^2} = -16.0\frac{cm}{s^2}$$

The x acceleration of the object is $-16\frac{cm}{s^2}$. (The magnitude of the acceleration is $16.0\frac{cm}{s^2}$.)

10. A jet plane lands with a velocity of $100\frac{m}{s}$ and can accelerate at a maximum rate of $-5.0\frac{m}{s^2}$ as it comes to rest. (a) From the instant it touches the runway, what is the minimum time needed before it stops? (b) Can this plane land at a small airport where the runway is 0.80 km long?

Solution:

(a) The data given in the problem is illustrated in Fig. The minus sign in the acceleration indicates that the sense of the acceleration is opposite that of the motion, that is, the plane is decelerating.

The plane will stop as quickly as possible if the acceleration does have the value $-5.0\frac{m}{s^2}$, so we use this value in finding the time t in which the velocity changes from $v_0 = 100\frac{m}{s}$ to v = 0. Eq. 2.6 tells us:

$$t = \frac{v - v_0}{a}$$

Substituting, we find:

$$t = \frac{\left(0 - 100\frac{m}{s}\right)}{\left(-5.0\frac{m}{s^2}\right)} = 20s$$

a = –5.0 m/s^2

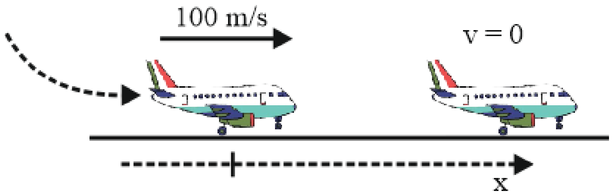


Fig. Plane touches down on runway at $100\frac{m}{s}$ and comes to a halt.

The plane needs 20s to come to a halt.

(b) The plane also travels the shortest distance in stopping if its acceleration is $-5.0\frac{m}{s^2}$. With $x_0 = 0$, we can find the plane's final x coordinate using Eq., using t = 20 s which we got from part (a):

$$x = x_0 + \frac{1}{2}(v_0 + v)t = 0 + \frac{1}{2}\left(100\frac{m}{s} + 0\right)(20s)$$

= 1000 m = 1.0 km

The plane must have at least 1.0 km of runway in order to come to a halt safely. 0.80 km is not sufficient.

11. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest if it faces constant resistance to motion?

(a) 1.5 cm **(b) 1.0 cm**
(c) 3.0 cm **(d) 2.0 cm**

Solution:

Case I:

$$\left[\frac{u}{2}\right]^2 - u^2 = 2.a.3$$

or $-\frac{3u^2}{4} = 2.a.3 \Rightarrow a = -\frac{u^2}{8}$

Case II:

$0 - \left[\frac{u}{2}\right]^2 = 2.a.x$ or $\frac{u^2}{4} = 2\left[-\frac{u^2}{8}\right]x\ x$

=> x = 1 cm

Alternative method: Let K be the initial energy and E be the resistive force. Then according to work – energy theorem,

W = ΔKE

i.e., $3F = \frac{1}{2}mv^2 - \frac{1}{2}m\left[\frac{v}{2}\right]^2$

$3F = \frac{1}{2}mv^2\left[1 - \frac{1}{4}\right]$

$3F = \frac{3}{4}\left[\frac{1}{2}mv^2\right]$ ………….(1)

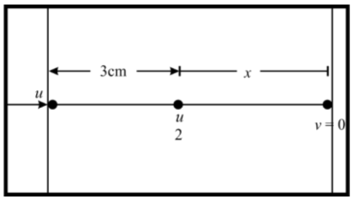
And $Fx = \frac{1}{2}m\left[\frac{v}{2}\right]^2 - \frac{1}{2}m(0)^2$

i.e., $\frac{1}{4}\left[\frac{1}{2}mv^2\right] = Fx$ …….(2)

Comparing eqns. (1) and (2)

F = Fx

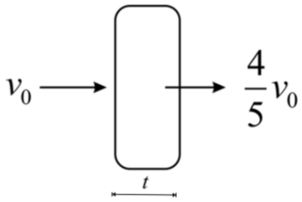
Or x = 1 cm



12. A bullet traveling horizontally loses $\frac{1}{5}$th of its velocity crossing wooden plank. How many such planks are required to stop the bullet?

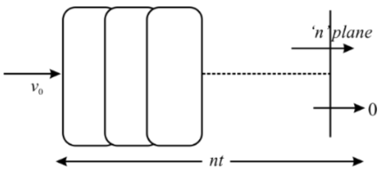
(a) ≈ 3 **(b) ≈ 4**
(c) ≈ 5 **(d) ≈ 6**

Solution:



$\left(\frac{4}{5}V_0\right)^2 - V_0^2 = 2a(t)$ -----------(I)

Where, t is the thickness of single plank



$0 - V_0^2 = 2a(nt)$ ------------(II)

Divide Eqn (I) amd (II),

$$\frac{\left(\frac{16}{25} - 1\right)V_0^2}{-V_0^2} = \frac{2at}{2ant}$$

=> n≅ 3

13. The initial velocity of a body moving along a straight line is 7 m/s. It has a uniform acceleration of 4 m/s^2. The distance covered by the body 5th second of its motion is

(a) 25 m **(b) 35 m**
(c) 50 m **(d) 85 m**

Solution:

Given,

u = 7m/s

a = 4m/s^2

n = 5

The distance covered by the body in nth is given by

$S_n = u + \frac{a}{2}(2n-1)$

$S_5 = 7 + \frac{4}{2}(2x5-1)$

S_5 = 25m

The correct option is A.

14. A particle travels 10*m* in first 5 *s* and 10*m* in next 3 *s*. Assuming constant acceleration what is the distance travelled in next 2 *s*

(a) 8.3 *m* **(b) 9.3 *m***

(c) 10.3 *m* **(d) None of above**

Solution:

Correct option is A.

If initial velocity be u and acceleration be a then distance in first 5 second will be according to

S = ut + $\frac{at^2}{2}$ i.e 10 = 5u + $\frac{25a}{2}$ …………(1)

And distance in 8 second will be

S = 8u+ $\frac{64a}{2}$ = 8u + 32a

So distance between t = 5 and t = 8 will be

S – 10 = 8u + 32a – 10

which is given as 10m so 10 = 8u+32a-10

or 20=8u+32a ……(2)

Solving equation 1 and equation 2 we get

a = $\frac{1}{3}$ and u = $\frac{35}{30}$

Now distance in 10 second is

$S_1 = 10u + \frac{100a}{2} = 10u + 50a$

So distance between t = 8s and t = 10s i.e. distance in last two seconds will be

$S_1 - S = 10u + 50a - (8u + 32a)$

= 2u + 18a = $\frac{70}{30} + \frac{18}{3} += \frac{25}{3}$meter which is nearly 8.3 meter

Expert **Equation of Motion**

1. An electron with initial velocity $v_0 = 1.50\times10^5 \frac{m}{s}$ enters a region 1.0 cm long where it is electrically accelerated, as shown in fig. It emerges with velocity $v = 5.70\times10^6 \frac{m}{s}$. What was its acceleration, assumed constant? (Such a process occurs in the electron gun in a cathode-ray tube, used in television receivers and oscilloscopes.)

Solution:

We are told that the acceleration of the electron is constant, so that Eqs. can be used.

Here we know the initial and final velocities of the electron (v_0 and v). If we let its initial coordinate be x_0 = 0 then the final coordinate is x = 1.0 cm = 1.0 × 10^{-2} m. We don't know the time t for its travel through the accelerating region and of course we don't know the (constant) acceleration, which is what we're being asked in this problem.

We see that we can solve for a if we use Eq.:

$$v^2 = v_0^2 + 2a(x - x_0) \implies a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

Substitute and get:

$$a = \frac{\left(5.70\times10^6 \frac{m}{s}\right)^2 - \left(1.50\times10^5 \frac{m}{s}\right)^2}{2\left(1.0\times10^{-2} m\right)}$$

$$= 1.62\times10^{15} \frac{m}{s^2}$$

The acceleration of the electron is $1.62\times10^{15} \frac{m}{s^2}$ (while it is in the accelerating region).

2. A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in

(a) $\frac{20}{3}$ *m* **(b) 20 m**

(c) 60 m **(d) 180 m**

Solution:

Correct option is D.

Final velocity is zero and acceleration is -ve.

$0 = u^2 - 2\ x\ a\ x\ s$

s = $\frac{u^2}{2a}$

s α u^2

If car moving with a velocity of 10m/s can be stopped by the application of a constant force F in a distance of 20m, since the velocity triples stopping distance will become $3^2 = 9$ times.

New stopping distance = 9 * 2 = 180 m

3. In a journey of 4 km a train travels with a constant acceleration for the first three-tenths of a km, at a uniform speed of 30 km/hr for the next 3 km and with a constant retardation for the rest of the distance. Find the time taken for the journey in minutes.

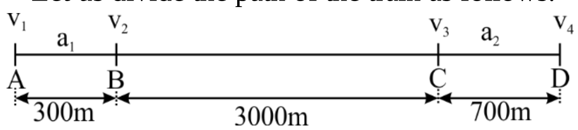
Solution:

Total distance = 4km = 4000m

Initial velocity = 0

The train covers $\frac{3}{10}^{th}$ of a km, at a uniform acceleration = 300 m

∴ Let us divide the path of the train as follows.



$v_1 = v_4 = 0$

$v_2 = v_3$

a_1 is positive acceleration from A to B

a_2 is retardation from C to B.

$v_2^2 - v_1^2 = 2a_1(AB)$

$v_2 = \sqrt{2a_1 \, x \, 300}$

$v_2 = 10\sqrt{6a_1}$

$v_2 = v_3 = 10\sqrt{6a_1}$ ………(I)

$v_4^2 - v_3^2 = -2a_2(CD)$

=> $v_3 = \sqrt{1400a_2}$

$v_3 = 10\sqrt{14a_2}$ ……(II)

(I) = (II) => $10\sqrt{14a_2} = 10\sqrt{6a_1}$

$\frac{a_1}{a_2} = \frac{14}{6} = \frac{7}{3}$ => $a_2 = \frac{3}{7}a_1$

$t_1 = \frac{v_2 - v_1}{a_1} = \frac{v_2}{a_1} = \frac{10\sqrt{6a_1}}{a_1} = \frac{10\sqrt{6}}{\sqrt{a_1}}$ …..(III)

$t_2 = \frac{BC}{v_2} = \frac{3000}{10\sqrt{6a_1}} = \frac{50\sqrt{6}}{\sqrt{a_1}}$ …..(IV)

$t_3 = \frac{v_4 - v_3}{-a_2} = \frac{v_3}{a_2} = \frac{10\sqrt{14a_2}}{a_2} = \frac{10\sqrt{14}}{\sqrt{a_2}}$ …..(V)

Substituting the value of a_2 in (V)

$t_3 = \frac{10\sqrt{14} \, x \, 7}{\sqrt{3a_1}} = \frac{70\sqrt{2}}{\sqrt{3a_1}} = \frac{70\sqrt{6}}{3\sqrt{a_1}}$

Given v_2 = 30 km/hr = $\frac{25}{3}$ m/s

∴ $\frac{25}{3} = 10\sqrt{6a_1}$

$\frac{25}{36} = 6a_1$ => $a_1 = \frac{\frac{25}{216}m}{s^2}$

∴ Total time = $t_1 + t_2 + t_3$

= $\sqrt{\frac{6}{25} \, x \, 216}\left(\frac{30+150+70}{3}\right)$

= $\frac{36}{5} \, x \left(\frac{250}{3}\right)$

= 600 s

= 10 min

4. A bullet of mass 20g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5 × 10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to :

(a) 0.1 ms^{-1} **(b) 0.7 ms^{-1}**
(c) 0.3 ms^{-1} **(d) 0.4 ms^{-1}**

Solution:

(b) From the third equation of motion

v2 – u2 = 2aS

But, $a = \frac{F}{m}$

∴ $v^2 = u^2 - 2\left(\frac{F}{m}\right)S$

$v^2 = (1)^2 - (2)\left[\frac{2.5\times10^{-2}}{20\times10^{-3}}\right]\frac{20}{100}$

$v^2 = 1 - \frac{1}{2} \Rightarrow v = \frac{1}{\sqrt{2}}$ m/s = 0.7 m/s

5. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time lapse is /seconds, Evaluate.
(a) Maximum velocity reached, and
(b) Total distance travelled

Solution:

(1) Let t_1 be the time taken by the car to attain the maximum velocity v_m while it is acceleration.

Using v = u +at$_2$

V_m = 0 + αt_1 or $t_1 = \frac{v_m}{\alpha}$........(1)

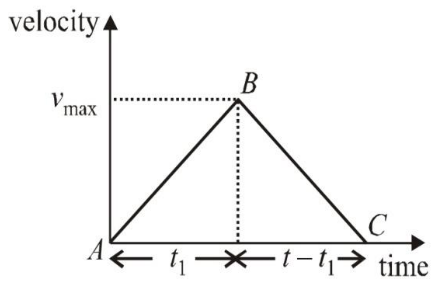
Since the total time elapsed is t, the car decelerators for time $t_2 = (t - t_1)$ to come by rest , a = $-\beta$ and v = 0

0 = $v_m - \beta(t - t_1)$ or $t_1 = t + \frac{v_m}{\beta}$.......(2)

Using (1) and (2) , we get

$\frac{v_m}{\alpha} = t - \frac{v_m}{\beta}$ or $t = v_m(\frac{1}{\alpha} + \frac{1}{\beta})$

Or $v_m = \frac{t\alpha\beta}{(\alpha+\beta)}$ ......(3)



(2) Total distance travelled = area of Δ ABC

$= \frac{1}{2} \times$ base $\times$ altitude $= \frac{1}{2} \times t \times v_{max}$

$= \frac{1}{2} \times t \times \frac{\alpha\beta}{\alpha+\beta} t = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right) t^2$

Pro **Equation of Motion**

1. A car, starting from rest, accelerates at the rate

f through a distance S, then continues at constant speed for time t and then decelerates at the rate $\frac{f}{2}$ to come to rest. If the total distance traversed is 15 S, then

(a) $S=\frac{1}{2}ft^2$ **(b)** $S=\frac{1}{4}ft^2$

(c) $S=\frac{1}{72}ft^2$ **(d)** $S=\frac{1}{6}ft^2$

Solution:

If t_1 be the time for which the car accelerates at the rate f from rest, the distance traveled in time t_1 is

s = s_1=0(t_1)+$\frac{1}{2}ft_1^2 = \frac{1}{2}ft_1^2$ ----(1)

(using formula $s = ut + \frac{1}{2}at^2$)

And the velocity at time t_1 is v_1 = ft_1

So, the distance traveled in time t will be

$s_2 = v_1 t = f t_1 t$ ------(2)

If t_2 be the time for which the car decelerates at the rate f/2 to come rest, the distance traveled in time t_2 is given by,

$0^2 - v_1^2 = 2\left(\frac{-f}{2}\right)s_3$

(using formula $v^2 - u^2 = 2as$)

Or $s_3 = \frac{(ft_1)^2}{f} = ft_1^2$ -------(3)

Using (1), $s_3 = 2s_1 = 2s$

Given, $s_1 + s_2 + s_3$ = 15s

or s + f$t_1 t$+2s = 15s

or ftt_1= 12s

or 12s = ftt_1 --------(4)

Eqn.(4)/(1) => $\frac{12s}{s} = \frac{\text{f}tt_1}{\left(\frac{1}{2}\right)ft_1^2}$ or $t_1 = \frac{t}{6}$

From (1), s = $\left(\frac{1}{2}\right)f\left(\frac{t}{6}\right)^2 = \frac{ft^2}{72}$

2. A body *A* starts from rest with an acceleration a_1. After 2 seconds, another body *B* starts from rest with an acceleration a_2. If they travel equal distances in the 5th second, after the start of A, then the ratio $a_1 : a_2$ is equal to

(a) 5 : 9 **(b) 5 : 7**

(c) 9 : 5 **(d) 9 : 7**

Solution:

Correct option is A.

Initial speed u = 0

Distance covered in nth second after starting from rest $S = \frac{1}{2}(2n-1)$

For A: n=5

So, $S_A = \frac{a_1}{2}(2 \ x \ 5 - 1) = \frac{9}{2}a_1$

For B: n = 3

So, $S_B = \frac{a_2}{2}(2 \ x \ 3 - 1) = \frac{5}{2}a_2$

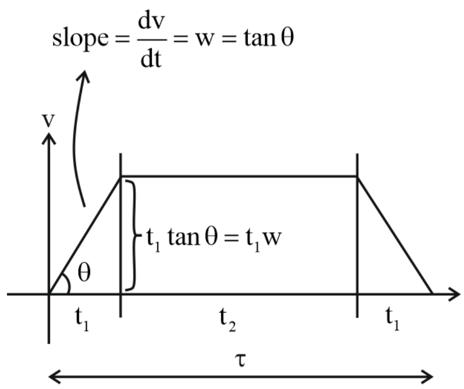
But $S_A = S_B$

Or $\frac{9}{2}a_1 = \frac{5}{2}a_2$

=> $a_1 : a_2 = 5 : 9$

3. A car starts moving rectilinearly, first with acceleration w = 5.0 ms^{-2} (the initial velocity is equal to zero), then uniformly, and finally, decelerating at the same rate w, comes to a stop. The total time of motion equals τ = 25 s. The average velocity during that time is equal to <v> = 72 kmh^{-1}. How long does the car move uniformly?

Solution:



Average velocity:

$$\vec{v} = \frac{area}{time}$$

$$\Rightarrow \vec{v} = \frac{1}{2}\frac{(\tau + t_2)(\omega t_1)}{\tau} \quad (2t_1 + t_2) = \tau$$

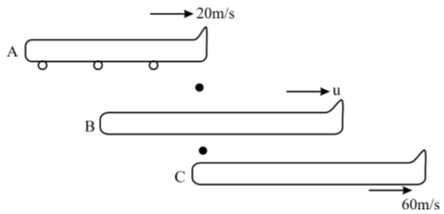
$$= \frac{1}{2}\frac{(\tau + t_1)}{\tau}\frac{\omega(\tau - t_1)}{\tau}$$

$$\Rightarrow 4\tau\vec{v} = \tau^2 - t_2^2$$

$$\Rightarrow t_2 = \sqrt{\tau\left(\tau - \frac{4v}{\omega}\right)}$$

4. Two ends of a train moves with speed 20 m/s and 60 m/s while crossing a given point. Calculate the velocity of train, when the midpoint of the train crosses the given point.

Solution:



A → B

$v^2 - 20^2 = 2a\frac{l}{2}$ ……….(I)

B → C

$60^2 - v^2 = 2a\frac{l}{2}$ ……….(II)

Divide (I) / (II),

$\frac{v^2 - 400}{3600 - v^2} = 1$

=> v^2-400 = 3600 - v^2

$2v^2$ = 4000

v^2 = 2000

v = $20\sqrt{5}$ m/s

5. A body is moving from rest under constant acceleration and let S_1 be the displacement in the first $(p-1)$ sec and S_2 be the displacement in the first p sec. The displacement in $(p^2-p+1)^{th}$ sec. will be

(a) $S_1 + S_2$ **(b)** $S_1 S_2$

(c) $S_1 - S_2$ **(d)** S_1 / S_2

Solution:

The initial velocity of body is zero, u = 0

2nd equation of motion,

S = ut $+\frac{1}{2}at^2$

S = $\frac{1}{2}at^2$ --------(1)

The displacement in time (P -1) sec is

$S_1 = \frac{1}{2}a(P-1)^2$ -------(2)

The displacement in time P sec,

$S_2 = \frac{1}{2}aP^2$ -------(3)

Adding equation (2) and (3), we get

$S_1 + S_2 = \frac{1}{2}a(2P^2 - 2P + 1)$-----(4)

The displacement at time $(p^2 - P + 1)^{th}$ sec

$S = \frac{1}{2}a(2P^2 - 2P + 1)$-----(5)

From equation (4) and (5),

S = $S_1 + S_2$

The correct option is A.

Beginner **Motion under Gravity**

1. A body is projected vertically upwards with 40 m/s from ground. Find the followings:

(a) Time of flight

(b) Maximum height

(c) Average speed and velocity in 5 sec

(d) Distance travelled in last second of its motion

Solution:

(a) v = u +at (going up)

0 = 40 – 10t

t = 4 sec

Total time = 2t

= 8sec

(b) $v^2 - u^2 = 2as$

0 - 40^2 = 2(-10) h

h = 80 m

(c) In 5 Sec,

Total distance = $d_1 + d_2$

d_1 = 80m (t = 0 to t = 4)

d_2 = 5m (t = 4 to t = 5)

d_{Total} = 85

Displacement = 75

Avg speed = $\frac{85}{5}$ = 17 m/s

Avg velocity = $\frac{75}{5}$ = 15 m/s

(d) $d_{last} = d_8 - d_7$

$= d_4 - d_3$

= 80 - $\left(\frac{1}{2}x10x3^2\right)$

= 80 – 45 = 35m

2. If a 15kg ball takes five seconds to strike the ground when released from rest, at what height was the ball dropped?Possible Answers:

(a) 50m (b) 75m

(c) 125m (d) 250m

Solution:

(c) Using the equation d = v_0t + (1/2)a(t^2) we can find the distance at which the ball was dropped. Notice that the mass of the ball does not matter in this problem. We are told that the ball is dropped from rest making, v_0 = 0, thus we have d = (1/2)a(t^2). When we plug in our values, and assuming that the acceleration is equal to gravity (10m/s^2) we find that d = (1/2)(10)(5^2) = 125 m.

3. A person on top of a 200m tall building drops a rock. How long will it take for the rock to reach the ground? Ignore air resistance. Possible Answers:

(a) t = 40.8s **(b) t = 6.4s**

(c) t = 7.3s **(d) 113.4s**

Solution:

(b) Use the following kinematic equation:

$$y = y_0 + v_0 t + \frac{1}{2}at^2.$$

We can choose the ground to be the zero distance so that y = 0 and y_0 = 200 m.

Also, the initial speed is zero.

$v_0 = 0$

The kinematic equation simplifies using these values.

$$0 = y_0 + \frac{1}{2}at^2$$

Solve to isolate the time variable.

$$t = \sqrt{\frac{-2y_0}{a}}$$

We know that the acceleration is the acceleration due to gravity. Now we can plug in the known values and solve.

$$t = \sqrt{\frac{-2(200m)}{-9.8\frac{m}{s^2}}}$$

$$t = \sqrt{40.8s^2} = 6.4s$$

4. An object is released from rest and falls a distance h during the first second of time. How far will it fall during the next second of time?

(a) h (b) 2h

(c) 3h (d) 4h

Solution:

From the equation d = 1/2 at^2, displacement is proportional to time squared. Traveling from rest for twice the time gives 4 times the displacement (or 4 h). Since the object already travelled h in the first second, during the time interval from 1 s to 2 s the object travelled the remaining 3h

5. Starting from rest, object 1 falls freely for 4.0 seconds, and object 2 falls freely for 8.0 seconds. Compared to object 1, object 2 falls:

(a) Half as far (b) Twice as far

(c) three times as far (d) four times as far

Solution:

Since (from rest) d = 1/2 gt^2, distance is proportional to time squared. An object falling for twice the time will fall four times the distance.

Hence option d is correct.

6. A ball which is dropped from the top of a building strikes the ground with a speed of 30 m/s. Assume air resistance can be ignored. The height of the building is approximately:

(a) 15 m **(b) 30 m**

(c) 45 m **(d) 75 m**

Solution:

To reach a speed of 30 m/s when dropped takes (using v = at) about 3 seconds. The distance fallen after three seconds is found using d = 1/2 at^2.

7. Two bodies of different mass m_a and m_b are dropped from two different heights a and b. The ratio of the time taken by the two to cover these distances are

(a) a : b **(b) b : a**

(c) $\sqrt{a} : \sqrt{b}$ **(d) $a^2 : b^2$**

Solution:

Correct option is C.

Acceleration due to gravity acting on both the bodies is same i.e. ± g.

Initial speed of the bodies u = 0

Distance covered by a body S = ut + $\frac{1}{2}at^2$

Thus we get S = $\frac{1}{2}gt^2$

Or t = $\sqrt{\frac{2S}{g}}$

=> t ∝ $\sqrt{S}$

Thus $\frac{t_a}{t_b} = \sqrt{\frac{a}{b}}$

8. A body starts to fall freely under gravity. The distances covered by it in first, second and third *seconds* are in ratio

(a) 1:3:5 **(b)** 1:2:3

(c) 1:4:9 **(d)** 1:5:6

Solution:

Correct option is A

According to equation of motion, distance covered in nth sec.

$$S_n = u = \frac{a}{2}(2n-1)$$

$$S_n = \frac{a}{2}(2n-1)$$

$$\therefore S_1 : S_2 : S_3 = \{2(1)-1\}:\{2(2)-1\}:\{2(3)-1\}$$

= 1 : 3 : 5

9. A body falls freely from rest, covers as much distance in the last second of its motion as it covers in the first three seconds. The body has fallen for a time of

(a) 3 s **(b) 5 s**

(c) 7 s **(d) 9 s**

Solution:

Correct option is B.

Displacement in the nth second of the free fall is

$S_{n^{th}} = \frac{1}{2}g(t_1^2 - t_2^2)$

Given that $(t_n - t_{n-1}) = 1$

Displacement in 3 seconds of the free fall

$S = \frac{1}{2}gt^2$

$S = \frac{1}{2}x\ 10\ x\ 3^2$

S = 45m

Given that $S_{n^{th}}$ = 45

On solving that we get:

t_1 = 5sec

10. A body is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time t/2 sec

(a) At h/2 from the ground

(b) At h/4 from the ground

(c) Depends upon mass and volume of the body

(d) At 3h / 4 from the ground

Solution:

Correct option is D.

$H = \frac{1}{2}gt^2$

At $\frac{t}{2}$, s = $\frac{1}{2}g\left(\frac{t}{2}\right)^2 = \frac{1}{4}\left(\frac{1}{2}gt^2\right)$

s = $\frac{1}{4}H$

Therefore, the height from ground is given by,

x = H – s = H - $\frac{1}{4}H = \frac{3}{4}H$

11. A ball is released from the top of a tower of height *h* meters. It takes *T* seconds to reach the ground. What is the position of the ball in *T*/3 seconds?

(a) *h*/9 *meters* from the ground

(b) 7*h*/9 *meters* from the ground

(c) 8*h*/9 *meters* from the ground

(d) 17*h*/18 *meters* from the ground

Solution:

Correct option is C.

The acceleration of the ball will be g. Initial velocity will be O.

In T sec body travels h mts.

By applying equations of motion we get

s = ut + $\frac{1}{2}gT^2$

h = $\frac{1}{2}gT^2$ -------(1)

in $\frac{T}{3}$ sec

$h_1 = \frac{1}{2}gT^2 = \frac{1}{2}g\left(\frac{T}{3}\right)^2 = \frac{1}{2}g\left(\frac{T^2}{9}\right)$ ------(2)

From

(1) and (2) we get $h_1 = \frac{h}{9}$ distance from point of release.

Therefore, distance from ground is h - $\frac{h}{9} = \frac{8h}{9}$

12. A stone is dropped into water from a bridge $44.1\ m$ above the water. Another stone is thrown vertically downward 1 *sec* later. Both strike the water simultaneously. What was the initial speed of the second stone?

(a) 12.25 *m / s* **(b)** 14.75 *m / s*

(c) 16.23 *m / s* **(d)** 17.15 *m / s*

Solution:

Correct option is A)

Step 1: Time taken (t) by stone 1.

(Taking downward positive)

Since acceleration is constant, therefore applying equation of motion

$h = ut + \frac{1}{2}gt^2$

=> 44.1 = 0 + $\frac{1}{2}(9.8)t^2$

=> t = 3s

Step 2: Initial Speed Calculation

Let the initial speed of stone 2 be u

Both the stones strike the water simultaneously but the stone 2 is thrown 1s later which means it takes 1s less as compared to stone 1 to cover the same distance.

Therefore, time taken by stone 2, t_1= t-1 =3-1s =2s

Since acceleration is constant

∴ Applying equation of motion (Taking downward positive)

$h = ut_1 + \frac{1}{2}g{t_1}^2$

=> 44.1 = u x 2 + $\frac{1}{2}$ x 9.8 x $(2)^2$

=> u = 12.25 m/s

Hence option A is correct.

13. Water drops fall at regular intervals from a tap which is $5\ m$ above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant?

(a) 2.50 *m* **(b) 3.75 *m***

(c) 4.00 *m* **(d) 1.25 *m***

Solution:

Correct option is B.

Height of tap = 5m and (g) = 10m/sec^2

For the first drop, 5 = ut + gt^2

= (0 x t) x 10t^2 = 5t^2 or t^2=1 ot t = 1.

It mean that the third drop leaves after one second of the first drop. Or, each drop leaves after every 0.5sec.

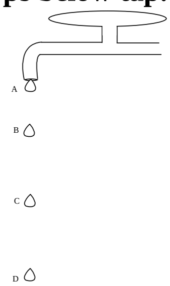
Distance covered by the second drop in 0.5 sec.

= $(0.5)^2 = 1.25m$

Therefore, distance of the second drop above the ground

= 5 – 1.25 = 3.75m

14. Water drops fall at regular interval from a tap. At an instant, when the 4th drop is about to leave tap, find ratio of separation between 3 successive drops below tap.



Solution:

Let assume time interval for each successive drop = Δt

Time taken by 1st drop = 3Δt

Time taken by 2nd drop = 2Δt

Time taken by 3rd drop = Δt

h_1= total distance travelled by 1st drop

= $\frac{1}{2}g(3\Delta t)^2$

= 4.5gΔt^2

h_2= $\frac{1}{2}g(2\Delta t)^2$

= 2gΔt^2

h_3= $\frac{1}{2}g(\Delta t)^2$

= 0.5gΔt^2

Required Ratio:

$h_3 : (h_2 - h_3) : (h_1 - h_2)$

= 0.5gΔt^2: 1.5gΔt^2: 2.5gΔt^2

= 1 : 3 :5

15. A ball is thrown vertically downward with a velocity of 20 m/s from the top of a tower. It hits the ground after some time with a velocity of 80 m/s. The height of the tower is (g =10 m/s^2)

(a) 340 m **(b) 320 m**
(c)300 m **(d) 360 m**

Solution:

(c) Given, u = 20 m/s, v = 80 m/s and h= ?

From kinematic equation of motion,

$$v^2 = u^2 + 2gh$$

$$h = \frac{v^2 - u^2}{2g}$$

$$= \frac{(80)^2 - (20)^2}{2 \times 10} \quad (\because \text{ given, } g = 10 \text{ m/s}^2)$$

= 300 m

Hence, correct option is (c).

Expert **Motion under Gravity**

1. A body is projected vertically upwards with 40m/s from the tower of height 80m. Find the followings:

(a) Maximum height

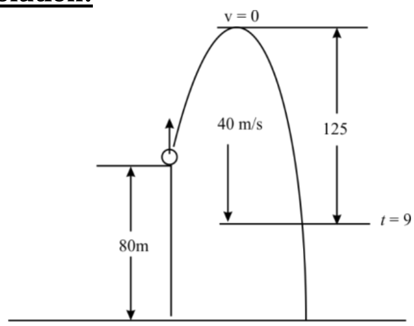
(b) Time of flight

(c) Velocity of strike

(d) Distance travelled in last second

(e) Average Speed & Average Velocity in 9 seconds

Solution:



(a) $v^2 - u^2 = 2as$

0 - 40^2 = 2 (-10) h

h = 80m

$H_{max} = 80 + 80 = 160m$

(b) 160 = $\frac{1}{2} x\ 10\ x\ t_1^2$

$t_1 = 9\sqrt{2}$ sec

v = u + at_2

0 = $u_0 - 10t_2$

$t_2 = 4$ sec

Total time = $t_1 + t_2$

= 4(1+$\sqrt{2}$) sec

(c) v_{strike} = u + at

= 0 + 10 x 4$\sqrt{2}$

= 40$\sqrt{2}$m/s

(d) $d_{last} = d_{4\sqrt{2}} - d_{4\sqrt{2}-1}$

$= 160 - \left(\frac{1}{2} x\ 10\ x \left(4\sqrt{2} - 1\right)^2\right)$

$= 160 - (5)\left(4\sqrt{2} - 1\right)^2$

$= \left(40\sqrt{2} - 5\right)$

(e) Distance travelled in 9 sec = $d_1 + d_2$

$= 80 + \left(\frac{1}{2} x\ 10\ x\ (5)^2\right)$

= 80 + 125

= 205 m

Displacement in 9 sec = 45m

$V_{avg} = \frac{205}{9} = 22.7\ m/s$

$\overrightarrow{V_{avg}} = \frac{45}{9} = 5\ m/s$

2. A stone is thrown vertically upward with an initial velocity v_o. Find out the distance travelled in time $4v_o/3g$.

Solution:

Lets consider,

v_0= initial velocity of the stone

g = acceleration due to gravity

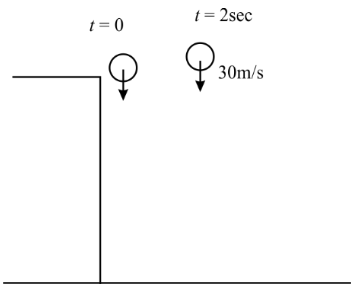
$t = \frac{4v_0}{3g}$

2nd equation of motion,

s = ut + $\frac{1}{2}gt^2$

s = $v_0 \frac{4v_0}{3g} - \frac{1}{2} g \left(\frac{4v_0}{3g}\right)^2$

s = $\frac{4v_0^2}{3g} - \frac{8v_0^2}{9g}$



s = $\frac{4v_0^2}{9g}$

3. A stone is dropped from rest from top of a cliff. A 2nd stone is thrown vertically down from same point with 30 m/s velocity 2s later. At what distance from top of the hill, will the two stones meet. (g = 10 m/s^2)

Solution:

t = 0, t = 2sec

Let say after 't_0' they meet

$h_1 = \frac{1}{2} g(2 + t_0)^2$

$h_2 = 30t_0 + \frac{1}{2} g{t_0}^2$

$h_1 = h_2$

$\frac{1}{2} g(2 + t_0)^2 = 30t_0 + \frac{1}{2} g{t_0}^2$

$5(2 + t_0)^2 = 30t_0 + 5{t_0}^2$

$5(4+{t_0}^2+4t_0) = 30t_0 + 5{t_0}^2$

$4 + {t_0}^2 + 4t_0 = 6t_0 + {t_0}^2$

$2t_0 = 4$

t_0 = 2 sec

$h_2 = (30\ x\ 2) + \frac{1}{2}\ x\ 10\ x\ 2^2$

= 60 + 20

= 80 m

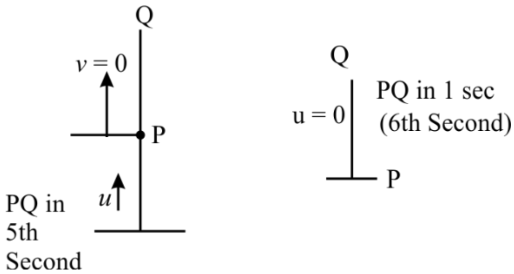
4. A body is thrown vertically upwards with velocity *u*. The distance travelled by it in the fifth and the sixth seconds are equal. The velocity *u* is given by (*g* = 9.8 *m/s*^{2})

(a) 24.5 *m/s* **(b) 49.0 *m/s***

(c) 73.5 *m/s* **(d) 98.0 *m/s***

Solution:

Correct option is B.



Distance travelled in 5th second

PQ is PQ = $D_5 = u + \frac{a}{2}(2n - 1)$

=> PQ = $u - \frac{g}{2}(2\ x\ 5 - 1)$

=> PQ = u - $\frac{9g}{2}$ --------(1)

Distance QP is => QP = ut + $\frac{1}{2}gt^2$

=> QP = 0 + $\frac{1}{2}g(1)^2$

=> QP = $\frac{g}{2}$ -------(2)

Now PQ = QP

So from (1) & (2) we get u - $\frac{9g}{2} = \frac{g}{2}$

u = 5g = 5 x 9.8 = 49 m/s

u = 49 m/s

5. A particle is dropped under gravity from rest from a height h ($g = 9.8\ m/sec^2$) and it travels a distance 9h/25 in the last second, the height h is

(a) 100 m **(b) 122.5 m**

(c) 145 m **(d) 167.5 m**

Solution:

The correct option is B 122.5m

Let h distance is covered in n sec

=> = $\frac{1}{2}gn^2$ -------------(i)

Distance covered in the nth sec = $\frac{1}{2}g(2n-1)$

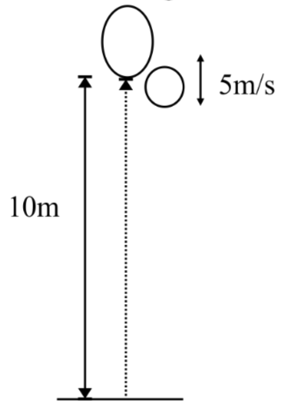
=> $\frac{9h}{25} = \frac{g}{2}(2n-1)$ ---------(ii)

From (i) and (ii), h=122.5m

6. A balloon rises from rest on the ground with constant acceleration $\frac{g}{8}$. A stone is dropped from balloon when the balloon has risen to a height of h = 10 m. Find the time taken by the stone to reach the ground. $\left(g = 10 m/s^2\right)$

Solution:

v = velocity of stone
at 10m height



v = velocity of stone at 10m height

$v^2 = \frac{2g}{8}x10$

v^2 = 25

v = 5 upward

When the stone is dropped:

10 = -5t + $\frac{1}{2}$ 10 t^2

t^2- t – 2 = 0

t^2-2t + t – 2 = 0

t (t-2) +1(t-2) = 0

t = 2sec

7. A body A of mass 4 kg is dropped from a height of 100 m. Another body B of mass 2 kg is dropped from a height of 50m at the same time.

(a) Both the bodies reach the ground simultaneously

(b) A takes nearly 0.7th of time required by B.

(c) B takes nearly 0.7th of time required by A.

(d) A takes double the time required by B.

Solution:

M_A = 4kg

H_A = 100m

M_B = 2kg

H_B = 50m

$H_A = \frac{1}{2}gt_A^2$

100 = 5 t_A^2

$t_A = \sqrt{20}s$

$H_B = \frac{1}{2}gt_B^2$

50 = 5 t_B^2

$t_B = \sqrt{10}s$

$\frac{t_A}{t_B} = \frac{\sqrt{20}}{\sqrt{10}} = \sqrt{2}$

$t_B = \frac{t_A}{\sqrt{2}}$

$t_B = \frac{\sqrt{2}}{2}t_A$

$t_B = 0.7t_A$

∴ B takes 0.7th of time required by A.

∴ Option (c)

8. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$ s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is ________.

Solution:

Let the ball takes time t to reach the ground

Using, $S = ut + \frac{1}{2}gt^2$

$S = 0 \times t + \frac{1}{2}gt^2$

200 = gt^2 [∵ 2S = 100m]

$t = \sqrt{\frac{200}{g}}$ ... (i)

In last $\frac{1}{2}s$, body travels a distance of 19 m, so in $\left(t - \frac{1}{2}\right)$

distance travelled = 81

Now, $\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81$

∴ $g\left(t - \frac{1}{2}\right)^2 = 81 \times 2$

$\left(t - \frac{1}{2}\right) = \sqrt{\frac{81 \times 2}{g}}$

$\therefore\ \frac{1}{2}=\frac{1}{\sqrt{g}}\left(\sqrt{200}-\sqrt{81\times 2}\right)$ from eq. (i)

$\sqrt{g}=2\left(10\sqrt{2}-9\sqrt{2}\right)$

$\sqrt{g}=2\sqrt{2}$

$\therefore$ g = 8 m/s^2

9. A person sitting in the ground floor of a building notices through the window of height 1.5 m, a ball dropped from the roof of the building crosses the window in 0.1 s. What is the velocity of the ball when it is at the topmost point of the window? (g = 10 m/s^2)

(a) 15.5 m/s (b) 14.5 m/s

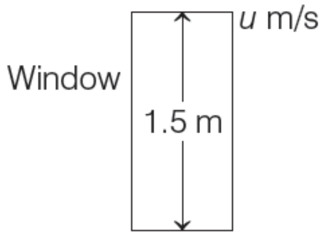
(c) 4.5 m/s (d) 20 m/s

Solution:

(b) According to question, time taken by the ball to cross the window,

t = 0.1 s

h = 1.5 m



If u be the velocity at the top most point of the window, then from equation of motion,

$$h=ut+\frac{1}{2}gt^2$$

$$1.5=u\times 0.1+\frac{1}{2}\times 10\times(0.1)^2$$

$$1.5=0.1u+0.05$$

$$u=\frac{1.5-0.05}{0.1}=\frac{1.45}{0.1}$$

= 14.5 m/s

10. A stone falls freely under gravity. It covers distances h_1, h_2, and h_3, in the first 5s, the next 5s and the next 5s respectively. The relation between h_1, h_2, and h_3, is

(a) $h_1 = 2h_2 = 3h_3$ (b) $h_1=\frac{h_2}{3}=\frac{h_3}{5}$

(c) $h_2 = 3h_1$ and $h_3 = 3h_2$ (d) $h_1 = h_2 = h_3$

Solution:

(b) For free fall from a height, u = 0

$\therefore$ Distance covered by stone in first 5 s,

$$h_1=0+\frac{1}{2}g(5)^2=\frac{25}{2}g \quad \text{... (i)}$$

$\therefore$ Distance covered in first 10 s,

$$s_2=0+\frac{1}{2}g(10)^2=\frac{100}{2}g$$

$\therefore$ Distance covered in second 5 s

$$h_2=s_2-h_1=\frac{100}{2}g-\frac{25}{2}g=\frac{72}{2}g \quad \text{... (ii)}$$

Distance covered in first 15 s,

$$s_3=0+\frac{1}{2}g(15)^2=\frac{225}{2}g$$

$\therefore$ Distance covered in last 5 s,

$$h_3=s_3-s_2=\frac{225}{2}g-\frac{100}{2}g=\frac{125}{2}g \quad \text{... (iii)}$$

From Eqs. (i), (ii) and (iii), we get

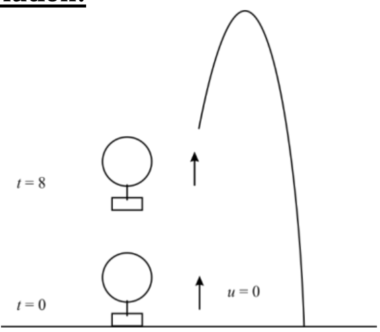
$$h_1:h_2:h_3=\frac{25}{2}g:\frac{75}{2}g:\frac{125}{2}g = 1:3:5$$

$$h_1=\frac{h_2}{3}=\frac{h_3}{5}$$

Pro Motion under Gravity

1. A balloon starts from ground with 1.25 m/s^2 acceleration. After 8 s, a stone is released from balloon. Find time in which stone will strike ground. Find distance covered by stone and its displacement from point from where it was released. Find also height of balloon when the stone strikes ground. g = 10 m/s^2.

Solution:



At t = 8 sec,

Vel. of balloon = u + at

= 0 + 1.25 x 8

= 10 m/s

For stone,

v = u + at

0 = 10 – 10t => t =1 sec

Height travelled by balloon in 8sec,

$$h=\frac{1}{2}x\ 1.25\ x\ 8^2$$

= 40 m

$40 + 5 = =\frac{1}{2}x\ g\ x\ t^2$

t = 3 sec

∴ Total time = 3 + 1 = 4sec

Distance = $\left(\frac{1}{2} x\ 10\ x\ 1^2\right) + 45$

= 50 m

Displacement = 40 m

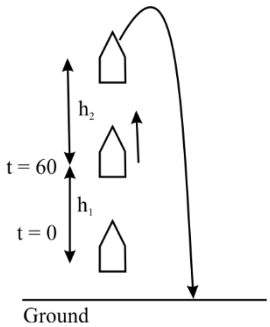
After 12 sec,

h = $\frac{1}{2} x\ 1.25\ x\ 12^2$

= 90 m

2. A rocket is fired vertically upwards with an acceleration 20 m/s^2. After 1 min of its flight, all its fuel gets used up. Find the maximum height reached by the rocket and total time taken by the rocket to reach the ground. Also find the velocity of strike. (velocity at which it will hit the ground)

Solution:



h = $\frac{1}{2}\ x\ 20\ x\ 60^2$

= 36 km

$v_{t=60}$ = u + at = 0 + 20 x 60

= 1200 m/s

$v^2 - u^2 = 2as$

0 - $(1200)^2$ = 2 (-10) h

h_2 = 72 km

Maximum height = $h_1 + h_2$

= 36 + 72

= 108 km

$t_1 = \frac{v-u}{g} = \frac{0-1200}{-10}$ = 120 sec

108000 = $\frac{1}{2}\ x\ 10\ x\ t^2$

t = $\sqrt{21600}$

= $60\sqrt{6} sec$

Total time = 60 + 120 + $60\sqrt{6}$

= (180 + $60\sqrt{6}$) sec

Vol. of strike = u + at

= 0 + 10 x $60\sqrt{6}$

= $600\sqrt{6}$ m/s

3. A man with a balloon rising vertically with an acceleration of 4.9 m/sec^2 releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reached by the ball is (g = 9.8 m/sec^2)

(a) 14.7 m **(b) 19.6 m**

(c) 9.8 m **(d) 24.5 m**

Solution:

Correct option is A.

a = 4.9 m/s^2

Initial velocity = 0

Velocity of balloon and ball after 2 sec.

v = u + at = 9.8 m/s^2

Height at this point

h = $\frac{1}{2} at^2$

= 0.5 x 4.9 x 4

= 9.8 m

The ball will attain further height = s

$\frac{1}{2} mv^2$ = mgs

s = $\frac{v^2}{2g} = 4.9m$

Greatest height that can be achieved,

H = h + s = 9.8 + 4.9 = 14.7m

4. A very large number of balls are thrown vertically upwards in quick succession in such a way that the next ball is thrown when the previous one is at the maximum height. If the maximum height is 5*m*, the number of balls thrown per minute is (take $g = 10 ms^{-2}$)

(a) 120 **(b) 80**

(c) 60 **(d) 40**

Solution:

Maximum height gained by ball is 5m

H = 5m

Initial velocity of ball be u

=> $v^2 = u^2 + 2aH$

0 = u^2- 2gH [v = 0]

u = $\sqrt{2gH}$ = 10m/s

Time taken by ball to gain height H is

v = u + at

10 – 10t = 0

t = 1sec.

The time interval for balls between one another is 1 second. So, 60 balls are thrown in one minute.

5. A stone is dropped from a height h. Simultaneously another stone is thrown up from the ground with such a velocity that it can reach a height of 4h. Find the time when the two stones cross each other.

Solution:

∵ The stone can reach height (H) = 4h,

Its initial velocity

v = $\sqrt{2(4h)g}$ (v = $\sqrt{2gH}$)

Let time of flight be t

Distance travelled by upper stone

$D_1 = \frac{1}{2}gt^2$

Distance travelled by lower stone

$D_2 = vt - \frac{1}{2}gt^2$

$D_1 + D_2$ = h

=> $\frac{1}{2}gt^2$ + vt - $\frac{1}{2}gt^2$ = h

=> h = vt => h = $\sqrt{8hg.}t$

=> t = $\frac{h}{\sqrt{8hg}} = \sqrt{\frac{h}{8g}}$

Hence time when two stones cross is $\sqrt{\frac{h}{8g}}$.

6. A balloon is ascending vertically with an acceleration of 0.2 m/s ^{2}. Two stones are dropped from it at an interval of 2 sec. Find the distance between them 1.5 sec after the second stone is released. (Use g=9.8m/s ^{2})

Solution:

The first stone travels for 2 + 1.5 = 3.5 sec.

The second stone travels for 1.5 secs.

Let first stone be dropped at t = 0

Thus, distance travelled by it in 0.5 seconds is

$\frac{1}{2}gt^2 = 4.9(3.5)^2$ = 60.025 m

Distance travelled by balloon upwards in 2 seconds

= $\frac{1}{2}at^2$ = 0.5 x 0.2 x 2^2 = 0.4 m

Distance travelled by stone 2 after being released

= 0.5 x 9.8 x 1.5^2 = 11.025 m

Thus, distance between the 2 stones at required time

= 60.025 – (-0.4 + 11.025) = 49.4 m

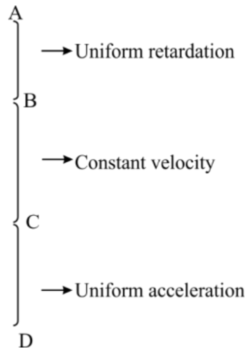
7. A lift ascends with a constant acceleration of 4 m/s^2, then with a constant velocity v, and then moves with a constant retardation of 4 m/s2 to finally come to rest. If the total height ascended is 20 m and the total time taken is 6 s, then the time during which the lift was moving with constant velocity v is

(a) 2 s **(b) 3 s**

(c) 4 s **(d) 5 s**

Solution:

We can divide the path of the lift into 3 parts



Given, AD = 20m

Total time T = 6s

In path CD,

u = 0

a = 4m/s^2

v = v

v^2 = 2as

=> CD = $\frac{v^2}{8}$

v – u = at

=> $t_1 = \frac{v-u}{a} = \frac{v}{4}s$

BC = s_2 = vt_2

AB => u’ = v

v’ = 0

a = -4m/s^2

v^2 = 8s

$\frac{v^2}{8} = s_3$

v – u = at_3

$t_3 = \frac{v-u}{a} = \frac{v}{4}s$

$s_1 + s_2 + s_3$ = 20 m

$\frac{v^2}{8} + \frac{v^2}{8}$+ v$t_2$ = 20

$\frac{v^2}{4}$+ vt_2 = 20

=> vt_2 = 20 - $\frac{v^2}{4}$

$t_2 = \frac{80-v^2}{4v}$

$t_1 + t_2 + t_3$ = 6s

$\frac{v}{4} + \frac{v}{4} + \frac{80-v^2}{4v}$ = 6

$\frac{2v^2-v^2+80}{4v}$ = 6

$v^2 - 24v + 80$ = 0

$v^2 - 20v - 4v + 80$ = 0

v (v – 20) – 4(v – 20) = 0

(v – 4) (v – 20) = 0

v = 4 or v = 20m/s

∴ v cant be 20 m/s then t will become -ve, which is not possible.

$t_2 = \frac{80-v^2}{4v}$

$t_2 = \frac{80-16}{4x4} = \frac{64}{16}$

t_2 = 4 s [constant velocity]

∴ Option (c)

8. A ball is thrown upward from the ground with velocity u. It is at a point 100 m high at two times t_1 and t_2 respectively. If g = 10 m/s^2, then

(a) $t_1 . t_2 = 20$ **(b)** $t_1 + t_2 = 20$

(c) $t_1 t_2 = \frac{u}{5}$ **(d)** $t_1 + t_2 = \frac{u}{5}$

Solution:

(a, d)

9. A particle is projected vertically upward, and it reaches a height h from the ground in t_1 sec and reaches ground from that same height is t_2 sec, then

(a) Initial velocity of projection is $\frac{g(t_1+t_2)}{2}$

(b) Velocity at half of its maximum height will be $\frac{g(t_1+t_2)}{2\sqrt{2}}$

(c) Maximum distance travelled by particle in vertical direction is $\frac{g}{2}(t_1+t_2)^2$

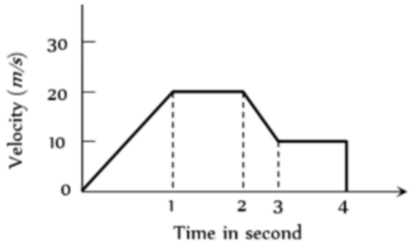
(d) All of the above

Solution:

(a, b)

Beginner **Graphs in Motion**

1. The variation of velocity of a particle with time moving along a straight line is illustrated in the following figure. The distance travelled by the particle in four second is



(a) 60 m **(b) 55 m**

(c) 25 m **(d) 30 m**

Solution:

Correct option is B.

REF. Image.

Distance up to 4sec

Distance (0 to 1sec) = $\frac{1}{2} x 1 x 20$ = 10m

(1 – 2sec) = 20 x (2-1) = 20m

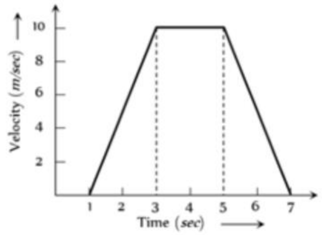
(2-3sec) = (3-2) x 10 + $\frac{1}{2}$ (3-2) x (20-10)

= 1 x 10 +$\frac{1}{2}$ x 1 x 10 = 15m

(3-4sec) = (4-3) x 10 = 10m

Total distance = 10 + 20 + 15 + 10 = 55m

2. In the following graph, distance travelled by the body in meters is



(a) 200 **(b) 250**

(c) 300 **(d) 400**

Solution:

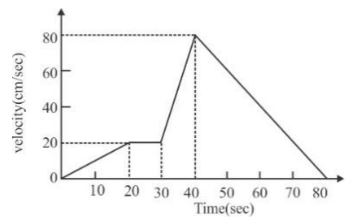
Correct option is A.

Area under the graph gives distance

Distance = $\frac{1}{2}$ x 10 x 10 +10 x 10 + $\frac{1}{2}$ x 10 x 10

= 50 + 100 + 50 = 200

3. The v – t graph of a moving object is given in figure. The maximum acceleration is



Solution:

Acceleration in the 1st part,

$a_1 = \frac{20-0}{20} = 1\ cm/s^2$

$a_2 = 0$

$a_3 = \frac{80-20}{40-30} = \frac{60}{10} = 6 cm/s^2$

$a_4 = \frac{0-80}{80-40} = -\frac{80}{40} = -2cm/s^2$

Max. acceleration is $6cm/s^2$

4. The displacement-time graph for two particles *A* and *B* are straight lines inclined at angles of 30^o and 60^o with the time axis. The ratio of velocities of $V_A : V_B$ is

(a) $1:2$ **(b)** $1:\sqrt{3}$

(c) $\sqrt{3}:1$ **(d)** $1:3$

Solution:

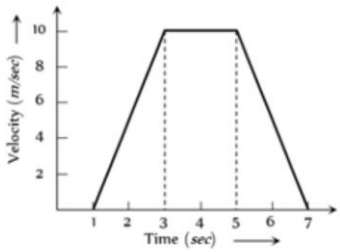
Correct option is D.

$V_A = tan30° = \frac{1}{\sqrt{3}}$

$V_B = tan60° = \sqrt{3}$

$\frac{V_A}{V_B} = \frac{1}{3} = 1 : 3$

5. For the velocity-time graph shown in figure below the distance covered by the body in last two seconds of its motion is what fraction of the total distance covered by it in all the seven seconds



(a) $\frac{1}{2}$ **(b) $\frac{1}{4}$**

(c) $\frac{1}{3}$ **(d) $\frac{2}{3}$**

Solution:

Correct option is B

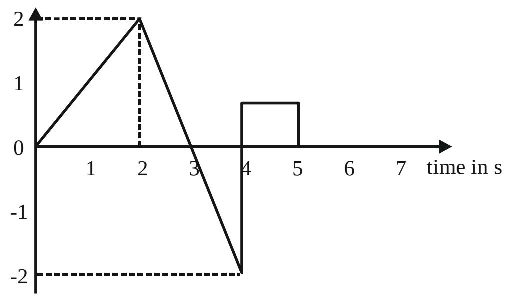
Distance covered is equal to the area under velocity – time graph.

Distance in last two seconds $s_2 = \frac{1}{2}\ x\ 8\ x\ 2$ = 8m

Total distance = $\frac{1}{2}\ x\ 8\ x\ (6 + 2)$ = 32m

Fraction of the distance covered by the body in the last two seconds of its motion to total distance traveled by it in all the seven seconds is 8/32 = 1 :4

6. The velocity-time graph of a body moving along a straight line is as follows:



The displacement of the body in 5 s is

(a) 5 m **(b) 2 m**

(c) 4 m **(d) 3 m**

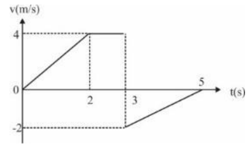
Solution:

Correct option is D.

Displacement is area under v-t graph:

= $\frac{1}{2}$ x 3 x 2 + $\frac{1}{2}$ x (1) x (-2) + 1 x 1 = 3m

7. For a particle moving along x-axis, velocity-time graph is as shown in figure. Find the distance travelled and displacement of the particle. Also find the average velocity of the particle in interval 0 to 5 second.



Solution:

Displacement = Area under graph

= $\left(\frac{1}{2}\ x\ 2\ x\ 4\right)$ + (1 x 4) - $\left(\frac{1}{2}\ x\ 2\ x\ 2\right)$

= 4 + 4 – 2

= 6m

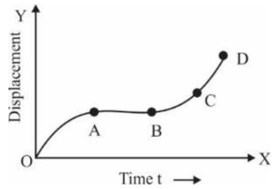
Distance = 4 + 4 + 2

= 10 m

$\overrightarrow{V_{avg}} = \frac{displacement}{time} = \frac{6}{5}$

= 1.2 m/s

8. The graph between the displacement *x* and time *t* for a particle moving in a straight line is shown in figure. During the interval *OA, AB, BC and CD, the acceleration of the particle for interval OA, AB, BC, CD are*



(a) + , 0 , + , + **(b) – , 0 , + , 0**

(c) + , 0 , – , + **(d) – , 0 , – , 0**

Solution:

Correct option is B

Slope of the displacement – time graph gives the instantaneous velocity of the particle.

Region OA: Slope is (+) and decreasing i.e v decreases in positive direction.

=> The particle is retarding i.e a is negative (-)

Region AB: Slope is zero i.e v = constant.

=> The particle is moving with zero acceleration i.e a = 0

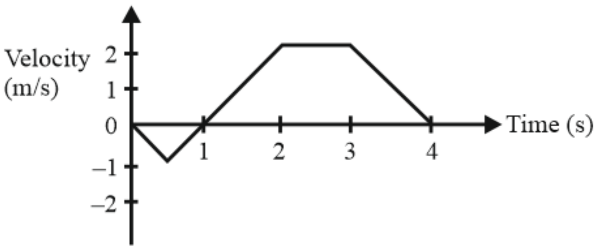
Region BC: Slope is (+) and increasing i.e v increases in positive direction.

=> The particle is accelerating i.e a is positive (+)

Region CD: Slope is positive but constant i.e v = constant.

=> The particle is moving with zero acceleration i.e a = 0

9.



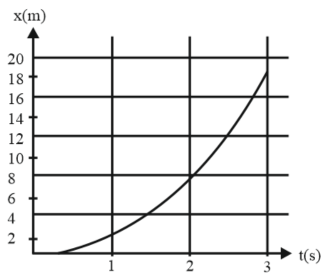
The graph above shows the velocity versus time for an object moving in a straight line. At what time after t = 0 does the object again pass through its initial position?

(a) 1 s
(b) Between 1 and 2 s
(c) 2 s
(d) Between 2 and 3 s

Solution:

Area bounded by the curve is the displacement by inspection the negative area between 0 and 1 s will be countered by an equal negative area sometime between 1 and 2 s.

10.



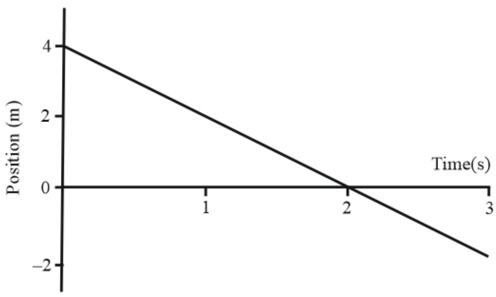
The graph above represents position x versus time t for an object being acted on by a constant force. The average speed during the interval between 1 s and 2 s is most nearly

(a) 2 m/s
(b) 4 m/s
(c) 5 m/s
(d) 6 m/s

Solution:

Average speed = total distance/total time = (8 m – 2 m)/(1 second)

11. The position vs. time graph for an object moving in a straight line is shown below. What is the instantaneous velocity at t = 2 s?

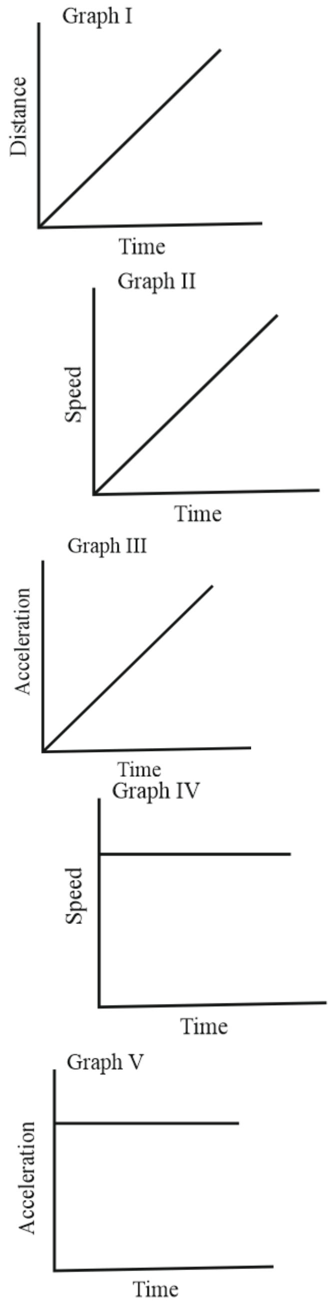


(a) –2 m/s
(b) 1/2 m/s
(c) 0 m/s
(d) 2 m/s

Solution:

Instantaneous velocity is the slope of the line at that point

12. Which of the following graphs could represent the motion of an object moving with a constant speed in a straight line?



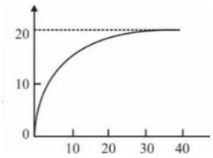
(a) Graph I only
(b) Graphs II and V only
(c) Graph II only
(d) Graphs I and IV only

Solution:

Constant speed is a constant slope on a position-time graph, a horizontal line on a velocity time graph or a zero value on an acceleration-time graph

13. The dispalacement of a particle as a function

of time is shown in the figure, the figure shows that



(a) The particle starts with certain velocity, but the motion is retarded and finally the particle stops
(b) The velocity of the particle is constant throughout.
(c) The acceleration of the particle is constant throughout.
(d) The particle starts with constant velocity, then motion is accelerated and finally the particle moves with another constant velocity.

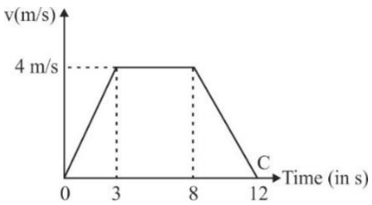
Solution:

Correct option is A

Initially the slope is decreasing and the slope = 0.

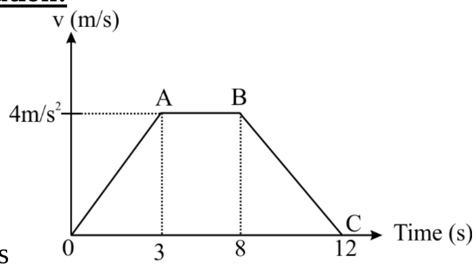
Thus velocity is decreasing as time increases, i.e. there is retardation and ultimately the particle stops. It must have obviously started with some velocity.

14. From the velocity – time graph given of a particle moving in a straight line, one can conclude that



(a) Its average velocity during the 12 second interval is 24/7 m/s
(b) Its velocity for the first 3 seconds is uniform and is equal to 4 m/s
(c) The body has a constant velocity between t = 3s and t = 8s
(d) The body has a uniform velocity from t = 8s to t = 12s

Solution:



s

From the given graph

-> The body has a constant acceleration between t = 0 and t = 3s

-> The body has a constant acceleration of zero between t = 3s and t = 8s

The body has constant retardation between t = 8s to t = 12

TO calculate average velocity, we need

$V_{avg} = \frac{displacement}{total\ time}$

Displacement can be calculated by calculating area of v −t graph.

Displacement = $\frac{1}{2}\ x\ 3\ x\ 4^2 + \frac{1}{2}\ x\ 4\ x\ 4^2 + 5\ x\ 4$

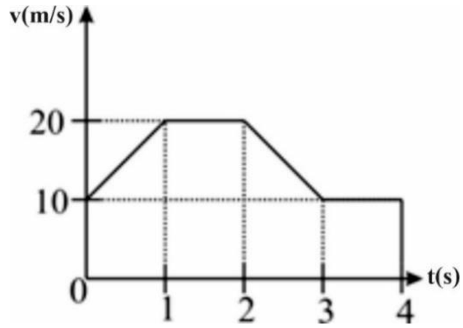
= 20 + 6 + 8 = 34m

Total time = 12s

∴ Avg velocity = $\frac{34}{12}$ = $\frac{17}{6}$m/s

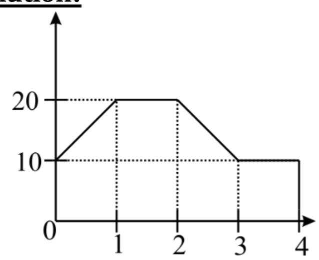
∴ Correct option is (c)

15. Figure given below shows the variation of velocity with time for a particle moving along a straight line, the average velocity, during the entire motion is



(a) 15 m/s **(b) 7.5 m/s**
(c) 6.25 m/s **(d) 13.75 m/s**

Solution:



For avg. velocity,

$V_{avg} = \frac{displacement}{total\ time}$

We can calculate displacement by calculating area of $\vec{V}/t$ graph

Displacement = 10 + $\frac{1}{2}$ x 10 x 1 + 20 x 1 + $\frac{1}{2}$ x 10 x 1 + 10 + 10

= 20 + 30 + 10

= 60 m

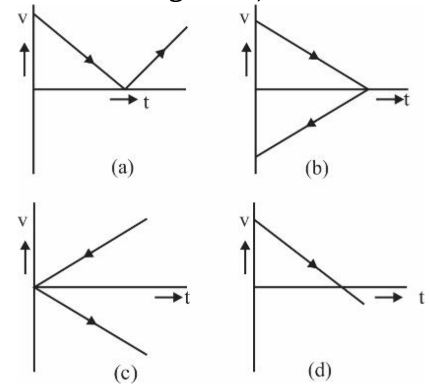
Total time = 4s

$\therefore V_{avg} = \frac{60}{4}$ = 15 m/s

$\therefore$ Option (a)

Expert **Graphs in Motion**

1. A ball is thrown vertically upwards, which of the following graph/graphs represents velocity-time graph of the ball during its flight (air resistance is neglected)

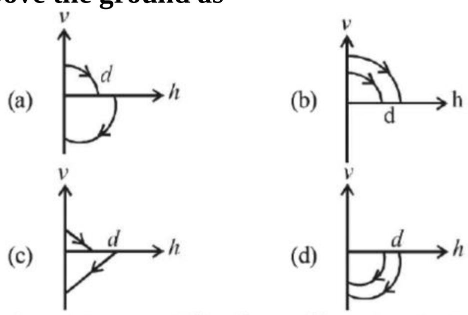


(a) a **(b) b**
(c) c **(d) d**

Solution:

In the positive region the velocity decreases linearly (during rise) and in the negative region velocity increases linearly (during fall) and the direction is opposite to each other during rise and fall, hence fall is shown in the negative region. Hence, the correct option is (D).

2. A ball is dropped vertically from a height d above the ground. It hits the ground and bounces up vertically to a height d/2. Neglecting subsequent motion and air resistance, its velocity v varies with the height h above the ground as



Solution:

Correct option is A

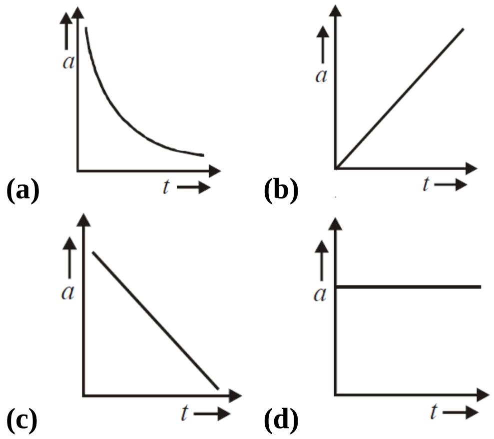
Using the equation $v^2 - u^2 = 2as$

We get v^2 = 2as Since u = 0

Clearly the graph will be parabolic.

The velocity will be negative while the ball falls while it will be positive when it bounces back i.e. they will have opposite direction, also the maximum height will be $\frac{d}{2}$

3. The distance travelled by a body moving along a line in time t is proportional to t^3.
The acceleration-time (a, t) graph for the motion of the body will be



Solution:

(b) Distance along a line i.e., displacement (s)

= t3 (∵ s ∝ t3 given)

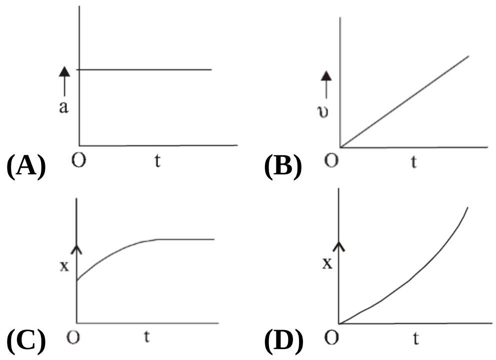
By double differentiation of displacement, we get acceleration,

$V = \frac{ds}{dt} = \frac{dt^3}{dt} = 3t^2$ and $a = \frac{dv}{dt} = \frac{d3t^2}{dt} = 6t$

a = 6t or a ∝ t

Hence graph (b) is correct.

4. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represents the motion qualitatively (a = acceleration, v = velocity, x = displacement, t = time)



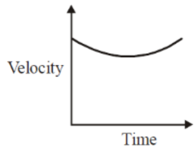
(a) (B), (C) **(b) (A)**
(c) (A), (B), (C) **(d) (A), (B), (D)**

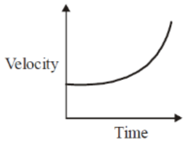
Solution:

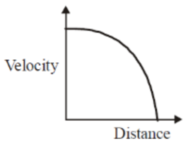
(d) For constant acceleration, there is straight line parallel to t-axis on $\vec{a}-t$.

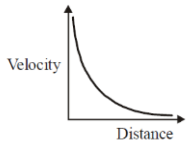
Inclined straight line on $\vec{v}-t$, and parabola on $\vec{x}-t$.

5. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity ?

(a) 

(b) 

(c) 

(d) 

Solution:

(c) According to question, object is moving with constant negative acceleration i.e., a = – constant (C)

$$\frac{vdv}{dx} = -C$$

vdv = – Cdx

$$\frac{v^2}{2} = -Cx + k$$

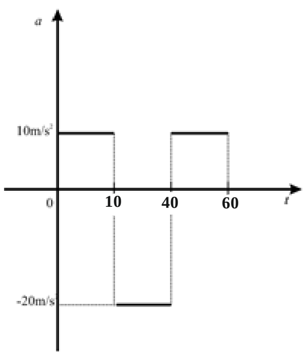
$$x = -\frac{v^2}{2C} + \frac{k}{C}$$

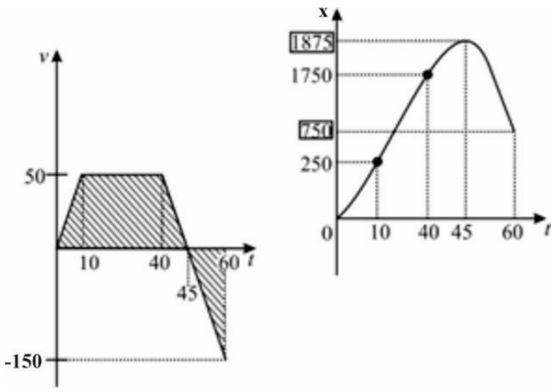
Hence, graph (3) represents correctly.

Pro **Graphs in Motion**

1. A body starts from rest with an acceleration 5 m/s^2 for 10 second. Then it moves with constant speed for next 30 second. After that it experiences a constant retardation of 10 m/s^2 for the next 20 seconds. Plot x-t, v-t & a-t graphs.

Solution:





2. A ball is dropped from a height of 19.6 m above the ground. It rebounds from the ground and raises itself up to the same height. Take the starting point as the origin and vertically downward as the positive x-axis. Draw approximate plots of x versus t, v versus t and a versus t. Neglect the small interval during which the ball was in contact with the ground.

Solution:

Let take downward as positive a(t) = 0

h = $\frac{1}{2}gt^2$

h = 19.6 m

g = 9.8 m

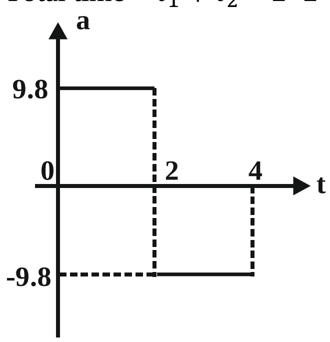
=> $t_1 = \sqrt{\frac{2h}{g}}$ (t_1= time of descent)

= $\sqrt{\frac{2(19.16)}{9.8}}$ = 2 sec

Time of ascent (t_2) = time of descent (t_1)

∴ t_2=2 sec

Total time = $t_1 + t_2$ = 2+2 = 4sec



v = gt

$0 \le t < 2$

For ascent u_{int} = -v (2)

= -g(2)

u = 19.6ms^{-1}

v = u + g(t-2)

= 19.6 + g (t-2);

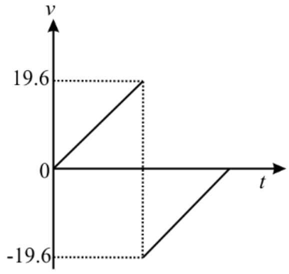
v = gt;

= 19.6 + g(t – 2);

$2 < t < 4$

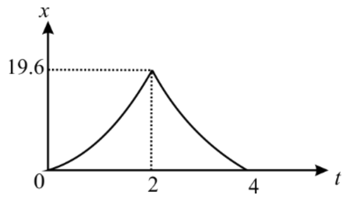
$0 \le t \le 2$

$2 < t \le 4$



$S = \frac{1}{2}gt^2$ $\quad 0 \le t < 2$

The motion of ball is symmetric about t =2 i.e., if we make t = 4 as initial point, and reverse the motion of time ascent looks like decent.

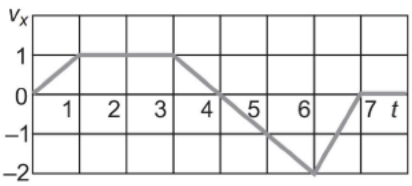


Graph is symmetric about 2

$S = \frac{1}{2}gt^2$ $\quad 0 \le t < 2$

$\frac{1}{2}g(4-t)^2$ $\quad 2 < t \le 4$

3. A point travels along the x axis with a velocity whose projection v_x is presented as a function of time by the plot in Fig. Assuming the coordinate of the point x = 0 at the moment t = 0, draw the approximate time dependence plots for the acceleration w_x, the x coordinate, and the distance covered s.



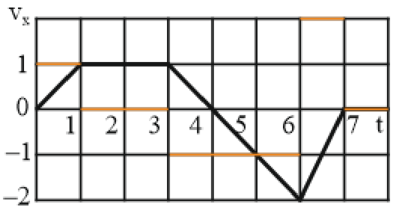
Solution:

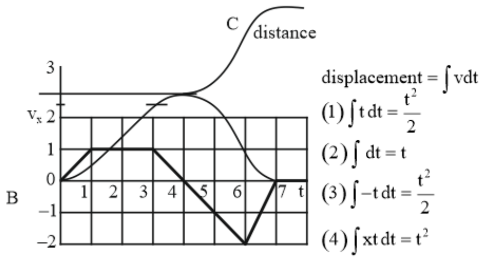
Acceleration : slope of v-t

Displacement: Area under v – t

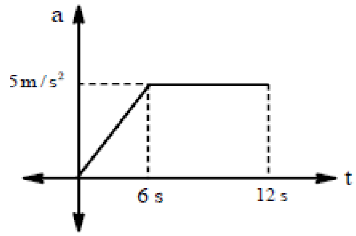
Distance: Magnitude of Area under v-

(a) Acceleration:





4. Given below is an a – t graph, the particle starts from rest at origin. Find displacement at t =12s.



<u>Solution:</u>

0-6 second

$a = \frac{5}{6}t$

$\frac{dv}{dt} = \frac{5}{6}t$

$\int_0^v dv = \int_0^t \frac{5}{6}t\,dt$

$v = \frac{5}{6} x \frac{1}{2} x t^2 = \frac{5}{12}t^2$

$v_{t=0} = \frac{5}{12} x\ 6\ x 6$ = 15m/s

$\frac{ds}{dt} = \frac{5}{12}t^2$

$\int_0^S ds$ = s = $\frac{5}{12} x \frac{1}{3} x\ t^3$

$s = \frac{5}{36}t^3$

$s_{t=0} = \frac{5}{36} x\ 6\ x\ 6\ x\ 6$

= 30m

u = 15 m/s

a = 5 m/s^2

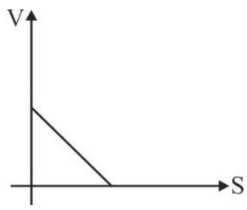
s = (15 x 16) + $\frac{1}{2}\ x\ 5\ x\ 6^2$

= 180 m

Total displacement = 30 +180

= 210 m

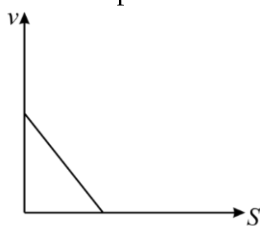
5. Velocity-displacement graph of a particle moving in a straight line is as shown in the figure.



(a) Magnitude of acceleration of particle is decreasing
(b) Magnitude of acceleration of particle is increasing
(c) Acceleration versus displacement graph is straight line
(d) Acceleration versus displacement graph is parabola

Solution:

Correct options are A & C.



Acceleration = v. $\frac{dv}{ds}$

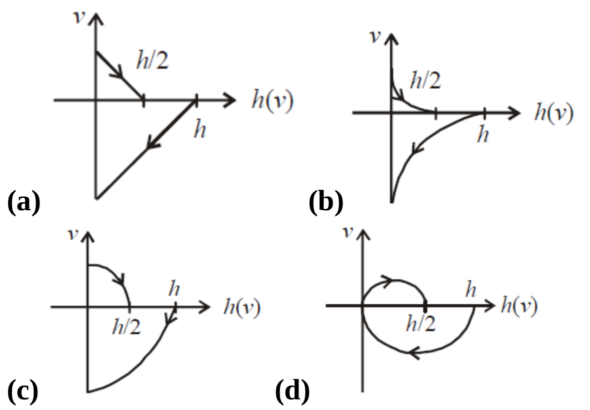
$\because \frac{dv}{ds}$ is negative,

acceleration is negative.

Acceleration versus displacement graph will be a straight line.

∴ Option (a) and (c)

6. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by:(graph are drawn schematically and on not to scale)

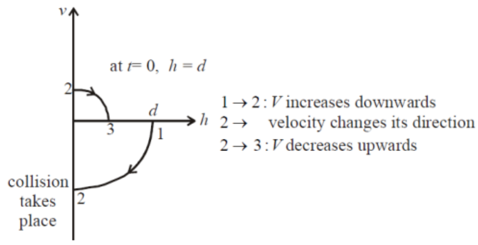


Solution:

(c) For uniformly accelerated/deaccelerated motion :

$v^2 = u^2 \pm 2gh$

As equation is quadratic, so, v-h graph will be a parabola



Initially velocity is downwards (–ve) and then after collision it reverses its direction with lesser magnitude, i.e. velocity is upwards (+ve).

Note that time t = 0 corresponds to the point on the graph where h = d.

Next time collision takes place at 3.

Beginner General Relative Motion 1D

1. Two trains, each 50 *m* long are travelling in opposite direction with velocity 10 *m/s* and 15 *m/s*. The time of crossing is

(a) $2s$ **(b)** $4s$

(c) $2\sqrt{3}s$ **(d)** $t=\frac{v_1-v_2}{a}$

Solution:

Correct option is B.

Total length to be crossed = 50 +50 = 100m

Relative velocity = 10 +15 = 25m/s

Time taken = $\frac{100}{25}$ = 4s

2. Two cars *A* and *B* at rest at same point initially. If *A* starts with uniform velocity of 40 *m/sec* and *B* starts in the same direction with constant acceleration of $4m/s^2$, then *B* will catch *A* after how much time?

(a) 10 *sec* **(b) 20 *sec***
(c) 30 *sec* **(d) 35 *sec***

Solution:

Correct option is B.

Let A and B meet after time t sec. It means distance travelled by both will be equal.

$S_A = ut = 40t$ and $S_B = \frac{1}{2}at^2 = \frac{1}{2}\ x\ 4\ x\ t^2$

$S_A = S_B$=> 40t = $\frac{1}{2}4t^2$ => t = 20sec.

3. A body *A* moves with a uniform acceleration a and zero initial velocity. Another body *B* starts from the same point moves in the same direction with a constant velocity v. The two bodies meet after a time t. The value of t is

(a) $\frac{2v}{a}$ **(b)** $\frac{v}{a}$

(c) $\frac{v}{2a}$ **(d)** $\sqrt{\frac{v}{2a}}$

Solution:

Correct option is A.

$S_B = vt$ Position of B

$S_A = ut + \frac{1}{2}at^2 = \frac{1}{2}at^2$ Portion of A

When they meet

$S_A = S_B$

$vt = \frac{1}{2}at^2$

$t = \frac{2v}{a}$

4. A body *A* is projected upwards with a velocity of $98\,m/s$. A second body *B* is projected upwards with the same initial velocity, but after 4 *sec* of projection of body A. Both the bodies will meet after

(a) 6 sec **(b) 8 sec**

(c) 10 sec **(d) 12 sec**

Solution:

Correct option is D.

Let t be the time of flight of the body A when they meet.

Then the time of flight of the body B will be (t-4) seconds.

Since, the displacement of both the bodies from the ground will be the same,

So, $h_1 = h_2$ where h_1 = displacement of body A and h_2 = displacement of body B.

Using equation of motion,

$h = 98t + \frac{1}{2}gt^2,$

$h_1 = 98t - \frac{1}{2}gt^2$ and

$h_2 = 98(t-4) - \frac{1}{2}g(t-4)^2$

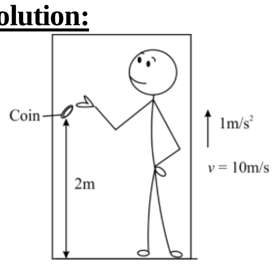
and since $h_1 = h_2$

$\therefore 98t - \frac{1}{2}gt^2 = 98(t-4) - \frac{1}{2}g(t-4)^2$

On solving, we get t = 12 seconds.10.

5. An elevator, in which a man is standing, is moving upward with a constant acceleration of 1 m/s^2. At some instant when speed of elevator is 10 m/s, the man drops a coin from a height of 2 m. Find the time taken by the coin to reach the floor. (g = 9.8 m/s^2)

Solution:



$a_{rel} = 1 + 10 = 11 m/s^2$

$u_{rel} = 0$

$d_{rel} = 2m$

$d_{rel} = \frac{1}{2}a_{rel}t^2$

$2 = \frac{1}{2} x\ 11\ x\ t^2$

$t = \sqrt{\frac{4}{11}}sec$ = 0.61 sec

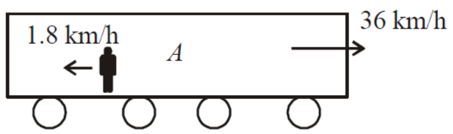
6. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour and 72 km hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(a) 29.5 ms^{-1} **(b) 28.5 ms^{-1}**

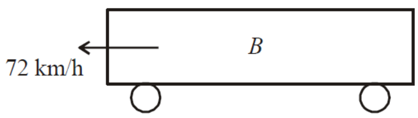
(c) 31.5 ms^{-1q} **(d) 30.5 ms^{-1}**

Solution:

(a) According to question, train A and B are running on parallel tracks in the opposite direction.



VA = 36 km/h = 10 m/s



V_B = – 72 km/h = –20 m/s

V_{MA} = – 1.8 km/h = – 0.5 m/s

$V_{man,\,B} = V_{man,\,A} + V_{A,\,B}$

$= V_{man,\,A} + V_A - V_B$ = – 0.5 + 10 – (– 20)

= – 0.5 + 30 = 29.5 m/s.

Expert **General Relative Motion 1D**

1. An elevator car, whose floor to ceiling distance is equal to 2.7 *m*, starts ascending with constant acceleration of 1.2*m/s*^{2}. 2 *sec* after the start, a bolt begins falling from the ceiling of the car. The free fall time of the bolt is

(a) $\sqrt{0.54}\,s$ **(b)** $\sqrt{6}\,s$

(c) 0.7 *s* **(d) 1 *s***

Solution:

Correct option is C.

Let the total time of free fall be t

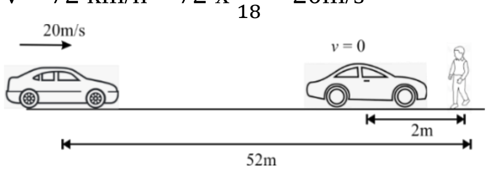
Then at t = 2s

$v_B = 2.4m/s$ $v_C = 2.4m/s$
$a_C = 1.2\ m/s^2$ $a_B = -10\ m/s^2$
wrt car.
Distance travelled by bolt = 2.7m
Then
-2.7m = $(v_{B/C})t + a_{B/C}\left(\frac{t^2}{2}\right)$
=> -2.7 = (2.4 – 2.4)t $\left(\frac{-10-1.2}{2}\right)t^2$
=> 2.7 = $\frac{11.2}{2}t^2$
=> $t^2 = \left(\frac{2.7}{11.2}x2\right)$ = $0.482s^2$
Note: $v_B = v_C$ till t = 2 s before the bolt starts to fall.
$v_B = v_C = a_C t = (1.2)(2)$ = 2.4 m/s
Let the displacement of the bolt be h. Then,
h = v_B(t) + $a_{B/2}t^2$
h = (2.4)$\sqrt{0.482}\,\frac{-(10)}{2}$(0.482)
h = -0.743
h = ≈ - 0.7 m
$|h| \approx 0.7\ m$

2. A motorist while driving at a speed of 72 km/hr sees a boy standing on the road at a distance of 52 m. He applies the brake and stops his car at a distance of 2 m from the boy. Find the acceleration caused due to the application of brake and time taken to stop the car.

Solution:

v = 72 km/h = 72 x $\frac{5}{18}$ = 20m/s



d = 50m
$v^2 - u^2 = 2as$
0 - 20^2 = 2(a)(50)
a = $\frac{20\ x\ 20}{100} = 4m/s^2$
t = $\frac{v-u}{a}$ = $\frac{0-20}{-4}$ = 5 sec

3. A student is standing at a distance of 50 metres from the bus. As soon as the bus begins its motion with an acceleration of 1*ms*$^{-2}$, the student starts running towards the bus with a uniform velocity u. Assuming the motion to be along a straight road, the minimum value of u, so that the student can catch the bus is

(a) 5 *ms*$^{-1}$ **(b) 8 *ms*$^{-1}$**
(c) 10 *ms*$^{-1}$ **(d) 12 *ms*$^{-1}$**

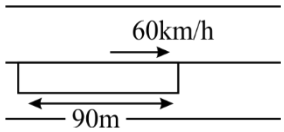
Solution:

Correct option is C.
Acceleration of the bus a = 1 ms^{-2}
Let the distance traveled by bus be x when the student caught it after time t.
Velocity of the student = u
Initial velocity of bus u_1 = 0
$\therefore x = \frac{1}{2}at^2$ => x = $\frac{t^2}{2}$ $(\because a = 1ms^{-2})$
Distance traveled by student = x + 50
$\therefore x + 50 = ut$
$\frac{t^2}{2} + 50 = ut$ => t^2 – 2ut + 100 = 0
The above relation must have real roots i.e its discriminant ≥0
$(2u)^2 - 4\ x\ 100\ \geq 0$
Or
$u^2 - 100\ \geq 0$ => u > 10 ms^{-1}
Thus the minimum velocity of the student must be 10 ms^{-1}

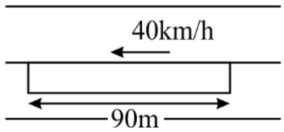
4. Two trains each of length 90 m moving in opposite directions along parallel tracks meet when their speeds are 60 km/hr and 40 km/hr. If their accelerations are 0.3 m/s^2 and 0.15 m/s^2 respectively, Find the time they take to pass each other.

(a) 8 s **(b) 4 s**
(c) 2 s **(d) 6.17 s**

Solution:



60km/h = 60 x $\frac{5}{18} = \frac{50}{3}$ m/s



40km/h = 40 x $\frac{5}{18} = \frac{100}{9}$ m/s
$V_1 = \frac{50}{3}\hat{\imath}$, $V_2 = \frac{-100}{9}\hat{\imath}$, $a_1 = 0.3\hat{\imath}$, $a_2 = 0.15\hat{\imath}$
$V_{rel} = V_1 - V_2$
$= \frac{150}{9} + \frac{100}{9} = \frac{250}{9}$
a_{rel} = 0.3 + 0.15
= 0.45
S_{rel} = 180m = $\frac{250}{9}t + \frac{1}{2}\frac{45}{100}t^2$
180 = $\frac{250}{9}t + \frac{9}{40}t^2$
180 x 9 x 40 = 250 x 40t + 81t^2
64800 = 10000 t + 81 t^2

$81t^2 + 10000t - 64800 = 0$

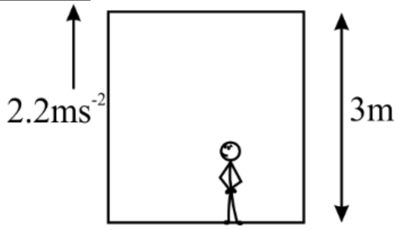
$t = \frac{-10000 \pm \sqrt{10^8 + 20995200}}{162} = \frac{-10000 \pm \sqrt{120995200}}{162}$

$t = \frac{999.8}{162} = 6.17$ Ans: D

5. A man standing in an elevator observes a screw falling from the ceiling. The ceiling is 3m above the floor.(a) If the elevator is moving upward with a speed of 2.2m/s, how long does it take for the screw to hit the floor?

(b) How long is the screw in the air if the elevator starts from rest when the screw falls and moves upward with a constant acceleration of a = 4.0m/s^2?

Solution:

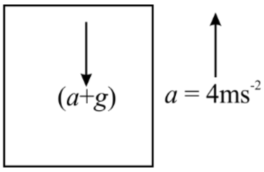


(a) for man standing inside:

$v^2 = ut + \frac{1}{2}at^2$

$3 = \frac{1}{2} x\ gt^2$ => [$t = \sqrt{\frac{3}{10}} = (0.78)sec$]

(b)



A = $4ms^{-2}$

$s = ut + \frac{1}{2}at^2$

$3 = \frac{1}{2}(g + a)t^2$

$t = \sqrt{\frac{6}{14}} = (0.69)sec$

6. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1}. If the muzzle speed of the bullet is 150 m/s. At what speed does the bullet hits the thief's car?

Solution:

Step 1: Velocity of bullet w.r.t ground [Refer Fig.]

Speed of Cars:

Police Car: $v_1 = 30\ x \frac{\frac{5}{18}m}{s} = \frac{25}{3}\ m/s$

Thief's Car: $v_2 = 190\ x\ \frac{5}{18} m/s = \frac{160}{3} m/s$

Concept of Muzzle Speed:

Muzzle speed is the speed of bullet with respect to the gum, just after firing. Here the speed of gun is same as that of Police car and bullet is fired in the same direction as the car.

$\therefore V_{muzzle} = V_{b/1}$

=> $V_b = V_{muzzle} + V_1$

$= 150$ m/s + $\frac{25}{3}$m/s

$= \frac{475}{3}$ m/s

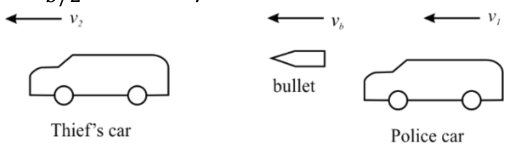
Step 2: Velocity of bullet w.r.t. thief's car

Velocity of bullet w.r.t. thief's car

$V_{b/2} = V_b - V_2$

$= \frac{475}{3} - \frac{160}{3}$ m/s

$\therefore V_{b/2} = 105 m/s$



Pro General Relative Motion 1D

1. A train of 150 m length is going towards north direction at a speed of $10 m/\sec$**. A parrot flies at the speed of** $5\ m/\sec$ **towards south direction parallel to the railway track. The time taken by the parrot to cross the train is**

(a) 12 *sec* **(b) 8 *sec***

(c) 15 *sec* **(d) 10 *sec***

Solution:

Correct option is D.

Relative velocity of the parrot w.r.t. the train

[10-(-5)] = (10-(-5)) m/s = 15 m/s

Time taken by the parrot to cross the train

$= \frac{150}{15} = 10$s.

2. Two trains travelling on the same track are approaching each other with equal speeds of 40 *m/s*. The drivers of the trains begin to decelerate simultaneously when they are just 2.0 *km* apart. Assuming the decelerations to be uniform and equal, the value of the deceleration to barely avoid collision should be

(a) 11.8 m/s^2 **(b) 11.0** m/s^2

(c) 2.1 m/s^2 **(d) 0.8** m/s^2

Solution:

Correct option is D.

Both trains will travel a distance of 1 km to come in rest.

In this case by using $v^2 = u^2 + 2as$

=> $0 = (40)^2 + 2a \times 1000 => a = -0.8 m/s^2$

3. A motorboat going downstream overcame a raft at a point A; τ = 60 min later it turned back and after some time passed the raft at a distance ℓ = 6.0 km from the point A. Find the flow velocity assuming the duty of the engine to be constant.

Solution:

∵ Raft is floating on river:

$v_{river} = v_{raft}$

Analysing w.r.t raft/river: which means raft/river are at rest.

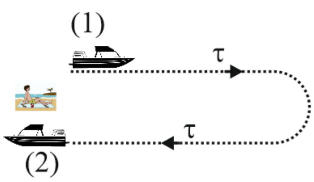
⇓

Boat travels to and fro with same velocity, just like a car on a stationary road

⇓

$t_{forward} = t_{backward} = \tau$

w.r.t raft



Now time varies equally in all reference frames.

∴ time between events 1 and 2 = time taken by raft to move distance

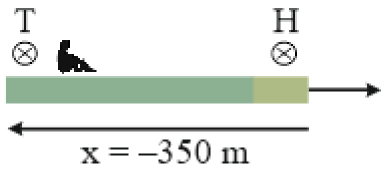
$\Rightarrow 2\tau = \dfrac{\ell}{v_{raft}}$

$\Rightarrow v_{raft} = \dfrac{\ell}{2\tau} = v_{river}$

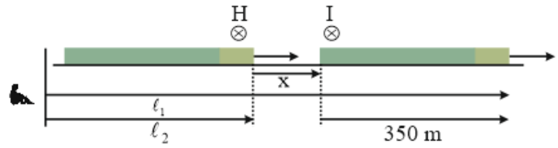
4. A train of length ℓ = 350 m starts moving rectilinearly with constant acceleration w = 3.0 × 10^{-2} ms^{-2}; t = 30 s after the start the locomotive headlight is switched on (event 1), and τ = 60 s after that event the tail signal light is switched on (event 2). Find the distance between these events in the reference frames fixed to the train and the Earth. How and at what constant velocity V relative to the Earth must a certain reference frame K move for the two events to occur in it at the same point?

Solution:

(a) w.r.t train



(b) w.r.t ground



$x = \ell_1 - \ell_2 - 350$

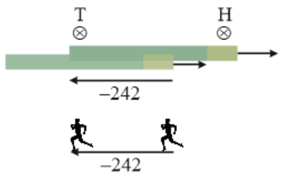
$= \frac{1}{2}(0.03)90^2 - \frac{1}{2}(0.03)30^2 - 350$

$= -242m$

-ve sign means overlap!

i.e., the train has not even travelled its own length yet.

⇓



c)For both events occurring at the same place to observer….

Observer's change in position = event's change in position.

$\therefore \vec{r}_{obsv} = -242$

$= -242/60 = -4m/s$

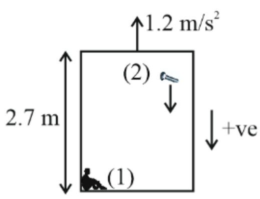
5. An elevator car whose floor-to-ceiling distance is equal to 2.7 m starts ascending with constant accelerating 1.2 ms^{-2}; 2.0 s after the start a bolt begins falling from the ceiling of the car. Find:

(a) The bolt's free fall time;

(b) The displacement and the distance covered by the bolt during the free fall in the reference frame fixed to the elevator shaft.

Solution:

(a) w.r.t elevator:



$$\vec{s}_{21} = \vec{u}_{21} + \frac{1}{2}\vec{a}_{21}t^2$$

$$+2.7 = 0 + \frac{1}{2}(9.8 - (1.2))t^2$$

$$\Rightarrow t = \sqrt{\frac{2 \times 2.7}{11}} = 0.7s$$

(b)

$$v_0 = 1.2 x 2 = 2.4 m/s$$

$$\Delta y = (2.4)0.7 + \frac{1}{2}(-9.8)(0.7)^2 = -0.7m$$

$$s = |\Delta y| + 2\left(\frac{v_0^2}{2g}\right) = 0.7 + \frac{2.4^2}{9.8} = 1.3m$$

6. A police jeep is chasing a thief with a velocity of 45 km/h, thief in another jeep is moving with velocity 153 km/h. Police fires a bullet with muzzle velocity of 180 m/s. The velocity with which it will strike the car of the thief is

(a) 150 m/s (b) 27 m/s

(c) 450 m/s (d) 250 m/s

Solution:

Correct option is A.

Effective speed of the bullet = speed of bullet + speed of police jeep

= 180 m/s + 45 km/h

= (180 + 12.5) m/s

= 192.5 m/s

Speed of thief's jeep = 153 km/h = 42.5 m/s

Velocity of bullet w.r.t. thief's car

= 192.5 - 42.5 = 150 m/s

7. Two trains each having a speed of 30 km/h are headed at each other on the same straight track. A bird that can fly at 60 km/h flies off from one train when they are 60 km apart and heads directly for the other train. On reaching the other train it flies directly back to the first, and so forth

(a) What is the total distance the bird travels?

(b) How many trips can the bird make from one train to the other before they crash?

Solution:

From the situation,

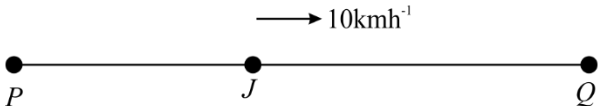
Recognizing the gap between the trains is closing at a constant rate of 60km/h, the total time that elapses before they crash is t = $\frac{60km}{60km/h}$ = 1.0 h.

During this time, the bird travels a distance of x = vt = (60km/h) (1.0h) = 60km.

8. Consider two cities P and Q between which consistent bus service is available in both directions every x minutes. A morning jogger is jogging towards Q from P with a speed of 10km/h. Every 18 mins a bus crosses this jogger in its own direction of motion and every 6 minutes another bus crosses in opposite direction. What is the time period between two consecutive buses and also find the speed of buses?

Solution:

The equations of motion are applicable in any frame of reference. Attaching the frame of reference at jogger, the velocity of bus is now required w.r.t. jogger. Suppose speed of buses = v kmh^{-1}



Distance between two buses on road: s = vx. The relative velocity of bus w.r.t. the jogger, for the buses moving from P to Q = (v – 10) kmh^1.

So we have

18 = $\frac{vx}{v-10}$ => vx = 18v – 180 (i)

Similarly for the buses moving from P to Q relative velocity = (v+10)kmh^{-1}. So we have

6 = $\frac{vx}{v \mp 10}$ => vx = 6 + 60 ……….. (ii)

Solving (i) and (ii) we find v = 20 kmh^{-1} and x = 9min.

9. Two bodies start moving in straight line simultaneously from point O. The first body moves with constant velocity of 40 m/s & and the second body starts from rest with a constant acceleration of 4 m/s^2

(a) Time that elapses before the second body catches up with the first body is 20s.

(b) Greatest distance between the two bodies prior to their meeting is 200m.

(c) Time elapsed when the distance between them is maximum is 10s.

(d) All above statements are false.

Solution:

Let two bodies A & B have initial velocity.

$u_A = 40m/s$

$u_B = 0m/s$

Acceleration of these bodies

$a_A = 0\ m/s^2$ & $a_B = 4m/s^2$

Distance equation for these bodies

$S_A = u_A t$

$S_B = \frac{1}{2}a_B t^2$

∴ When $S_A = S_B$,

$u_A t = \frac{a_B}{2} t_1^2$

$\frac{40 \ x \ 2}{4} = t$

t = 20s

∴ Option (a) is correct.

Distance between the 2 bodies is given by

$\Delta s = s_A - s_B = u_A t - \frac{a_B t^2}{2}$

For Δs = maximum,

$\frac{d(\Delta s)}{dt} = u_A - a_B t = 0$

$u_A = a_B t$

40 = 4t

=> t = 10s

∴ Option (c) is correct

=> $\Delta s_{max} = 40(10) - \frac{1}{2}(4)(10)^2$

= 400 – 200

= 200 m

∴ Option (b) is correct

∴ (a), (b) & (c) are correct.

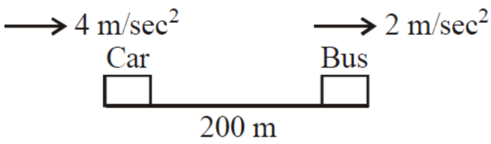
10. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m/s^2 and the car has acceleration 4 m/s^2. The car will catch up with the bus after a time of:

(a) $\sqrt{110}s$ **(b)** $\sqrt{120}s$

(c) $10\sqrt{2}s$ **(d) 15 s**

Solution:

(c)



Given, $u_C = u_B = 0$, $a_C = 4$ m/s^2, $a_B = 2$m/s^2

Hence relative acceleration, $a_{CB} = 2$ m/sec^2

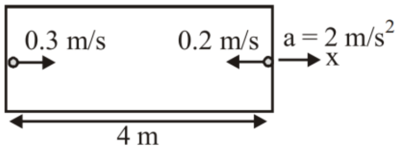
Now, we know, $s = ut + \frac{1}{2}at^2$

$200 = \frac{1}{2} \times 2t^2$ $\because$ u = 0

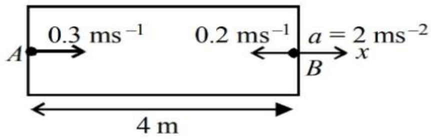
Hence, the car will catch up with the bus after time

$t = 10\sqrt{2}$ second

11. A rocket is moving in a gravity free space with a constant acceleration of 2ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end the chamber in + x direction with a speed of 0.3m/s related to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2m/s from its right end relative to the rocket. The time in seconds when the two balls hit each other is



Solution:



For ball A

$u_1 = 0.3$m/s, $a_1 = -2$ms^{-2}, $s_1 = x$, $t_1 = t$

∴ $s_1 = u_1 t_1 + \frac{1}{2} u_1 t_1^2$

x = 0.3t - t^2 ....(1)

For ball B

$u_2 = 0.2$m/s, $a_2 = 2$ms^{-2}, $s_2 = 4 - x$, $t_2 = t$

∴ $s_2 = u_2 t_2 + \frac{1}{2} u_2 t_2^2$

4 - x = 0.2t - t^2 ....(2)

Adding eq. (1) and (2)

4 = 0.5t ∴ t = $\frac{4}{0.5}$ = 8s.

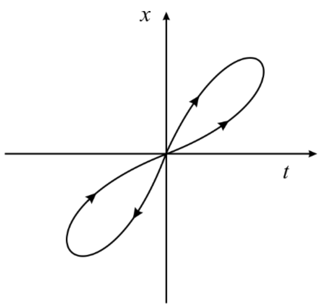
Test Yourself

Beginner Test - I

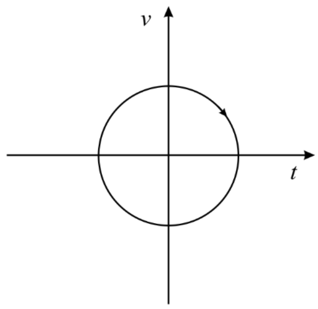
1. What is the condition for an object to be considered as a point object?
2. Does the displacement of an object depend on the choice of the position of origin of the coordinate system?
3. Draw position-time graph for a body at rest.
4. The displacement-time graph for two particles X and Y are straight lines making angles of 30° and 60° with the time axis. What is the radio of the velocities of Y and X?
5. Speed of a particle cannot be negative. Why?
6. The position coordinate of a moving particle is given by $x = 6 + 18t + 9t^2$, where x is in meters and t in seconds. What is the velocity at $t = 2s$?
   We know that, velocity is rate of change of displacement i.e. $v = \frac{dx}{dt}$
7. For which condition, the magnitude of average velocity is equal to the average speed for a particular motion?
8. Draw position-time graph for a non-uniform motion when body starts from origin with increasing velocity.
9. From the given example. Find if the motion is one or two or three-dimensional.
   (i). A kite flying in the sky
   (ii). A cricket ball hit by a player
   (iii). Moon revolving around the earth and
   (iv). The motion of a stone in a circle
10. A drunkard walking in a narrow lane takes 5 steps forwards and 3 steps backward, followed again 5 steps forwards and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Determine how long the drunkard takes to fall in a pit 13 m away from that start.
11. Explain how an object could have zero average velocity but non-zero average speed?
12. If the displacement of a body is zero, is distance necessarily zero? Answer with one example.
13. The data regarding the motion of the two different object P and Q are given in the following table. Examine them carefully and state whether the motion of the objects is uniform or non-uniform.

Time	**Distance travelled by object *P* (in m)**	**Distance travelled by object Q (in m)**
9:30am	**10**	**12**
9:46am	**20**	**19**
10:00am	**30**	**23**
10:15am	**40**	**35**
10:30am	**50**	**37**
10:45am	**60**	**41**
11:00am	**70**	**44**

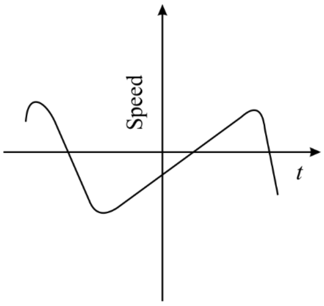
14. A body travels with a velocity v_1 for time t_1 and with a velocity v_2 for time t_2 find the average velocity of the body for the total time.
15. Is earth inertial or non-inertial frame of reference?
16. Look at the graph (a) to (d) carefully and state with reasons, which of these cannot possibly represent one-dimensional motion of a particle?



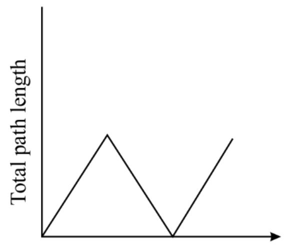
(a)



(b)



(c)

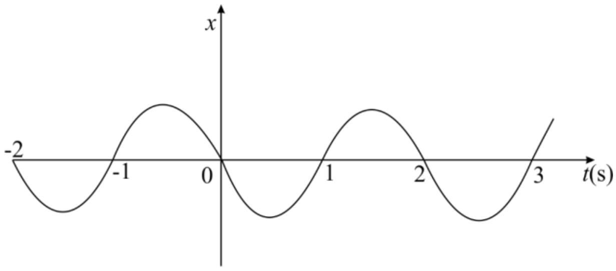


(d)

17. The position of an object moving along *x*-axis is given by $x = a + bt^2$, where $a = 8.5m$, $b = 2.5m$ and *t* is measured in seconds. What is its velocity at $t = 0\ s$ and $t = 2.0\ s$? What is the average velocity between $t = 0\ s$ and $= 4.0\ s$?
18. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km/h. Finding the market closed, he instantly turns and walks back with a speed of 7.5 km/h. What is the (i) magnitude of average velocity and (ii) average speed of the man, over interval of time (a) 0 to 30 min (b) 0 to 50 min (c) 0 to 40 min?
19. A particle moving in a straight line covers half the distance with a speed of 3 m/s. The other half of the distance is covered in two equal intervals of time with speeds of 4.5 m/s and 7.5 m/s, respectively. Find the average speed of the particle during this motion.
20. If position of a particle at instant *t* is given by $x = 2t^3$, find the acceleration on the particle.
21. Give an example of uniformly accelerated linear motion.
22. Draw *v*-*t* graph for non-uniform accelerated motion.
23. A car starts accelerated from rest for sometime, maintains the velocity for sometime and then comes to rest with uniform deceleration. Draw *v*-*t* graph.
24. Find the acceleration and velocity of a ball at the instant it reaches its highest point if it is thrown up with velocity *v*.
25. Two particles *A* and *B* are moving along the same straight line. *B* is ahead of *A*. Velocities remaining unchanged, what would be effect on the magnitude of relative velocity if *A* ahead of *B?*
26. The displacement of a particle is given by at^2. What is dependency of acceleration on time?
27. Point *P, Q* and *R* are in a vertical line such that $PQ = QR$. A ball at *P* is allowed to fall freely. What is the ratio of the times of descent through *PQ* and *QR* ?
28. Which of the following is true for displacement?
    (i). It cannot be zero.
    (ii). Its magnitude is greater than the distance travelled by the object.
29. The velocity of a particle is given by equation $v = 4 + 2\ (C_1 + C_2 t)$ where C_1 and C_2 are constant. Find the initial velocity and acceleration of the particle.
30. At $t = O$, a particle is at rest at origin. Its acceleration is $2\ m/s^2$ for the first 3s and $-2\,m/s^2$ for next 3s. Plot the acceleration *versus* time and velocity *versus* time graph.
31. A police van moving on a highway with a speed of $30\ kmh^{-1}$ fires a bullet at a thief's car speeding \

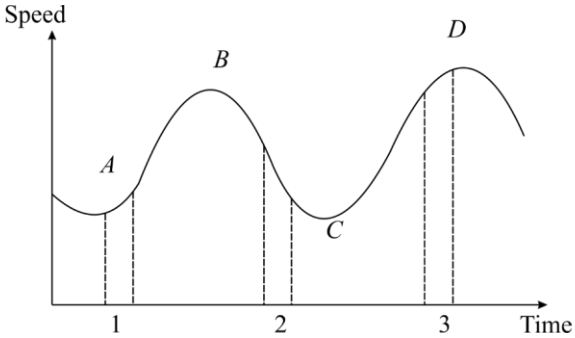
away in the same direction with a speed of 90 $km\ h^{-1}$.
If the muzzle speed of the bullet is 150 ms^{-1} with what speed does the bullet hit the thief's car? (Note, obtain that speed which is relevant for damaging the thief's car?)

32. Figure gives the *x*-*t* plot of a particle executing one-dimensional simple harmonic motion. Give the sign of position, velocity and acceleration variables of the particles at $t = 0.3s,\ 1.2s, -1.2s$.



In SHM, the acceleration $a = -\omega^2 x$ i.e. acceleration is directly proportional to the displacement and opposite in direction.

33. Figure gives a speed-time graph of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion, give the sign of *u* and *a* in the three intervals. What are the accelerations at the points *A*, *B*, *C* and *D*?



The slope of *v*-*t* graph represents the acceleration i.e. higher the slope of *v*-*t* graph, higher the acceleration.

34. Two trains *A* and *B* of length 400 m each are moving on two parallel tracks with a uniform speed of 72 $km\ h^{-1}$ in the same direction with *A* ahead of *B*. The driver of *B* decides to overtake *A* and accelerates by 1 ms^{-2}. If after 50 s, the guard of *B* just brushes past the driver of *A*, what was the original distance between them?

35. A jet plane beginning its take off moves down the runway at a constant acceleration of 4.00 m/s^2.
(i). Find the position and velocity of the plane 5.00 s after it begins to move.
(ii). If a speed of 70.0 m/s is required for the plane to leave the ground, how long a runway is required?

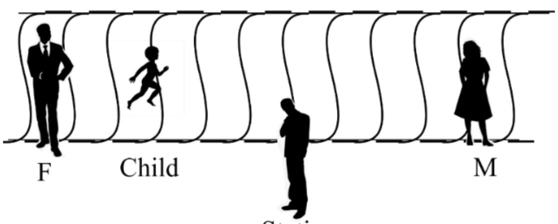
36. A player throws a ball upwards with an initial speed of 29.4 ms^{-1}.
(i). What is the direction of acceleration during the upward motion of the ball?
(ii). What are the velocity and acceleration of the ball at the highest point of its motion?
(iii). Choose $x = 0$ and $t = 0$ be the location and time at its highest point, vertically downward direction to be the positive direction of *x*-axis and give the sign of position, velocity and acceleration of the ball during its upward and downward motion.
(iv). To what height does the ball rise and after how long does the ball return to the player's hands? (Take $g = 9.8\ ms^{-2}$ and neglect air resistance)

37. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between $t = 0$ to 12s. ($g = 10\ ms^{-2}$)

38. State which of the following situations are possible and give an example for each of these?
(i). An object with a constant acceleration but with zero velocity.

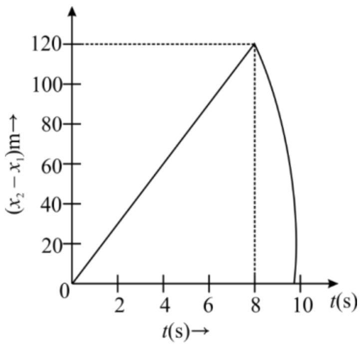
(ii). An object moving in a certain direction with acceleration in the perpendicular direction.

39. On a long horizontal moving belt (as shown in figure), a child runs to and fro with a speed 9 kmh^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 kmh^{-1}. For an observer on a stationary platform outside, what is the
(i). Speed of child running in the direction of motion of the belt?
(ii). Speed of the child running opposite to the direction of motion of the belt?
(iii). Time taken by the child in (i) and (ii) to cover the distance of 50m?
Which of the answer will alter if motion is viewed by one of the parents?



40. A train passes a station A at 40 kmh^{-1} and maintains its speed for 7 km and is then uniformly retarded, stopping at B which is 8.5 km from A. A second train starts from A at the instant the first train passes and being accelerated some part of the journey and uniformly retarded for the rest, stops at B at the same times as the first train. Calculate the maximum speed of the second train, use only the graphical method.

41. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 ms^{-1} and 30 ms^{-1}. Verify that the graph shown in figure, correctly represents the time variation of the relative position of the second stone with respect to the first.
Neglect the air resistance and assume that the stones do not rebound after hitting the ground. Take, $g = 10\ ms^{-1}$. Give the equations for the linear and curved part of the plot.



42. A motor car moving at a speed of 72 km/h cannot come to a stop is less than 3.0 s while for a truck time interval is 5.0 s. On a highway, the car is behind the truck both moving at 72 km/h. The truck gives a signal that it is going to stop at emergency. At what distance the car should be from the truck so that it does not bumb onto (collide with) the truck? Human response time is 0.5 s.

43. A ball is thrown upward with an initial velocity of 100 m/s. After how much time will it return? Draw velocity-time graph for the ball and find from the graph.
(i). Maximum height attained by ball and
(ii). Height of the ball after 15 s. Take, $g = 10\ ms^{-2}$

44. If a body moving with uniform acceleration in straight line describes successive equal distance in time interval t_1, t_2 and t_3, then show that

$$\frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3} = \frac{3}{t_1 + t_2 + t_3}$$

45. Can a body have zero velocity and finite acceleration?

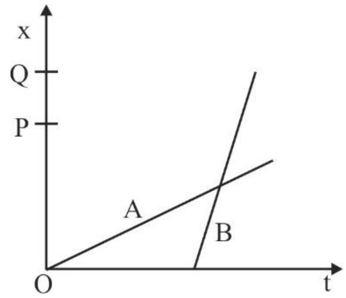
46. Plot a graph of velocity-time, for the condition if as object is moving with increasing acceleration and having zero initial velocity.
47. Give two examples for the formula $V_{AB} = V_A - V_B$ where the symbols have their usual meaning.

Answer Key

Refer. Solutions

Beginner Test - II

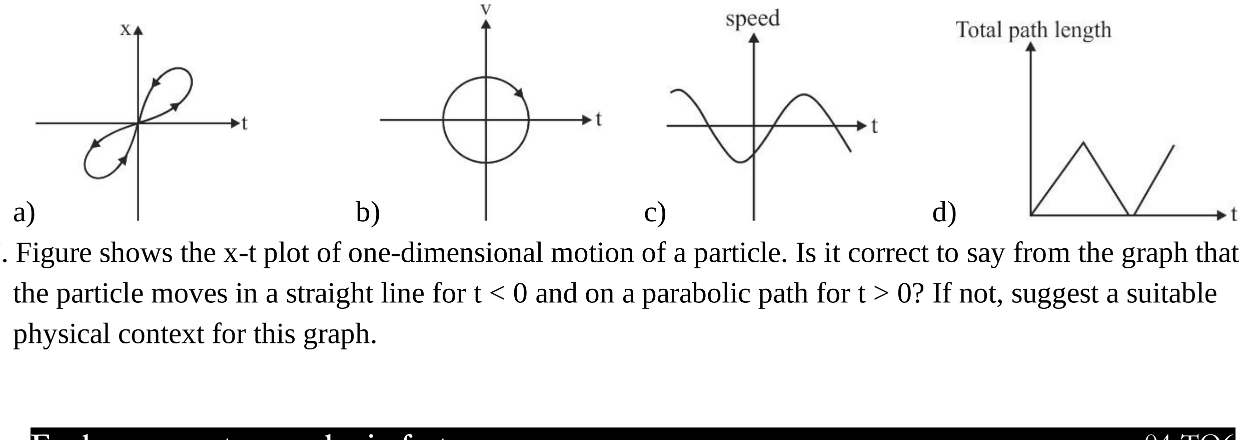
1. In which of the following examples of motion, can the body be considered approximately a point object:
   (a) A railway carriage moving without jerks between two stations.
   (b) A monkey sitting on top of a man cycling smoothly on a circular track.
   (c) A spinning cricket ball that turns sharply on hitting the ground.
   (d) A tumbling beaker that has slipped off the edge of a table.
2. The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in fig. Choose the correct entries in the brackets below



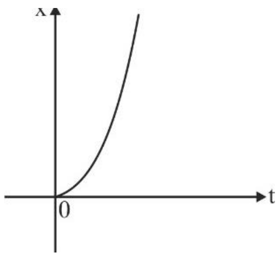
   (a) (A/B) lives closer to the school than (B/A)
   (b) (A/B) starts from the school earlier than (B/A)
   (c) (A/B) walks faster than (B/A)
   (d) A and B reach home at the (same/different) time e) (A/B) overtakes (B/A) on the road (once/twice).
3. A woman starts from her home at 9.00 am, walks with a speed of 5 km h^{-1} on a straight road up to her office 2.5 km away, stays at the office up to 5.00 pm, and returns home by an auto with a speed of 25 km h^{-1} . Choose suitable scales and plot the x-t graph of her motion.
4. A drunkard walking in a narrow lane takes 5 steps forward and 3 steps backward, followed again by 5 steps forward and 3 steps backward, and so on. Each step is 1 m long and requires 1 s. Plot the x-t graph of his motion. Determine graphically and otherwise how long the drunkard takes to fall in a pit 13 m away from the start.
5. A jet airplane travelling at the speed of 500 km h^{-1} ejects its products of combustion at the speed of 1500 km h^{-1} relative to the jet plane. What is the speed of the latter with respect to an observer on ground?
6. A car moving along a straight highway with a speed of 126 km h^{-1} is brought to a stop within a distance of 200 m. What is the retardation of the car (assumed uniform), and how long does it take for the car to stop?
7. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km h^{-1} in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m/s^2 . If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between them?
8. On a two-lane road, car A is travelling with a speed of 36 km h^{-1} . Two cars B and C approach car A in opposite directions with a speed of 54 km h^{-1} each. At a certain instant, when the distance AB is equal to AC, both being 1 km, B decides to overtake A before C does. What minimum acceleration of car B is required to avoid an accident?
9. Two towns A and B are connected by a regular bus service with a bus leaving in either direction every T

minutes. A man cycling with a speed of 20 km h^{-1} in the direction A to B notices that a bus goes past him every 18 min in the direction of his motion, and every 6 min in the opposite direction. What is the period T of the bus service and with what speed (assumed constant) do the buses ply on the road?

10. A player throws a ball upwards with an initial speed of 29.4 m s^{-1} . What is the direction of acceleration during the upward motion of the ball? What are the velocity and acceleration of the ball at the highest point of its motion?
    Choose the x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis, and give the signs of position, velocity and acceleration of the ball during its upward, and downward motion. To what height does the ball rise and after how long does the ball return to the player's hands? (Take g = 9.8 m s^{-2} and neglect air resistance).
11. Read each statement below carefully and state with reasons and examples, if it is true or false.
    A particle in one-dimensional motion
    (a) with zero speed at an instant may have non-zero acceleration at that instant
    (b) with zero speed may have non-zero velocity,
    (c) with constant speed must have zero acceleration,
    (d) with positive value of acceleration must be speeding up.
12. A ball is dropped from a height of 90 m on a floor. At each collision with the floor, the ball loses one tenth of its speed. Plot the speed-time graph of its motion between t = 0 to 12 s.
13. Explain clearly, with examples, the distinction between:
    (a) magnitude of displacement (sometimes called distance) over an interval of time, and the total length of path covered by a particle over the same interval.
    (b) magnitude of average velocity over an interval of time, and the average speed over the same interval. [Average speed of a particle over an interval of time is defined as the total path length divided by the time interval]. Show in both (a) and (b) that the second quantity is either greater than or equal to the first
    When is the equality sign true? [For simplicity, consider one-dimensional motion only].
14. A man walks on a straight road from his home to a market 2.5 km away with a speed of 5 km h^{-1} .
    Finding the market closed, he instantly turns and walks back home with a speed of 7.5 km h^{-1}. What is the
    (a) magnitude of average velocity, and
    (b) average speed of the man over the interval of time (i) 0 to 30 min, (ii) 0 to 50 min, (iii) 0 to 40 min?
    [Note: You will appreciate from this exercise why it is better to define average speed as total path length divided by time, and not as magnitude of average velocity. You would not like to tell the tired man on his return home that his average speed was zero!]
15. In Q13 and Q14, we have carefully distinguished between average speed and magnitude of average velocity. No such distinction is necessary when we consider instantaneous speed and magnitude of velocity. The instantaneous speed is always equal to the magnitude of instantaneous velocity. Why?
16. Look at the graphs (a) to (d) fig. carefully and state, with reasons, which of these cannot possibly represent one-dimensional motion of a particle.

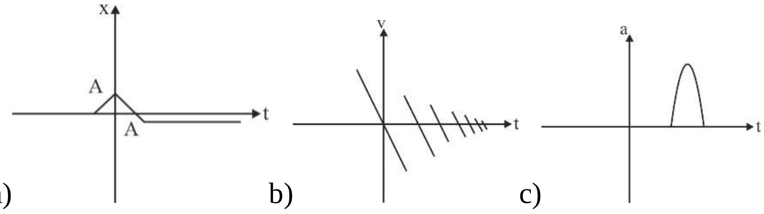


17. Figure shows the x-t plot of one-dimensional motion of a particle. Is it correct to say from the graph that the particle moves in a straight line for t < 0 and on a parabolic path for t > 0? If not, suggest a suitable physical context for this graph.

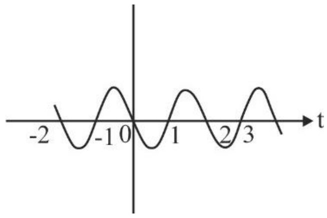


18. A police van moving on a highway with a speed of 30 km h^{-1} fires a bullet at a thief's car speeding away in the same direction with a speed of 192 km h^{-1} . If the muzzle speed of the bullet is 150 m s^{-1}, with what speed does the bullet hit the thief's car? (Note: Obtain that speed which is relevant for damaging the thief's car).

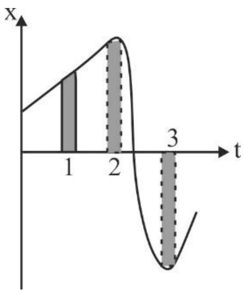
19. Suggest a suitable physical situation for each of the following graphs (Fig):



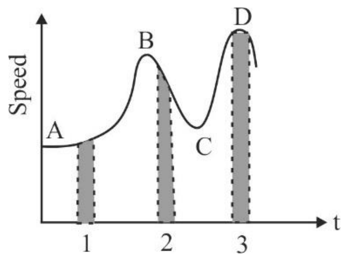
20. Figure gives the x-t plot of a particle executing one-dimensional simple harmonic motion. Give the signs of position, velocity and acceleration variables of the particle at t = 0.3 s, 1.2 s, – 1.2 s.



21. Figure gives the x-t plot of a particle in one-dimensional motion. Three different equal intervals of time are shown. In which interval is the average speed greatest, and in which is it the least? Give the sign of average velocity for each interval.



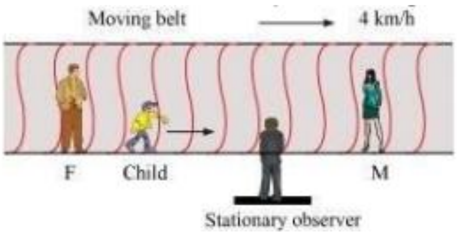
22. Figure gives a speed-time graph of a particle in motion along a constant direction. Three equal intervals of time are shown. In which interval is the average acceleration greatest in magnitude? In which interval is the average speed greatest? Choosing the positive direction as the constant direction of motion, give the signs of v and a in the three intervals. What are the accelerations at the points A, B, C and D?



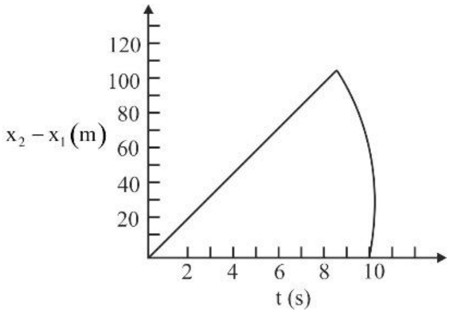
23. A three-wheeler starts from rest, accelerates uniformly with 1 m s^{-2} on a straight road for 10 s, and then moves with uniform velocity. Plot the distance covered by the vehicle during the n^{th} second (n = 1, 2, 3….) versus n. What do you expect this plot to be during accelerated motion: a straight line or a parabola?

24. A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m/s. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m/s and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands?

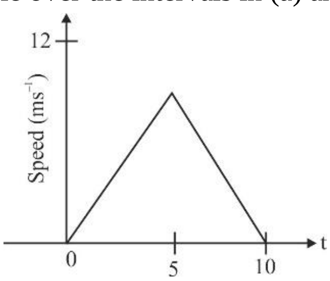
25. On a long horizontally moving belt (Fig.), a child runs to and fro with a speed 9 km h^{-1} (with respect to the belt) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km h^{-1}. For an observer on a stationary platform outside, what is the
(a) Speed of the child running in the direction of motion of the belt?
(b) Speed of the child running opposite to the direction of motion of the belt?
(c) Time taken by the child in (a) and (b)?
Which of the answers alter if motion is viewed by one of the parents?



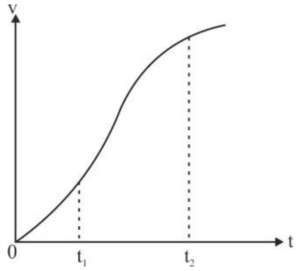
26. Two stones are thrown up simultaneously from the edge of a cliff 200 m high with initial speeds of 15 m/s and 30 m/s. Verify that the graph shown in Fig. correctly represents the time variation of the relative position of the second stone with respect to the first. Neglect air resistance and assume that the stones do not rebound after hitting the ground. Take g = 10 m/s^2. Give the equations for the linear and curved parts of the plot.



27. The speed-time graph of a particle moving along a fixed direction is shown in Fig. 3.28. Obtain the distance traversed by the particle between (a) t = 0 s to 10 s, (b) t = 2 s to 6 s. (Fig.)
What is the average speed of the particle over the intervals in (a) and (b)?



28. The velocity-time graph of a particle in one-dimensional motion is shown in Fig.:



Which of the following formulae are correct for describing the motion of the particle over the time-interval t_1 to t_2?

Answer Key

1. (a), (b)	2. *Refer Solution*	3. 6 min	4. *Refer Solution*
5. – 1000 km/h	6. 11.44 s	7. 1250 m	8. 1 m/s^2
9. 9min	10. 6 s	11. (a) True	(b) False
(c) True	(d) False	12. 12.01 sec	
13. *Refer Solution*	14. 5.625 km/h	15. Refer Solution	
16. Refer Solution	17. Refer Solution	18. 105 m/s	
19. Refer Solution	20. Refer Solution	21. Refer Solution	
22. *Refer Solution*	23. 10 s	24. 10 s	
25. (a) 13 km/h	(b) 5 km/	(c) 20s.	
26. Refer Solution	27. 9 m/s	28. *Refer Solution*	

Expert Test - I

1. A person travelling in a straight line moves with a constant velocity v_1 for certain distance 'x' and with a constant velocity v_2 for next equal distance. The average velocity v is given by the relation

(a) $\frac{1}{v}=\frac{1}{v_1}+\frac{1}{v_2}$
(b) $\frac{2}{v}=\frac{1}{v_1}+\frac{1}{v_2}$
(c) $\frac{v}{2}=\frac{v_1+v_2}{2}$
(d) $v=\sqrt{v_1 v_2}$

2. Preeti reached the metro station and found that the escalator was not working. She walked up the stationary escalator in time t_1. On other days, if she remains stationary on the moving escalator, then the escalator takes her up in time t_2. The time taken by her to walk up on the moving escalator will be

(a) $\frac{t_1+t_2}{2}$
(b) $\frac{t_1 t_2}{t_2-t_1}$
(c) $\frac{t_1 t_2}{t_2+t_1}$
(d) $t_1 - t_2$

3. If the velocity of a particle is $v = At + Bt^2$, where A and B are constants, then the distance travelled by it between 1s and 2s is

(a) 3A + 7B
(b) $\frac{3}{2}A+\frac{7}{3}B$
(c) $\frac{A}{2}+\frac{B}{3}$
(d) $\frac{3}{2}A+4B3$

4. Two cars P and Q start from a point at the same time in a straight line and their positions are represented by

$X_P(t) = at + bt^2$ and $X_Q(t) = ft - t^2$. At what time do the cars have the same velocity?

(a) $\frac{a-f}{1+b}$
(b) $\frac{a+f}{2(b-1)}$
(c) $\frac{a+f}{2(1+b)}$
(d) $\frac{f-a}{2(1+b)}$

5. A car starts from rest and accelerates at 5 m/s^2. At t = 4s, a ball is dropped out of a window by a person sitting in the car. What is the velocity and acceleration of the ball at t = 6 s?
(Take, g =10 m/ s^2)

(a) 20 m/s, 5 m/s^2
(b) 20 m/s, 0
(c) $20\sqrt{2}$ m/s, 0
(d) $20\sqrt{2}$ m/s, 10 m/s^2

6. A small block slides down on a smooth inclined plane, starting from rest at time t = 0. Let s_n be the distance travelled by the block in the interval t = n – 1 to t = n. Then, the ratio $\frac{s_n}{s_{n+1}}$ is

(a) $\frac{2n-1}{2n}$
(b) $\frac{2n-1}{2n+1}$
(c) $\frac{2n+1}{2n-1}$
(d) $\frac{2n}{2n-1}$

7. A person sitting in the ground floor of a building notices through the window of height 1.5 m, a ball dropped from the roof of the building crosses the window in 0.1 s. What is the velocity of the ball when it is at the topmost point of the window? (g = 10 m/s^2)

(a) 15.5 m/s
(b) 14.5 m/s
(c) 4.5 m/s
(d) 20 m/s

8. A ball is thrown vertically downward with a velocity of 20 m/s from the top of a tower. It hits the ground after some time with a velocity of 80 m/s. The height of the tower is (g =10 m/s^2)

(a) 340 m
(b) 320 m
(c)300 m
(d) 360 m

9. A person standing on the floor of an elevator drops a coin. The coin reaches the floor in time t_1 if the elevator is at rest and in time t_1 if the elevator is moving uniformly. The which of the following option is correct?

(a) $t_1 < t_2$ or $t_1 > t_2$ depending upon whether the lift is going up or down
(b) $t_1 < t_2$
(c) $t_1 > t_2$
(d) $t_1 = t_2$

10. A toy car with charge q moves on a frictionless horizontal plane surface under the influence of a uniform electric field **E**. Due to the force q**E**, its velocity increases from 0 to 6 m/s in one second duration. At that instant, the direction of the field is reversed. The car continues to move for two more seconds under the influence of this field. The average velocity and the average speed of the toy car between 0 to 3 seconds are respectively

(a) 1 m/s, 3.5 m/s
(b) 1 m/s, 3 m/s
(c) 2 m/s, 4 m/s
(d) 1.5 m/s, 3 m/s

11. A stone falls freely under gravity. It covers distances h_1, h_2, and h_3, in the first 5s, the next 5s and the next 5s respectively. The relation between h_1, h_2, and h_3, is

(a) $h_1 = 2h_2 = 3h_3$
(b) $h_1 = \frac{h_2}{3} = \frac{h_3}{5}$
(c) $h_2 = 3h_1$ and $h_3 = 3h_2$
(d) $h_1 = h_2 = h_3$

Answer Key

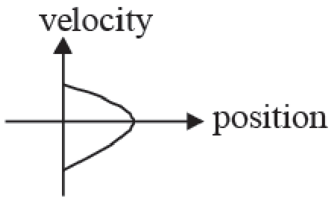
1. (b)	2. (c)	3. (b)	4. (*d*)
5. (d)	6. (b)	7. (b)	8. (*c*)
9. (d)	10. (b)	11. (b)	

Expert Test - II

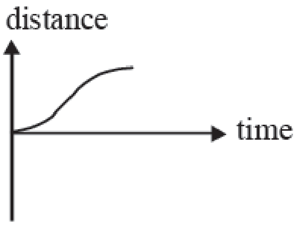
1. A particle is moving with speed v = $\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time t = τ (assume that the particle is at origin at t = 0).

(a) $\frac{b^2\tau}{4}$
(b) $\frac{b^2\tau}{2}$
(c) $b^2\tau$
(d) $\frac{b^2\tau}{\sqrt{2}}$

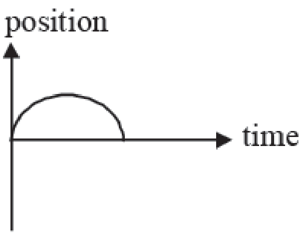
2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.



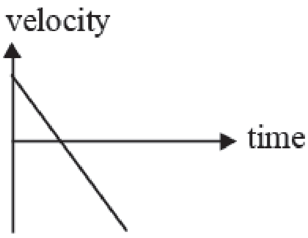
(a)



(b)

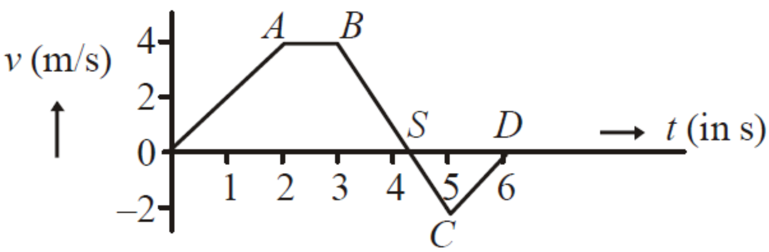


(c)



(d)

3. The velocity (v) and time (t) graph of a body in a straight line motion is shown in the figure. The point S is at 4.333 seconds. The total distance covered by the body in 6 s is:



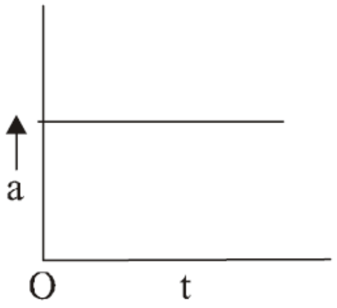
(a) $\frac{37}{3} m$
(b) 12 m

(c) 11 m (d) $\frac{49}{4} m$

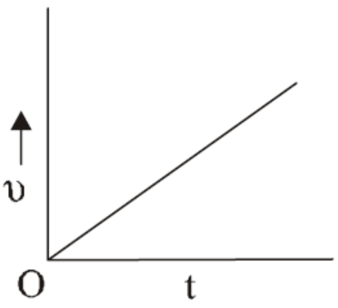
4. A bullet of mass 20g has an initial speed of 1 ms^{-1}, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2} N, the speed of the bullet after emerging from the other side of the wall is close to :
(a) 0.1 ms^{-1} (b) 0.7 ms^{-1}
(c) 0.3 ms^{-1} (d) 0.4 ms^{-1}

5. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$
where, a, b and c are constants. When the particle attains zero acceleration, then its velocity will be:
(a) $a+\frac{b^2}{4c}$ (b) $a+\frac{b^2}{3c}$
(c) $a+\frac{b^2}{c}$ (d) $a+\frac{b^2}{2c}$

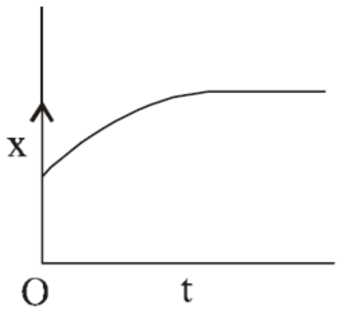
6. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represents the motion qualitatively (a = acceleration, v = velocity, x = displacement, t = time)



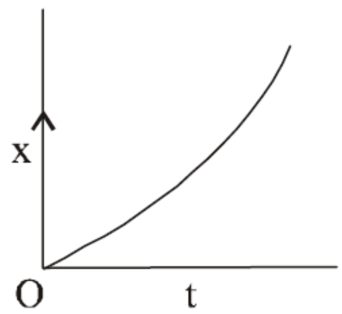
(A)



(B)



(C)

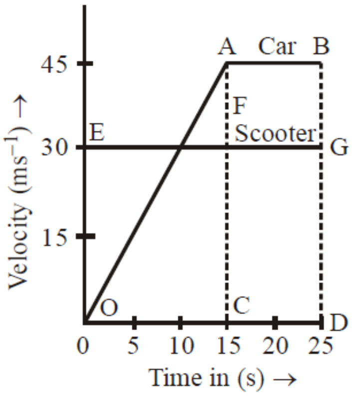


(D)

(a) (B), (C) (b) (A)
(c) (A), (B), (C) (d) (A), (B), (D)

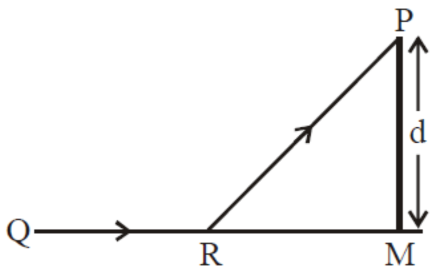
7. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to:
(a) $\frac{2a_1a_2}{a_1+a_2}t$ (b) $\sqrt{2a_1a_2}t$
(c) $\sqrt{a_1a_2}t$ (d) $\frac{a_1+a_2}{2}t$

8. The velocity-time graphs of a car and a scooter are shown in the figure. (i) the difference between the distance travelled by the car and the scooter in 15 s and (ii) the time at which the car will catch up with the scooter are, respectively



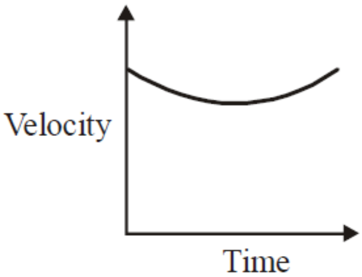
(a) 337.5m and 25s
(b) 225.5m and 10s
(c) 112.5m and 22.5s
(d) 11.2.5m and 15s

9. A man in a car at location Q on a straight highway is moving with speed v. He decides to reach a point P in a field at a distance d from highway (point M) as shown in the figure. Speed of the car in the field is half to that on the highway. What should be the distance RM, so that the time taken to reach P is minimum?

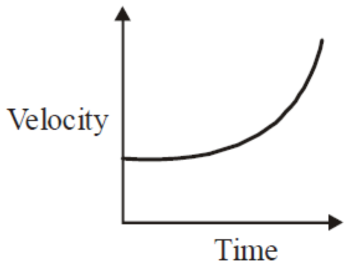


(a) $\frac{d}{\sqrt{3}}$
(b) $\frac{d}{2}$
(c) $\frac{d}{\sqrt{2}}$
(d) d

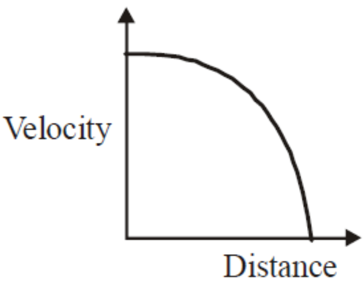
10. Which graph corresponds to an object moving with a constant negative acceleration and a positive velocity ?



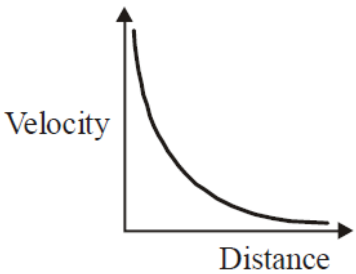
(a)



(b)

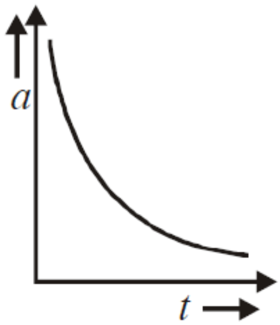


(c)

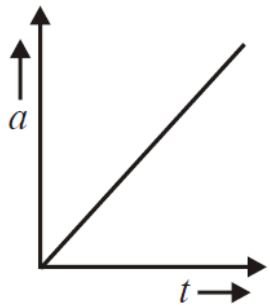


(d)

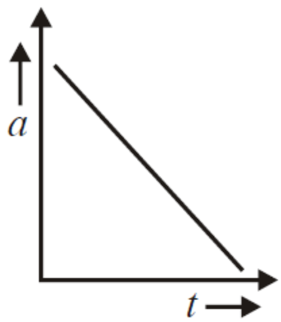
11. The distance travelled by a body moving along a line in time t is proportional to t^3.
The acceleration-time (a, t) graph for the motion of the body will be



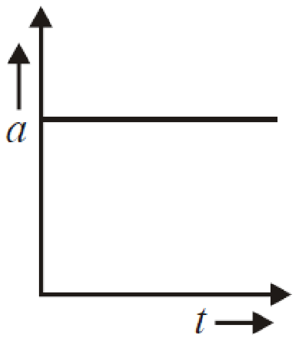
(a)



(b)

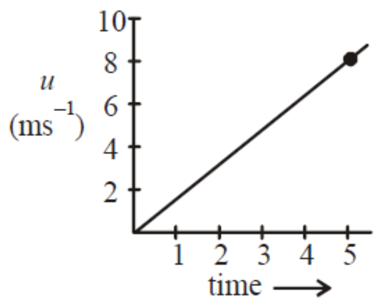


(c)



(d)

12 The speed verses time graph for a particle is shown in the figure. The distance travelled (in m) by the particle during the time interval t = 0 to t = 5 s will be ___________________.



(s)

13. The distance x covered by a particle in one dimensional motion varies with time t as $x^2 = at^2 + 2bt + c$. If the acceleration of the particle depends on x as x^{-n}, where n is an integer, the value of n is ____________.

14. Train A and train B are running on parallel tracks in the opposite directions with speeds of 36 km/hour \ and 72 km hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(a) 29.5 ms^{-1}
(b) 28.5 ms^{-1}
(c) 31.5 ms^{-1q}
(d) 30.5 ms^{-1}

15. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, speed of the plane is:

(a) $\frac{\sqrt{3}}{2}v$
(b) $\frac{2v}{\sqrt{3}}$
(c) v
(d) $\frac{v}{2}$

16. A car is standing 200 m behind a bus, which is also at rest. The two start moving at the same instant but with different forward accelerations. The bus has acceleration 2 m/s^2 and the car has acceleration 4 m/s^2. The car will catch up with the bus after a time of:

(a) $\sqrt{110}s$ (b) $\sqrt{120}s$

(c) $10\sqrt{2}s$ (d) 15 s

17. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator?

(a) 37 s (b) 27 s

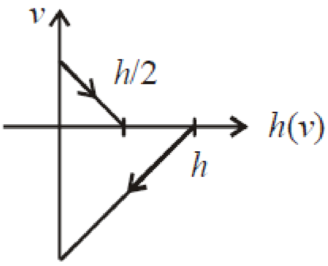
(c) 24 s (d) 45 s

18. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity] :

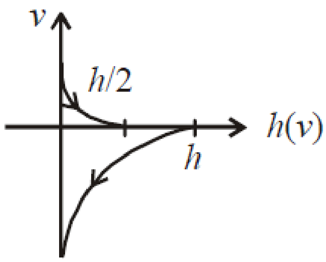
(a) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (b) $t = 1.8\sqrt{\frac{h}{g}}$

(c) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (d) $t = \sqrt{\frac{2h}{3g}}$

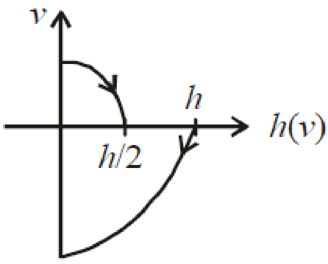
19. A Tennis ball is released from a height h and after freely falling on a wooden floor it rebounds and reaches height $\frac{h}{2}$. The velocity versus height of the ball during its motion may be represented graphically by:(graph are drawn schematically and on not to scale)



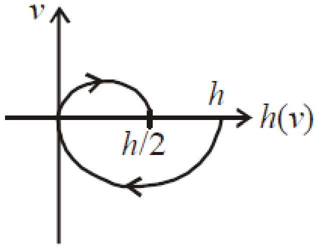
(a)



(b)

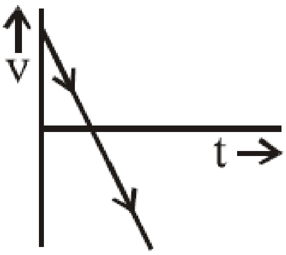


(c)

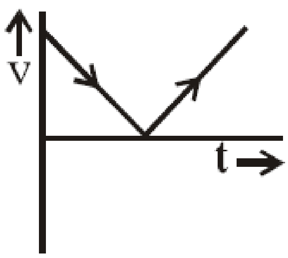


(d)

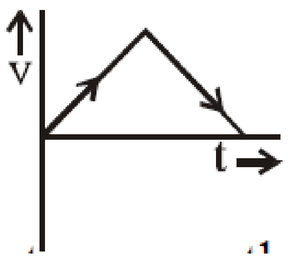
20. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

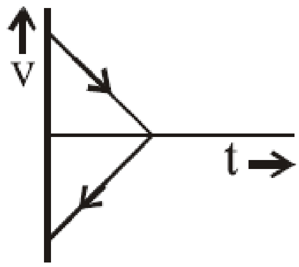


(a)

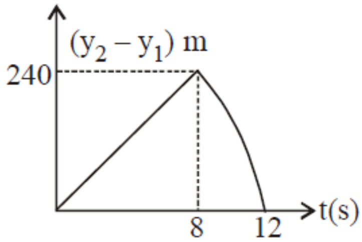


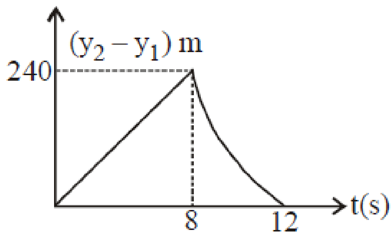
(b)

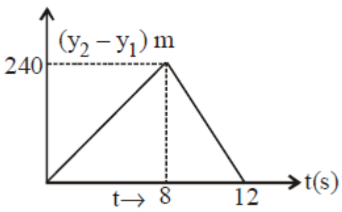
(c) 

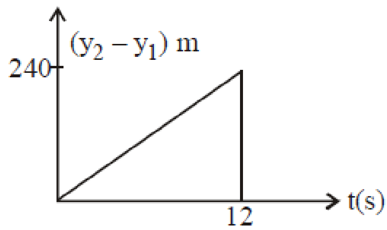
(d) 

21. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take g = 10 m/s^2)
(The figures are schematic and not drawn to scale)

(a) 

(b) 

(c) 

(d) 

22. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is:
(a) $2gH = n^2u^2$
(b) $gH = (n - 2)^2 u^2d$
(c) $2gH = nu^2 (n - 2)$
(d) $gH = (n - 2)u^2$

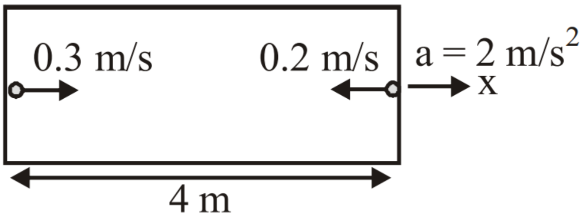
23. A ball is dropped from the top of a 100 m high tower on a planet. In the last $\frac{1}{2}$ s before hitting the ground, it covers a distance of 19 m. Acceleration due to gravity (in ms^{-2}) near the surface on that planet is ________.

Answer Key

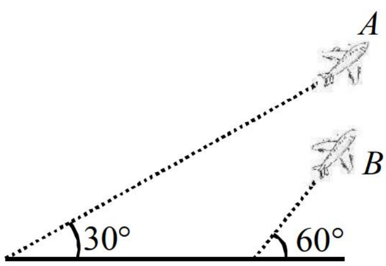
1. (b)	2. (b)	3. (a)	4. (*b*)
5. (b)	6. (d)	7. (c)	8. (*c*)
9. (a)	10. (c)	11. (b)	12. (20)
13. (3)	14. (a)	15. (d)	16. (*c*)
17. (c)	18. (c)	19. (c)	20. (*a*)
21. (b)	22. (c)	23. (a)	

Pro Test - I

1. A rocket is moving in a gravity free space with a constant acceleration of $2ms^{-2}$ along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end the chamber in + x direction with a speed of 0.3m/s related to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2m/s from its right end relative to the rocket. The time in seconds when the two balls hit each other is



2. A particle of mass m moves on the x axis as following: it start from rest at t = 2 from the point x = 0 and comes to rest at t = 1 at the point x = 1. No other information is available about its motion at intermediate time
($0<t<1$). If α denotes the instantaneous acceleration of the particle, then:
(a) α cannot remain positive for all t in the interval $0<t<1$.
(b) $|\alpha|$ cannot exceed 2 at any point in its path.
(c) $|\alpha|$ must be ≥ 4 at some point or points in its path.
(d) α must change sign during the motion, but no other assertion can be made with information given.
3. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant rate β to come to rest. If the total time lapse is /seconds, Evaluate.
(a) Maximum velocity reached, and
(b) Total distance travelled
4. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed A is $100\sqrt{3}$m/s. At time t = 0s, an observer in A finds B at a distance of 500m. The observer sees B moving with constant velocity perpendicular to the line of motion of A. If at t = $t_\circ$, A just escapes being hit by B, $t_\circ$ in seconds is



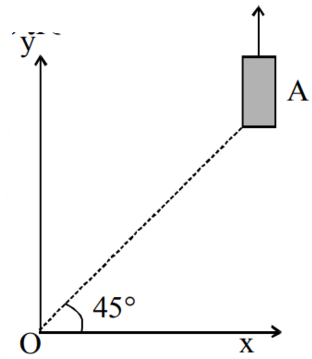
5. Four person's K,L,M,N are initially at the four corners of a square side d. Each person now moves with a uniform speed v in such as way that K always moves directly towards L,L directly towards M,M directly towards N, and N directly towards K. The four persons will meet at a time........

6. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley A is moving along

straight line parallel to the y-axis (see figure) with a constant velocity of ($\sqrt{3}$ - 1) m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown angle the surface from the origin of O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley

(a) The motion of the ball is observed from the frame of the trolley. Calculate the angle θ made by the velocity vector of the ball the x-axis in this frame.

(b) Find the speed of the ball with respect of the surface, if $\phi = 4\theta/4$.



Answer Key

1. (8)　　2. (a,c,d)　　3. (i) $v_m = \frac{t\alpha\beta}{(\alpha+\beta)}$　　(ii) $\frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2$

4. (5)　　5. $\left(\frac{d}{v}\right)$　　6. v_B = 2 m/s

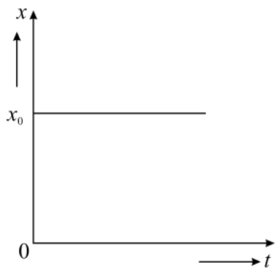
Solution:
Test Papers

Solutions : Beginner Test - I

1.
Sol:- An object can be considered as a point object, if the distance travelled by it is very large than its size.

2.
Sol:- No, the displacement of the object does not depend on the choice of the position of the origin.

3.
Sol:-



In the above x-t graph, body is at rest at position x_0.

4.
Sol:- $\frac{v_Y}{v_X} = \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{\sqrt{3}}{1/\sqrt{3}} = 3:1$

5.
Sol:- Speed is the distance travelled in unit time and distance cannot be negative.

6.
Sol:- Given, $x = 6 + 18t + 9t^2$

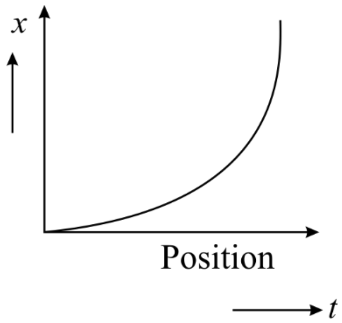
$$v_t = \frac{dx}{dt} = 18 + 18t$$

At $t = 2$,

$$v_2 = 18 + 18 \times 2$$
$$= 54\ m/s$$

7.
Sol:- When a particle is moving along a straight line with the fixed direction.

8.
Sol:-



In the above x-t graph, the slope of x-t graph is increasing it means that the velocity is increasing.

9.
Sol:- (i). A flying kite in the sky comes under three-dimensional motion.
(ii). A cricket ball hit by a player comes under two-dimensional motion.
(iii). Moon revolving around the sun-earth comes under two-dimensional motion.
(iv). The motion of the stone in circular motion comes under two-dimensional motion.

10
Sol:- The effective distance travelled by drunkard in 8 steps $= 5 - 3 = 2m$
Therefore, he takes 32 steps to move 8m.
Now, he well have to cover 5m more to reach the pit, for which he has to take only 5 forward steps.
Therefore, he well have to take $= 32 + 5 = 37$ steps to move 13 m. Thus, he well fall into the pit after taking 37 steps i.e., after 37 s from the start

11.
Sol:- Average velocity, $v_{avg} = \frac{Net\ displacement}{Total\ time\ taken}$
And average speed, $s_{avg} = \frac{Total\ distance\ travelled}{Total\ time\ taken}$
If an object moves along a straight line starting from origin and then returns back to origin.
Average velocity $= 0$
And Average speed $= \frac{2s}{t}$

12.
Sol:- No, because the distance covered by an object is the path length of the path covered by the object. The displacement of an object is given by the change in position between the initial position and final position.
e.g. A boy stats from his home and moves towards market along a straight path. Then, he returns to home from the same path. Here, displacement is zero but distance is non-zero.

13.
Sol:- We can see that the object P covers a distance of 10 m in every 15 min. In other

words, it covers equal distance in equal intervals of time. So, the motion of object P is uniform. On the other hand, the object Q covers 7 m from 9:30 am to 9:45 am, 4 m from 9:45 am to 10:00 am and so an. In other words, it covers unequal distances in equal intervals of time. So, the motion of object Q is non-uniform.

14.

Sol:- Displacement travelled in time

$$(t_1 + t_2) = s_1 + s_2 = v_1t_1 + v_2t_2$$

$\therefore$ Average velocity $= \frac{Net\ displacement}{Total\ time\ taken}$

$$= \frac{v_1t_1 + v_2t_2}{(t_1 + t_2)}$$

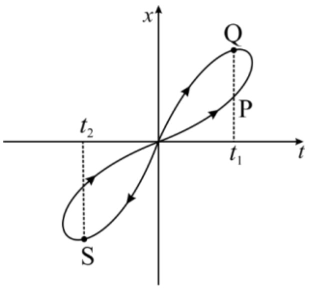
15.

Sol:- Since, earth revolves around the sun and also spins about its own axis, so it is an accelerated frame of reference. Hence, earth is a non-inertial frame of reference.

However, if we do not take large scale motion such as wind and ocean currents into consideration, we can say that approximation the earth is an inertial frame.

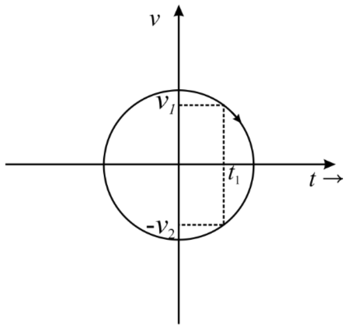
16.

Sol:- (a) No, graph (a) is not representing one-dimensional motion of a particle, because graph shows two different positions of the particle at same instant of time. (At time t_1, particle is at positions P and Q and at time t_2, particle is at positions R and S) which is not possible.



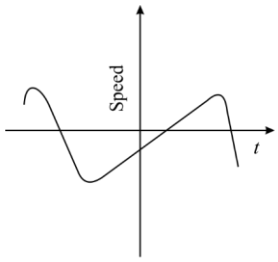
(a)

(b). No, graph (b) cannot represent dimensional motion of a particle, because graph shows one positive velocity (v_1) and another negative velocity (v_2) of the particle at the same instant of time (t_1) which is not possible.



(b)

(c). No, graph (c) cannot represent one-dimensional motion of a particle, because, graph shows negative speed of the particle but speed cannot be negative.



(c)

(d). No, graph (d) cannot represent one-dimensional motion of a particle, because graph shows that total path length increases from time $t = 0$ to $t = t_1$ but decreases from $t = t_1$ to $t = t_2$. But total path length of a moving particle can never decrease with time.

17.

Sol:- We know that, $v = \frac{dx}{dt}$

On differentiating w. r. t. t, we get

$v = \frac{d}{dt}(a + bt^2) = 2bt = 5t\ m/s$ [$\because b = 2.5m$]

At $t = 0, v = 0,\ t = 2s$ and $v = 10\ m/s$

Average velocity $= \frac{x(4)-x(2)}{4-2} = \frac{a+16\ b - a - 4b}{2}$

$$= 6 \times b = 6 \times 2.5 = 1.5\ m/s$$

18.

Sol:- Time taken by man to go from his home to market

$$t_1 = \frac{Distance}{Speed} = \frac{2.5}{5} = \frac{1}{2}\ h$$

Time taken by man to go from market to his home

$$t_2 = \frac{2.5}{7.5} = \frac{1}{3}\ h$$

∴ Total time taken, $t_1 + t_2 = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}\ h = 50min$

(i). 0 to 30 min

(a). Average velocity $= \frac{Displacement}{Time} = \frac{2.5}{1/2} = 5\ km/h$

(b). Average speed $= \frac{Distance}{Time} = \frac{2.5}{1/2} = 5\ km/h$

(ii). 0 to 50 min

Total distance travelled $= 2.5 + 2.5 = 5\ km$

Total displacement $= 2.5 - 2.5 = 0$

(a). Average velocity $= \frac{Displacement}{Time} = 0$

(b). Average speed $= \frac{Distance}{Time} = \frac{5}{5/6} = 6\ km/h$

(iii). 0 to 40 min

Distance moved in 30 min (from home to market)= $2.5\ km$.

Distance moved in 10 min (from market to home) with speed $= 7.5\ km/h = 7.5 \times \frac{10}{60} = 1.25\ km$

So, displacement $= 2.5 - 1.25 = 1.25 km$

Distance travelled $= 2.5 + 1.25 = 3.75\ km$

(a). Average velocity $= \frac{1.25}{(40/60)} = 1.875\ km/h$

(b). Average speed $= \frac{3.75}{(40/60)} = 5.625\ km/h$

19.

Sol:- Time to cover $\frac{s}{2}$ distance

$$t_1 = \frac{s/2}{3} = \frac{s}{6} s$$

Time to cover s_1 distance,

$$t_2 = \frac{s_1}{4.5} s$$

Time to cover s_2 distance,

$$t_3 = \frac{s_2}{7.5} s$$

Now, $s_1 + s_2 = \frac{s}{2} s$

∴ $\quad 4.5 t_2 + 7.5\ t_3 = \frac{s}{2}$

Since, $\quad t_2 = t_3$

$\Longrightarrow \quad 4.5\ t_2 + 7.5\ t_2 = \frac{s}{2}$

$\Rightarrow \quad t_2 = \frac{s}{24} s$

∴ Total time $= t_1 + t_2 + t_3 = \frac{s}{6} + \frac{s}{24} + \frac{s}{24}$

$$= \frac{6}{24} s = \frac{1}{4} s$$

Hence, average speed $s_{av} = \frac{s}{s/4} = 4\ m/s$

20.

Sol Given,$x = 2\ t^3$, velocity, $v = \frac{dx}{dt} = \frac{d(2t^3)}{dt} = 6t^2$

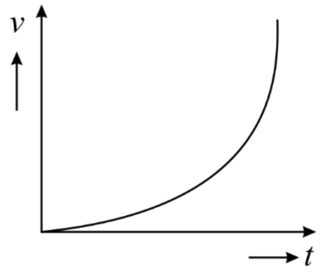
Acceleration,$a = \frac{dv}{dt} = \frac{d(6\ t^2)}{dt} = 12\ t$

21.

Sol Motion of a body under gravity.

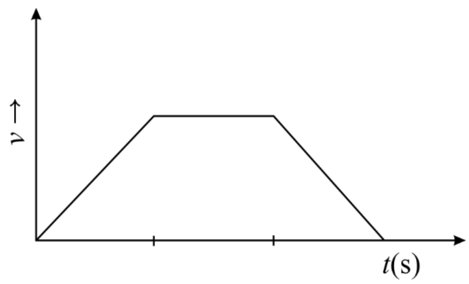
22.

Sol



Here, acceleration is increasing.

23



24.

Sol:-Acceleration is 9.8 m/m^2 (downwards) and velocity is zero at the highest point.

25.

Sol:-There will be no effect on the magnitude of relative velocity.

26.

Sol:-Let x be the displacement. Then, $x = at^2$

∴ Velocity of the object, $v = \frac{dx}{dt} = 2at$

Acceleration of the object, $a = \frac{dv}{dt} = 2a$

It means that a is constant.

27.

Sol:-Let $t_1$and t_2 be the times of descent through PQ and QR, respectively.

Let $PQ = QR = h$

Then, $h = \frac{1}{2} g t_1^2$ and $2h = \frac{1}{2} g (t_1 + t_2)^2$

By dividing, we get

$\frac{1}{2}=\frac{t_1^2}{\left((t_1+t_2)\right)^2}$ Or $\frac{1}{\sqrt{2}}=\frac{t_1}{(t_1+t_2)}$

Hence, $t_1 : t_2 = 1 : \left(\sqrt{2}-1\right)$

28.

Sol:-Both these statements are not true, because

(i)Its magnitude can be zero.

(ii)Its magnitude is either less than or equal to the distance travelled by the object.

29.

Sol:-The given equation is $v = 4 + 2(C_1 + C_2t)$

$\Rightarrow \quad v = (4+2C_1) + 2C_2t$

Comparing the above equations with equation of motion

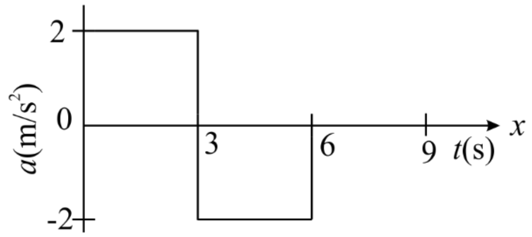
$v = u + at$

Initial velocity, $u = 4 + 2C_1$

Acceleration of the particle $= 2C_2$

30.

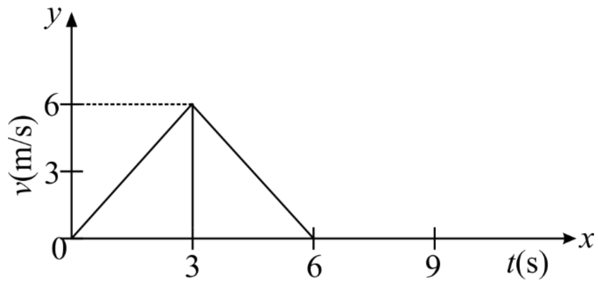
Sol:-The acceleration-time graph is



The area enclosed between $a - t$ curve gives change in velocity for the corresponding interval.

At $t = 0$, $v = 0$, hence final velocity at $t = 3\ s$ will increase to $6\ m/s$. In next 3 s, the velocity will decrease to zero.

Thus, the velocity time graph is



31.

Sol:-Muzzle speed of bullet, $v = 150\ ms^{-1}$

$$= 150 \times \frac{18}{5} = 540\ kmh^{-1}$$

Speed of police van, $v_P = 30\ km/h$

Speed of thief's car, $v_T = 192\ km/h$

Since the bullet is sharing the velocity of the police van, its effective velocity is

$$v_B = v + v_P$$
$$= 540 + 30 = 570\ km/h$$

The speed of the bullet w.r.t the thief's car moving in the same direction.

$$v_{BT} = v_B - v_T = 570 - 192 = 378\ km/h$$
$$= \frac{378 \times 1000}{60 \times 60} = 105\ ms^{-1}$$

32.

Sol:-In the SHM, acceleration$a = \omega^2 x$, where ω (i.e. angular frequency) is constant.

(i)At time $t = 0.3\ s$, x is negative, the slope of x-t plot is negative, hence position and velocity are negative. Since$a = \omega^2 x$, hence acceleration is positive.

(ii)At time $t = 1.2s$, x is positive, the slope of x-t plot is also positive; hence position and velocity are positive. Since$a = \omega^2 x$, hence acceleration is negative.

(iii)At $t = -1.2s$, x is negative, the slope of x-t plot is also negative. But since both x and t are negative here, hence velocity is positive. Finally, acceleration $\boldsymbol{a}$ is also positive.

33.

Sol:-We know that average acceleration in a small interval of time is equal to slope of velocity-time graph in that interval. As the slope of velocity-time graph is maximum in interval 2 as compared to other intervals 1and 3, hence the magnitude of average acceleration is greatest in interval 2. The average speed is greatest in interval 3 for obvious reasons.

In interval 1, the slope of velocity-time graph is positive, hence acceleration $\boldsymbol{a}$ is positive. The speed u is positive in this interval due to obvious reasons.

In interval 2, the slope of velocity-time graph is negative, hence acceleration $\boldsymbol{a}$ is negative. The speed u is positive in this interval due to obvious reasons.

In interval 3, the velocity-time graph is parallel to time axis, therefore acceleration $\boldsymbol{a}$ is zero in this interval but v is positive due to obvious reasons.

At points A, B, C and D, the velocity-time graph is parallel to time axis. Therefore, acceleration a is zero at all the four points.

34.

Sol:-For a train A, $u = 72kmh^{-1}$

$$= \frac{72 \times 1000}{60 \times 60} = 20\ ms^{-1}$$

$$t = 50\ s, a = 0, x = x_A$$

As, $x = ut + \frac{1}{2}at^2$

$$\therefore x_A = 20 \times 50 + \frac{1}{2} \times 0 \times 50^2 = 1000m$$

For train B, $u = 72km/h = 20\ ms^{-1}$

$$a = 1\ m/s^2, t = 50s, x = x_B$$

As, $x = ut + \frac{1}{2}at^2$

$$x_B = 20 \times 50 + \frac{1}{2} \times 1 \times 50^2 = 2250m$$

Taking the guard of the train B in the last compartment of the train B, it follows that original distance between the two trains=length of train A+ length of train B.

$= 800$

Or original distance between the two trains is given by

$$= x_B - x_A = 2250 - 1000 = 1250$$

Or original distance between the two trains

$$= 1250 - 800 = 450\ m$$

35.

Sol:-Because the acceleration is constant, we can apply the equations of motion derived above.

(i)We take the origin of the x-axis to be the initial position of the plane, so that $x_0 = 0$.

It is useful to begin by listing all the data gives is the problem.

$$a = 4.00\ m/s^2$$

$$v = 0, x = 0$$

The problem may be stated in terms of the symbols as follows

Find x and v at $t = 5.00\ s$

When x and u are zero, these two equations reduce to $v = at$ and $x = \frac{1}{2}at^2$

At $t = 5.00s$

$$v = (4.00\ m/s^2)(5.00s) = 20.0\ m/s$$

$$x = \frac{1}{2}(4.00\ m/s^2)\ (5.00s)^2$$

$$x = 50.0\ m$$

(ii)The problem here may be stated as

Find x when $v = 70.0\ m/s$

It contains the single unknown x, as well as and v,

Which are known with? $u = 0, v_x^2 = 2a_x x$

Solving for x, we obtain

$$x = \frac{v^2}{2a} = \frac{(70.00\ m/s)^2}{2(4.00\ m/s^2)} = 613\ m$$

36.

Sol: - (i) Since, the ball is moving under the effect of gravity, the direction of acceleration due to gravity is always vertically downwards.

(ii)At the highest point, the velocity of the ball becomes zero and acceleration is equal to the acceleration due to gravity = 9.8 ms^{-2} in vertically downward direction.

(iii)When the highest point is chosen as the location for $x = 0$ and $t = 0$ and vertically downward direction to be the positive direction of x-axis and upward direction as negative direction of x-axis.

During upward motion, sign of position is negative, sign of velocity is negative and sign of acceleration is positive. During downward motion, sign of position is positive, sign of velocity is positive and sign of acceleration is also positive.

(iv)Let t be the time taken by the ball to reach the highest point where height from ground be s. Taking vertical upward motion of the ball, we have

$u = -29.4ms^{-1}, a = 9.8\ ms^{-2}, v = 0, s = S, t = ?$

As, $v^2 - u^2 = 2as$

$\therefore\ 0 - (-29.4)^2 = 2 \times 9.8 \times S$

Or $S = \frac{-(29.4)^2}{2\times 9.8} = -44.1\ m$

Here, negative sign shows that the distance is covered in upward direction.

As, $v = u + at$

$\therefore 0 = -29.4 + 9.8 \times t$

Or $t = \frac{29.4}{9.8} = 3s$

It means time of ascent $= 3\ s$

When an object moves under the effect of gravity alone, the time of ascent is always equal to the time of descent.

Therefore, total time after which the ball returns to the player's hand $= 3 + 3 = 6s$

37.

Sol:- Taking vertical downward motion of ball from a height 90 m, we have

$$u = 0, a = 10\,m/s^2, s = 90m, t = ? : u = ?$$

$$t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 90}{10}} = 3\sqrt{2}\ s = 4.24s$$

$$t = \sqrt{2as} = \sqrt{2 \times 10 \times 90} = 30\sqrt{2}\ m/s$$

Rebound velocity of ball.

$$u' = \frac{9}{10}u = \frac{9}{10} \times 30\sqrt{2} = 27\sqrt{2}\ \ m/s$$

Time to reach the highest point is

$$t' = \frac{u'}{a} = \frac{27\sqrt{2}}{10} = 27\sqrt{2} = 27\sqrt{2} = 3.81s$$

Total time $= t + t' = 4.24 + 3.81 = 8.05s$

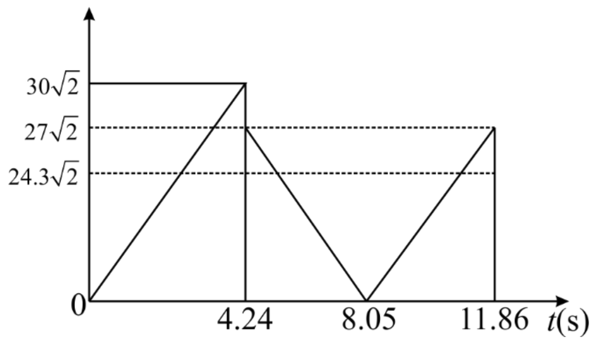
The ball will take further 3.81 s to fall back to floor, where its velocity before striking the floor $= 27\sqrt{2}\ m/s$

Velocity of ball after striking the floor

$$= \frac{9}{10} \times 27\sqrt{2} = 24.3\sqrt{2}\ m/s$$

Total time elapsed before upward motion of ball $= 8.05 + 3.81 = 11.86s$

Thus, the speed-time graph of this motion is shown in the figure.



38.

Sol:- Both the situations are possible

(i). When an object is projected upwards, its velocity at the top-most point is zero even though the acceleration on itis $9.8\,m/s^2$ (g).

(ii). When a stone tied to a string is whirled in a circular path, the acceleration acting on it is always at right angles i.e. perpendicular to the direction of motion of stone (we will study about it in chapter 'motion in a plane')

39.

Sol:- Let us consider left to right to be the positive direction of x-axis.

(i). Here, velocity of belt, $v_B = +\,4kmh^{-1}$, speed of child w. r. t. belt $v_C = +\,9\ kmh^{-1} = 5/2\ \ ms^{-1}$

Speed of the child w. r. t. stationary observer,

$$v'_C = v_C + v_B$$
$$= 9 + 4 = 13\ kmh^{-1}$$

(ii). Here, $v_B = +\,4\ kmh^{-1},\ v_C = -\,9kmh^{-1}$

Speed of the child w. r. t. stationary observer,

$$v'_C = v_C + v_B$$
$$= -9 + 4 = -5kmh^{-1}$$

Here, negative sign shows that the child will appear to run in a direction opposite to the direction of motion of the belt.

(iii). Distance between the parents, $s = 50m$

Since parents and child are located on the same belt, the speed of the child as observed by stationary observer in either direction (either from mother to father or from father to mother) will be $9kmh^{*-1}$.

Time taken by child in case (i) and (ii) is

$$t = \frac{50}{(5/2)} = 20\ s$$

If motion is observed by one of the parents, answer to case (i) or case (ii) will get altered. It is so because speed of child w. r. t.. either of mother or father is $9\ kmh^{-1}$. But answer (iii) remains unaltered due to the fact that parents and child are on the same belt and as such all are equally affected by the motion of the belt.

40.

Sol:- Area $AEFG = AE \times AG\ \Rightarrow 7 = 40 \times AG$

Or $AG = \frac{7}{40}h$

Area FGB gives the distance covered under retardation, it is $(8.5 - 7)km = 1.5km$

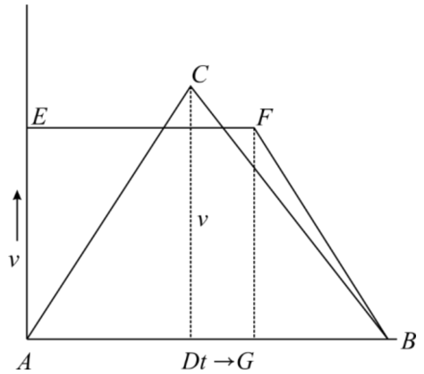
Area of $\Delta FGB = \frac{1}{2}GB \times FG\ \Rightarrow GB = \frac{2\times1.5}{40}h = \frac{3}{40}h$

Total time = $\left(\frac{7}{40}+\frac{3}{40}\right)h=\frac{1}{4}h$

Area of $\Delta ACB=\frac{1}{2}AB\times CD$

$$8.5=\frac{1}{2}\times\frac{1}{4}\times v$$

$$v=8.5\times 8\ kmh^{-1}=68kmh^{-1}$$



41.

Sol:- Taking vertical upward motion of the first stone for time t, we have

$$x_0=200m, u=15\,m/s$$

$$a=-10\,m/s^2, t=t, x=x$$

As, $x=x_0+ut+\frac{1}{2}\,at^2$

$\therefore \quad x_1=200+15t+\frac{1}{2}(-10)t^2$

Or $x_1=200+15t\ -5t^2$

Taking vertical upward motion of the second stone for time t, we have

$x_0=200m, u=30\,ms^{-1}$

$a=-10\,ms^{-2}, t=t, x=x_2$

Then, $x_2=200+30t-\frac{1}{2}\times 10\,t^2$

$=200+30t-5t^2$

When the first stone hits the ground, $x_1=0$, from Eq

So, $t^2-3t-40=0$

Or $(t-8)\,(t+5)=0$

$\therefore$ Either $t=8s$ or $-\ 5s$

Since, $t=0$ corresponds to the instant, when the stone was projected. Hence, negative time has no meaning in this case. So, $t=8s$.

When the second stone hits the ground, $x_2=0$, from Eq (ii).

So, $0=200+30\,t-5t^2$ or $t^2-6t-40=0$

Or $\quad (t-10)(t+4)=0$

Therefore, either $t=10s$, or $t=-4s$

Since, $t=-4s$ is meaningless, so $t=10$ s.

Relative position of second stone w. r. t. first is

$=x_2-x_1=15t$

Since, (x_2-x_1) and t are linearly related, therefore, the graph is a straight line till $t=8s$.

For maximum separation, $t=8s$, so maximum separation $=15\times 8=120m$

After 8s only, second stone would be in motion for 2s, so the graph is in accordance with the quadratic equation, $x_2=200+30\,t-5t^2$ for the interval of time 8s to 10 s.

42.

Sol:- Given, speed of the car as well as truck $=72\ km/h$

$=72\times\frac{5}{18}m/s=20\,m/s$

Retarded motion for truck

$v=u+a_t\,t$

$0=20+a_t\times 5$

Or $\quad a_t=-4\ m/s^2$

Retarded motion for the car

$v=u+a_c\,t$

$0=20+a_c\times 3$

Or $\quad a_c=-\frac{20}{3}\ m/s^2$

Let car be at distance x from truck, when truck gives the signal and t be the time taken to cover this distance.

As human response time is 0.5 s, therefore time of retarded motion of car is $(t-0.5)s$.

Velocity of car after time t,

$v_c=u-at=20-\left(\frac{20}{3}\right)\,(t-0.5)$

Velocity of truck after time t,

$v_t=20-4t$

To avoid the car bump onto the truck, $v_c=v_t$

$20-\frac{20}{3}(t-0.5)=20-4t$

Or $\quad 4t=\frac{20}{3}(t-0.5)$

Or $\quad t=\frac{5}{3}(t-0.5)$

Or $\quad 3t=5t-2.5$

Or $\quad t=\frac{2.5}{2}=\frac{5}{4}s$

Distance travelled by the truck in time t,

$s_t=u_t\,t+\frac{1}{2}\,a_t\,t^2$

$=20\times\frac{5}{4}+\frac{1}{2}\times(-4)\times\left(\frac{5}{4}\right)^2$

$s_t=25-3.125=21.875\,m$

Distance travelled by the car in time t
$=$ distance travelled by the car in 0.5 s (without retardation) $+$ Distance travelled by car in $(t-0.5)\ s$ (with retardation)

$$s_c = (20\times 0.5) + 20\left(\frac{5}{4} - 0.5\right) - \frac{1}{2}\left(\frac{20}{3}\right)\left(\frac{5}{4} - 0.5\right)^2$$

$= 23.125\ m$

$\therefore\ s_c - s_t = 23.125 - 21.875$

$= 1.250\ m$

Therefore, to avoid the bump onto the truck, the car must maintain a distance from the truck more than 1.250 m.

43.

Sol:- Here, $u = 100\ ms^{-1}, g = -10\ ms^{-1}$

At highest point, $v = 0$

As $\quad v = u + gt$

$\Rightarrow \quad 0 = 100 - 10\ \times t$

$\therefore$ Time taken to reach highest point

$$t = \frac{100}{10} = 10\ s$$

The ball will return to the ground at $t = 20\ s$.

Velocity of the ball at different instants of time will be as follows.

At $t = 0, \quad v = 100 - 10\times 0 = 100\ ms^{-1}$

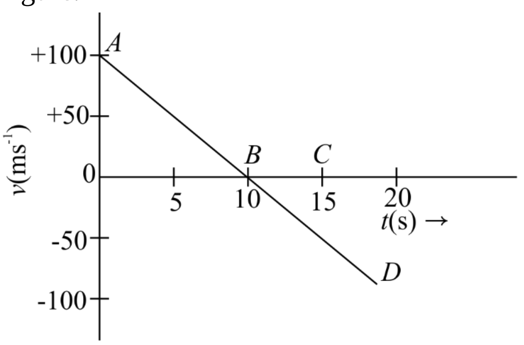
At $t = 5s, \ v = 100 - 10\times 5 = 50\ ms^{-1}$

At $t = 10s, v = 100 - 10\times 10 = 0$

At $t = 15s, \ v = 100 - 10\times 15 = -\ 50\ ms^{-1}$

At $t = 20s, \ v = 100 - 10\times 20 = -100\ ms^{-1}$

The velocity time-graph will be as shown in figure.



(i). Maximum height attained by ball

$=$ Area of ΔAOB

$= \frac{1}{2}\times 10s\times 100ms^{-1} = 500\ m$

(ii). Height attained after 15 s

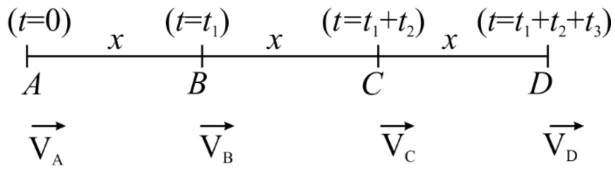
$= Area\ of\ \Delta AOb + Area\ of\ \Delta ACD$

$= 500 + \frac{1}{2}(15-10)\times(-50)$

$= 500 - 125 = 375m$

44.

Sol:- As shown in figure, let three successive equal distance be represented by AB, BC and CD



Let each distance be x, m. Let v_A, v_B, v_C and v_D be the velocity at points A, B, C and D respectively.

Average velocity between A and B = $\frac{v_A + v_B}{2}$

$\frac{v_A+v_B}{2}\ N\ t_A = x$ or $v_A + v_B = \frac{2x}{t_1}$

Similarly $v_B + v_C = \frac{2x}{t_2}$ and $v_C + v_D = \frac{2x}{t_3}$

Average velocity between A and D $\frac{v_A+v_D}{2}$

$$\therefore \quad \frac{v_A+v_D}{2}(t_1+t_2+t_3) = x + x + x$$

Or $\quad v_A + v_D = \frac{6x}{t_1+t_2+t_3}$

Hence, $v_A + v_D = (v_A + v_B) - (v_B + v_C) + (v_C + v_D)$

Or $\frac{6x}{t_1+t_2+t_3} = \frac{2x}{t_1} - \frac{2x}{t_2} + \frac{2x}{t_3}$

Or $\frac{3}{t_1+t_2+t_3} = \frac{1}{t_1} - \frac{1}{t_2} + \frac{1}{t_3}$

Solutions : Beginner Test - II

1.

(a), (b)

The size of a carriage is very small as compared to the distance between two stations. Therefore, the carriage can be treated as a point sized object.

The size of a monkey is very small as compared to the size of a circular track. Therefore, the monkey can be considered as a point sized object on the track.

The size of a spinning cricket ball is comparable to the distance through which it turns sharply on hitting the ground. Hence, the cricket ball cannot be considered as a point object.

The size of a beaker is comparable to the height of the table from which it slipped. Hence, the beaker cannot be considered as a point object.

2.

A lives closer to school than B.
A starts from school earlier than B.
B walks faster than A.
A and B reach home at the same time.
B overtakes A once on the road.
Explanation:
In the given x–t graph, it can be observed that distance OP < OQ. Hence, the distance of school from the A's home is less than that from B's home.
In the given graph, it can be observed that for x = 0, t = 0 for A, whereas for x = 0, t has some finite value for B. Thus, A starts his journey from school earlier than B.
In the given x–t graph, it can be observed that the slope of B is greater than that of A. Since the slope of the x–t graph gives the speed, a greater slope means that the speed of B is greater than the speed A.
It is clear from the given graph that both A and B reach their respective homes at the same time
B moves later than A and his/her speed is greater than that of A. From the graph, it is clear that B overtakes A only once on the road.

3.

Speed of the woman = 5 km/h
Distance between her office and home = 2.5 km

$$Time\ taken = \frac{Distance}{Speed}$$

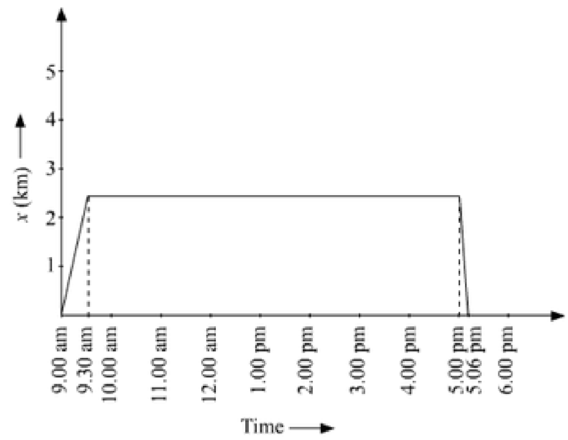
$$= \frac{2.5}{5} = 0.5\ h = 30\ min$$

It is given that she covers the same distance in the evening by an auto. Now, speed of the auto = 25 km/h

$$Time\ taken = \frac{Distance}{Speed}$$

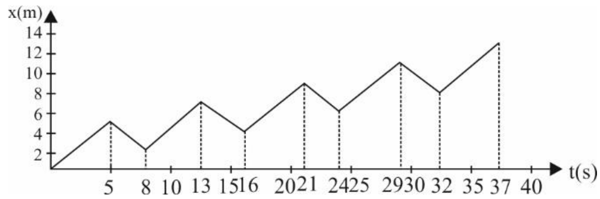
$$= \frac{2.5}{25} = \frac{1}{10} = 0.1\ h = 6\ min$$

The suitable x-t graph of the motion of the woman is shown in the given figure.



4.

Distance covered with 1 step = 1 m
Time taken = 1 s
Time taken to move first 5 m forward = 5 s
Time taken to move 3 m backward = 3 s
Net distance covered = 5 – 3 = 2 m
Net time taken to cover 2 m = 8 s
Drunkard covers 2 m in 8 s.
Drunkard covered 4 m in 16 s.
Drunkard covered 6 m in 24 s.
Drunkard covered 8 m in 32 s.
In the next 5 s, the drunkard will cover a distance of 5 m and a total distance of 13 m and falls into the pit. Net time taken by the drunkard to cover 13 m = 32 + 5 = 37 s
Net time taken by the drunkard to cover 13 m = 32 + 5 = 37 s



The x-t graph of the drunkard's motion can be shown as:

5.

Speed of the jet airplane, v_{jet} = 500 km/h
Relative speed of its products of combustion with respect to the plane,
v_{smoke} = – 1500 km/h
Speed of its products of combustion with respect to the ground = v'_{smoke}Relative speed of its products of combustion with respect to the airplane,
$v_{smoke} = v'_{smoke} - v_{jet}$
$1500 = v'_{smoke} - 500$

v'_{smoke} = – 1000 km/h
The negative sign indicates that the direction of its products of combustion is opposite to the direction of motion of the jet airplane.

6.

Initial velocity of the car, u = 126 km/h = 35 m/s
Final velocity of the car, v = 0
Distance covered by the car before coming to rest, s = 200 m
Retardation produced in the car = a
From third equation of motion, a can be calculated as:

$$v^2 - u^2 = 2as$$

$$(0)^2 - (35)^2 = 2 \times a \times 200$$

$$a = -\frac{35 \times 35}{2 \times 200} = -3.06\ m/s^2$$

From first equation of motion, time (t) taken by the car to stop can be obtained as:
V=u+at

$$t = \frac{v-u}{a} = \frac{-35}{-3.06} = 11.44s$$

7.

For train A:
Initial velocity, u = 72 km/h = 20 m/s
Time, t = 50 s
Acceleration, a_1 = 0 (Since it is moving with a uniform velocity) From second equation of motion, distance (s_I) covered by train A can be obtained as:

$$s_1 = ut + \frac{1}{2}a_1t^2$$

= 20 × 50 + 0 = 1000 m
For train B:
Initial velocity, u = 72 km/h = 20 m/s
Acceleration, a = 1 m/s2
Time, t = 50 s
From second equation of motion, distance (s_{II}) covered by train A can be obtained as:

$$s_{11} = ut + \frac{1}{2}at^2$$

$$= 20 \times 50 + \frac{1}{2} \times 1 \times (50)^2 = 2250\ m$$

Hence, the original distance between the driver of train A and the guard of train B = 2250 –1000 = 1250 m.

8.

Velocity of car A, v_A = 36 km/h = 10 m/s
Velocity of car B, v_B = 54 km/h = 15 m/s
Velocity of car C, v_C = 54 km/h = 15 m/s
Relative velocity of car B with respect to car A,
$v_{BA} = v_B - v_A$ = 15 – 10 = 5 m/s
At a certain instance, both cars B and C are at the same distance from car A i.e., s = 1 km = 1000 m
Time taken (t) by car C to cover 1000 m
$=\frac{1000}{25}$=40 sec
Hence, to avoid an accident, car B must cover the same distance in a maximum of 40 s. From second equation of motion, minimum acceleration (a) produced by car B can be obtained as:

$$s = ut + \frac{1}{2}at^2$$

$$1000 = 5 \times 40 + \frac{1}{2} \times a \times (40)^2$$

$$a = \frac{1600}{1600} = 1\ m/s^2$$

9.

Let V be the speed of the bus running between towns A and B.
Speed of the cyclist, v = 20 km/h
Relative speed of the bus moving in the direction of the cyclist
= V – v = (V – 20) km/h
The bus went past the cyclist every 18 min i.e., $\frac{18}{60}$ h (when he moves in the direction of the bus).
Distance covered by the bus =

$$(v-20)\frac{18}{60}km \ldots\ldots \text{(i)}$$

Since one bus leaves after every T minutes, the distance travelled by the bus will be equal to:

$$V \times \frac{T}{60} \ldots \text{(ii)}$$

Both equations (i) and (ii) are equal.

$$(v-20) \times \frac{18}{60} = \frac{VT}{60} \ldots\ldots \text{(iii)}$$

Relative speed of the bus moving in the opposite direction of the cyclist = (V + 20) km/h

Time taken by the bus to go past the cyclist = 6min $=\frac{6}{60}$h

$\therefore (v+20)\frac{6}{60}=\frac{VT}{60}$ ... ... (iv)

From equations (iii) and (iv), we get

$(v+20)\times\frac{6}{60}=(v-20)\times\frac{18}{60}$

$v+20=3v-60$

$2v=80$

$v=40\ km/h$

Substituting the value of V in equation (iv), we get

$(40+20)\times\frac{6}{60}=\frac{40T}{60}$

$T=\frac{360}{40}=9\ min$

10.

The direction of acceleration during the upward motion of the ball is downward.

Velocity = 0, acceleration = 9.8 m/s^2

x > 0 for both up and down motions,

v < 0 for up and v > 0 for down motion,

a > 0 throughout the motion

44.1 m, 6 s

Explanation:

Irrespective of the direction of the motion of the ball, acceleration (which is actually acceleration due to gravity) always acts in the downward direction towards the centre of the Earth.

At maximum height, velocity of the ball becomes zero. Acceleration due to gravity at a given place is constant and acts on the ball at all points (including the highest point) with a constant value i.e., 9.8 m/s^2 .

During upward motion, the sign of position is positive, sign of velocity is negative, and sign of acceleration is negative. During downward motion, the signs of position, velocity, and acceleration are all positive.

Initial velocity of the ball, u = 29.4 m/s

Final velocity of the ball, v = 0 (At maximum height, the velocity of the ball becomes zero)

Acceleration, a = – g = – 9.8 m/s^2

From third equation of motion, height (s) can be calculated as:

$v^2-u^2=2gs$

$s=\frac{v^2-u^2}{2g}$

$=\frac{(0)^2-(29.4)^2}{2\times(-9.8)}=44.1\ m$

From first equation of motion, time of ascent (t) is given as:

$v^2-u^2=2gs$

$s=\frac{v^2-u^2}{2g}$

$=\frac{(0)^2-(29.4)^2}{2\times(-9.8)}=44.1\ m$

Time of ascent = Time of descent Hence, the total time taken by the ball to return to the player's hands = 3 + 3 = 6 s.

11.

a) True

b) False

c) True

d) False

Explanation:

When an object is thrown vertically up in the air, its speed becomes zero at maximum height. However, it has acceleration equal to the acceleration due to gravity (g) that acts in the downward direction at that point.

Speed is the magnitude of velocity. When speed is zero, the magnitude of velocity along with the velocity is zero.

A car moving on a straight highway with constant speed will have constant velocity. Since acceleration is defined as the rate of change of velocity, acceleration of the car is also zero.

This statement is false in the situation when acceleration is positive and velocity is negative at the instant time taken as origin. Then, for all the time before velocity becomes zero, there is slowing down of the particle. Such a case happens when a particle is projected upwards.

This statement is true when both velocity and acceleration are positive, at the instant time taken as origin. Such a case happens when a particle is moving with positive acceleration or falling vertically downwards from a height.

12.

Ball is dropped from a height, s = 90 m

Initial velocity of the ball, u = 0

Acceleration, a = g = 9.8 m/s^2

Final velocity of the ball = v

From second equation of motion, time (t) taken by the ball to hit the ground can be obtained as:

$$s = ut + \frac{1}{2}at^2$$

$$90 = 0 + \frac{1}{2} \times 9.8t^2$$

$$t = \sqrt{18.38} = 4.29\ s$$

From first equation of motion, final velocity is given as: v = u + at = 0 + 9.8 × 4.29 = 42.04 m/s

Rebound velocity of the ball, ur =

$$\frac{9}{10}v = \frac{9}{10} \times 42.04 = 37.84 m/s$$

Time (t) taken by the ball to reach maximum height is obtained with the help of first equation of motion as: v = ur + at′

0 = 37.84 + (– 9.8) t′

$$t' = \frac{-37.84}{-9.8} = 3.86\ s$$

Total time taken by the ball = t + t′ = 4.29 + 3.86 = 8.15 s

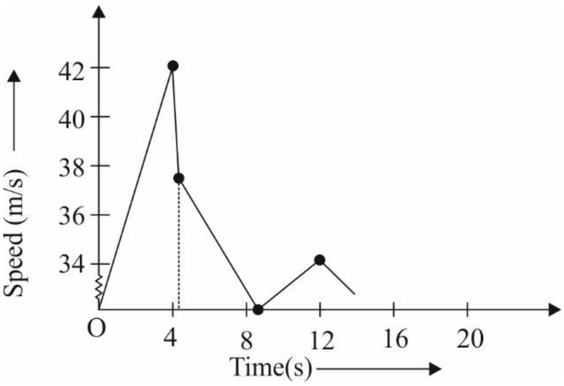
As the time of ascent is equal to the time of descent, the ball takes 3.86 s to strike back on the floor for the second time.

The velocity with which the ball rebounds from the floor

$$= \frac{9}{10} \times 37.84 = 34.05$$

Total time taken by the ball for second rebound = 8.15 + 3.86 = 12.01 s

The speed-time graph of the ball is represented in the given figure as:



13.

a)

The magnitude of displacement over an interval of time is the shortest distance (which is a straight line) between the initial and final positions of the particle. The total path length of a particle is the actual path length covered by the particle in a given interval of time.

For example, suppose a particle moves from point A to point B and then, comes back to a point, C taking a total time t, as shown below. Then, the magnitude of displacement of the particle = AC.

Whereas, total path length = AB + BC

It is also important to note that the magnitude of displacement can never be greater than the total path length. However, in some cases, both quantities are equal to each other.

b)$Magnitude\ of\ average\ velocity$

$$= \frac{Magnitude\ of\ displacement}{Time\ interval}$$

For the given particle,

$$Average\ velocity = \frac{AC}{t}$$

$$Average\ speed = \frac{Total\ path\ length}{Time\ interval}$$

$$= \frac{AB + BC}{t}$$

Since (AB + BC) > AC, average speed is greater than the magnitude of average velocity.

The two quantities will be equal if the particle continues to move along a straight line.

14.

Time taken by the man to reach the market from home,

$$t_1 = \frac{2.5}{5} = \frac{1}{2}h = 30\ min$$

Time taken by the man to reach home from the market,

$$t_2 = \frac{2.5}{7.5} = \frac{1}{3}h = 20\ min$$

Total time taken in the whole journey = 30 + 20 = 50 min

$$Average\ velocity = \frac{Displacement}{Time} = \frac{2.5}{\frac{1}{2}}$$

$$= 5km/h \ldots (a(\text{i}))$$

$$Average\ speed = \frac{Distance}{Time} = \frac{2.5}{\frac{1}{2}}$$

$$= 5km/h \ldots (b(\text{i}))$$

Time = 50 min = $\frac{5}{6}$ hr
Net displacement = 0
Total distance = 2.5 + 2.5 = 5 km

$$Average\ velocity = \frac{Displacement}{Time}$$
$$= 0 \ldots (a(\text{ii}))$$
$$Average\ speed = \frac{Distance}{Time} = \frac{5}{\frac{5}{6}}$$
$$= 6km/h \ldots (b(\text{ii}))$$

Speed of the man = 7.5 km
Distance travelled in first 30 min = 2.5 km
Distance travelled by the man (from market to home) in the next 10 min

$$= 7.5 \times \frac{10}{60} = 1.25km$$

Net displacement = 2.5 – 1.25 = 1.25 km
Total distance travelled = 2.5 + 1.25 = 3.75 km

$$Average\ velocity = \frac{1.25}{\frac{40}{60}} = \frac{1.25 \times 3}{2}$$
$$= 1.875km/h \ldots (a(\text{iii}))$$
$$Average\ speed = \frac{3.75}{\frac{40}{60}}$$
$$= 5.625km/h \ldots (b(\text{iii}))$$

15.
Instantaneous velocity is given by the first derivative of distance with respect to time i.e.,

$$v_{In} = \frac{dx}{xt}$$

Here, the time interval dt is so small that it is assumed that the particle does not change its direction of motion. As a result, both the total path length and magnitude of displacement become equal in this interval of time.
Therefore, instantaneous speed is always equal to instantaneous velocity.

16.
The given x-t graph, shown in (a), does not represent one-dimensional motion of the particle. This is because a particle cannot have two positions at the same instant of time.
The given v-t graph, shown in (b), does not represent one-dimensional motion of the particle. This is because a particle can never have two values of velocity at the same instant of time.
The given v-t graph, shown in (c), does not represent one-dimensional motion of the particle. This is because speed being a scalar quantity cannot be negative.
The given v-t graph, shown in (d), does not represent one-dimensional motion of the particle. This is because the total path length travelled by the particle cannot decrease with time.

17.
No. The x-t graph of a particle moving in a straight line for t < 0 and on a parabolic path for t > 0 cannot be shown as the given graph. This is because, the given particle does not follow the trajectory of path followed by the particle as t = 0, x = 0. A physical situation that resembles the above graph is of a freely falling body held for some time at a height

18.
Speed of the police van, v_p = 30 km/h = 8.33 m/s
Muzzle speed of the bullet, v_b = 150 m/s
Speed of the thief's car, v_t = 192 km/h = 53.33 m/s
Since the bullet is fired from a moving van, its resultant speed can be obtained as: = 150 + 8.33 = 158.33 m/s
Since both the vehicles are moving in the same direction, the velocity with which the bullet hits the thief's car can be obtained as: v_{bt}t = $v_b - v_t$ = 158.33 – 53.33 = 105 m/s

19.
(a) The given x-t graph shows that initially a body was at rest. Then, its velocity increases with time and attains an instantaneous constant value. The velocity then reduces to zero with an increase in time. Then, its velocity increases with time in the opposite direction and acquires a constant value. A similar physical situation arises when a football (initially kept at rest) is kicked and gets rebound from a rigid wall so that its speed gets reduced. Then, it passes from the player who has

kicked it and ultimately gets stopped after sometime.

(b) In the given v-t graph, the sign of velocity changes and its magnitude decreases with a passage of time. A similar situation arises when a ball is dropped on the hard floor from a height. It strikes the floor with some velocity and upon rebound, its velocity decreases by a factor. This continues till the velocity of the ball eventually becomes zero.

(c) The given a-t graph reveals that initially the body is moving with a certain uniform velocity. Its acceleration increases for a short interval of time, which again drops to zero. This indicates that the body again starts moving with the same constant velocity. A similar physical situation arises when a hammer moving with a uniform velocity strikes a nail.

20.

Negative, Negative, Positive (at t = 0.3 s)
Positive, Positive, Negative (at t = 1.2 s)
Negative, Positive, Positive (at t = –1.2 s)
For simple harmonic motion (SHM) of a particle, acceleration (a) is given by the relation:
$a = -\omega^2 x$ ω → angular frequency
……………..… (i)
t = 0.3 s
In this time interval, x is negative. Thus, the slope of the x-t plot will also be negative. Therefore, both position and velocity are negative. However, using equation (i), acceleration of the particle will be positive. t = 1.2 s
In this time interval, x is positive. Thus, the slope of the x-t plot will also be positive. Therefore, both position and velocity are positive. However, using equation (i), acceleration of the particle comes to be negative. t = – 1.2 s
In this time interval, x is negative. Thus, the slope of the x-t plot will also be negative. Since both x and t are negative, the velocity comes to be positive. From equation (i), it can be inferred that the acceleration of the particle will be positive.

21.

Interval 3 (Greatest), Interval 2 (Least)
Positive (Intervals 1 & 2), Negative (Interval 3)
The average speed of a particle shown in the x-t graph is obtained from the slope of the graph in a particular interval of time.
It is clear from the graph that the slope is maximum and minimum respectively in intervals 3 and 2. Therefore, the average speed of the particle is the greatest in interval 3 and is the least in interval 2. The sign of average velocity is positive in both intervals 1 and 2 as the slope is positive in these intervals. However, it is negative in interval 3 because the slope is negative in this interval.

22.

Average acceleration is greatest in interval 2
Average speed is greatest in interval 3 v is positive in intervals 1, 2, and 3 a is positive in intervals 1 and 3 and negative in interval 2 a = 0 at A, B, C, D
Acceleration is given by the slope of the speed-time graph. In the given case, it is given by the slope of the speed-time graph within the given interval of time. Since the slope of the given speed-time graph is maximum in interval 2, average acceleration will be the greatest in this interval. Height of the curve from the time-axis gives the average speed of the particle. It is clear that the height is the greatest in interval 3. Hence, average speed of the particle is the greatest in interval 3.
In interval 1:
The slope of the speed-time graph is positive. Hence, acceleration is positive. Similarly, the speed of the particle is positive in this interval.
In interval 2:
The slope of the speed-time graph is negative. Hence, acceleration is negative in this interval. However, speed is positive because it is a scalar quantity.
In interval 3:

The slope of the speed-time graph is zero. Hence, acceleration is zero in this interval. However, here the particle acquires some uniform speed. It is positive in this interval. Points A, B, C, and D are all parallel to the time-axis. Hence, the slope is zero at these points. Therefore, at points A, B, C, and D, acceleration of the particle is zero.

23.

Straight line

Distance covered by a body in n^{th} second is given by the relation

$D_n = u = \frac{a}{2}(2n-1)$ ... (i)

Where,

u = Initial velocity

a = Acceleration

n = Time = 1, 2, 3, ... , n

$\therefore D_n = \frac{1}{2}(2n-1)$ ... (ii)

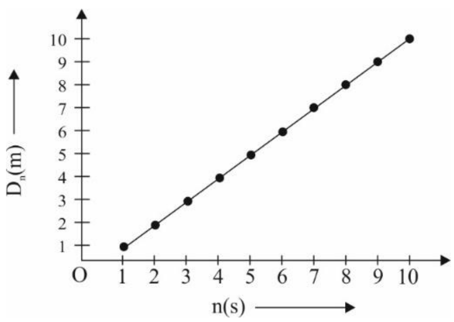
In the given case, u = 0 and a = 1 m/s^2

This relation shows that:

$D_n \propto$ n … (iii)

Now, substituting different values of n in equation (iii), we get the following table:

n	1	2	3	4	5	6	7	8	9	10
D_n	0.5	1.5	2.5	3.5	4.5	5.5	6.5	7.5	8.5	9.5



The plot between n and D_n will be a straight line as shown:

Since the given three-wheeler acquires uniform velocity after 10 s, the line will be parallel to the time-axis after n = 10 s.

24.

Initial velocity of the ball, u = 49 m/s

Acceleration, a = – g = – 9.8 m/s^2

Case I:

When the lift was stationary, the boy throws the ball.

Taking upward motion of the ball,

Final velocity, v of the ball becomes zero at the highest point.

From first equation of motion, time of ascent (t) is given as:

$v = u + at$

$t = \frac{v-u}{a}$

$= \frac{-49}{-9.8} = 5\ s$

But, the time of ascent is equal to the time of descent. Hence, the total time taken by the ball to return to the boy's hand = 5 + 5 = 10 s.

Case II:

The lift was moving up with a uniform velocity of 5 m/s. In this case, the relative velocity of the ball with respect to the boy remains the same i.e., 49 m/s. Therefore, in this case also, the ball will return back to the boy's hand after 10 s.

25.

Speed of the belt, v_B = 4 km/h

Speed of the boy, v_b = 9 km/h

a) Since the boy is running in the same direction of the motion of the belt, his speed (as observed by the stationary observer) can be obtained as:

v_BB = v_b + v_B = 9 + 4 = 13 km/h

b) Since the boy is running in the direction opposite to the direction of the motion of the belt, his speed (as observed by the stationary observer) can be obtained as: v_bB = v_b + (–v_B) = 9 – 4 = 5 km

c) Distance between the child's parents = 50 m

As both parents are standing on the moving belt, the speed of the child in either direction as observed by the parents will remain the same i.e., 9 km/h = 2.5 m/s.

Hence, the time taken by the child to move towards one of his parents is 50/2.5 = 20s.

If the motion is viewed by any one of the parents, answers obtained in (a) and (b) get altered. This is because the child and his parents are standing on the same belt and hence, are

equally affected by the motion of the belt.
Therefore, for both parents (irrespective of the direction of motion) the speed of the child remains the same i.e., 9 km/h.
For this reason, it can be concluded that the time taken by the child to reach any one of his parents remains unaltered.

26.

For first stone:
Initial velocity, u_I = 15 m/s
Acceleration, a = –g = – 10 m/s^2
Using the relation,
$x_1 = x_0 + u_1 t + \frac{1}{2}at^2$
Where, height of the cliff, x_0=200 m
$x_1 = 200 + 15t - 5t^2$ ... (i)
When this stone hits the ground, x_1 = 0
$\therefore -5t^2 + 15t + 200 = 0$
$t^2 - 3t - 40 = 0$
$t^2 - 8t + 5t - 40 = 0$
$t(t-8) + 5(t-8) = 0$
t = 8 s or t = -5 s
Since the stone was projected at time t = 0, the negative sign before time is meaningless.
∴t = 8 s
For second stone:
Initial velocity, u_{II} = 30 m/s
Acceleration, a = –g = – 10 m/s^2
$x_2 = x_0 + u_{11} t + \frac{1}{2}at^2$
$= 200 + 30t - 5t^2$ ... (ii)
Using the relation,
At the moment when this stone hits the ground;
x_2 = 0
$5t^2 + 30t + 200 = 0$
$t^2 - 6t - 40 = 0$
$t^2 - 10t + 4t + 40 = 0$
$t(t-10) + 4(t-10) = 0$
$t(t-10)(t+4) = 0$
t = 10 s or t = -4 s
Here again, the negative sign is meaningless.
∴t = 10 s
Subtracting equations (i) and (ii), we get

$x_2 - x_1 = (200 + 30t - 5t^2) - (200 + 15t - 5t^2)$
$x_2 - x_1 = 15t$ ... (iii)

$x_2 - x_1 = (200 + 30t - 5t^2) - (200 + 15t - 5t^2)$
$x_2 - x_1 = 15t$ ... (iii)
Equation (iii) represents the linear path of both stones. Due to this linear relation between ($x_2 - x_1$) and t, the path remains a straight line till 8 s.
Maximum separation between the two stones is at t = 8 s.
$(x_2 - x_1)$max = 15× 8 = 120 m
This is in accordance with the given graph.
After 8 s, only second stone is in motion whose variation with time is given by
the quadratic equation: $x_2 - x_1$ = 200 + 30t – $5t^2$
Hence, the equation of linear and curved path is given by
$x_2 - x_1$ = 15t (Linear path)
$x_2 - x_1$ = 200 + 30t – $5t^2$ (Curved path)

27.

Distance travelled by the particle = Area under the given graph
$= \frac{1}{2} \times (10-0) \times (12-0) = 60\ m$
$Average\ speed = \frac{Distance}{Time} = \frac{60}{10} = 6\ m/s$
Let s_1 and s_2 be the distances covered by the particle between time
t = 2 s to 5 s and t = 5 s to 6 s respectively.
Total distance (s) covered by the particle in time t = 2 s to 6
s s = s_1 + s_2 … (i)
For distance s_1:
Let u′ be the velocity of the particle after 2 s and a′ be the acceleration of the particle in t = 0 to t = 5 s.
Since the particle undergoes uniform acceleration in the interval t = 0 to t = 5 s, from first equation of motion, acceleration can be obtained as:
v = u + at
Where,
v = Final velocity of the particle
12 = 0 + a′ × 5
$a' = \frac{12}{5} = 2.4 m/s^2$
Again, from first equation of motion, we have

v = u + at

= 0 + 2.4 × 2 = 4.8 m/s

Distance travelled by the particle between time 2 s and 5 s i.e., in 3 s

For distance s2:

Let a″ be the acceleration of the particle between time t = 5 s and t = 10 s. From first equation of motion, v = u + at (where v = 0 as the particle finally comes to rest) 0 = 12 + a″ × 5

$a'' = \frac{-12}{5}$

$= -2.4 m/s^2$

Distance travelled by the particle in 1s (i.e., between t = 5 s and t = 6 s)

$s_2 = u''t + \frac{1}{2}at^2$

$= 12 \times a + \frac{1}{2}(-2.4) \times (1)^2$

$= 12 - 1.2 = 10.8\, m$ ... (iii)

From equations (i), (ii), and (iii), we get

s = 25.2 + 10.8 = 36 m

$\therefore Average\ speed = \frac{36}{4} = 9\ m/s$

28.

The correct formulae describing the motion of the particle are (c), (d) and, (f)

The given graph has a non-uniform slope. Hence, the formulae given in (a), (b), and (e) cannot describe the motion of the particle. Only relations given in (c), (d), and (f) are correct equations of motion.

Solutions: Expert Test - I

1.

(b) For distance x, the person moves with constant velocity v_1 and for another x distance, he moves with constant velocity of v_2, then

Total distance travelled, D = x + x = 2x

Total time-taken, T = $t_1 + t_2$

$= \frac{x}{v_1} + \frac{x}{v_2} \qquad \left[\because t = \frac{Distance}{Velocity}\right]$

The average velocity,

$V_{av} = \frac{total\ distance}{total\ time} = \frac{D}{T}$

$v = \frac{2x}{\frac{x}{v_1} + \frac{x}{v_2}} = \frac{2}{\frac{1}{v_1} + \frac{1}{v_2}} \qquad [\because v_{av} = v]$

$\frac{1}{v_1} + \frac{1}{v_2} = \frac{2}{v}$

2.

(c) Speed of walking = $\frac{h}{t_1}$ = v_1

Speed of escalator = $\frac{h}{t_2}$ = v_2

Time taken when she walks over running escalator

$t = \frac{h}{v_1 + v_2}$

$\frac{1}{t} = \frac{v_1}{h} + \frac{v_2}{h} = \frac{1}{t_1} + \frac{1}{t_2}$

$t = \frac{t_1 t_2}{t_1 + t_2}$

3.

(b) Velocity of the particle is given as

v = At + Bt^2

where A and B are constants.

$\frac{dx}{dt}$ = At + Bt^2

$\left[\because v = \frac{dx}{dt}\right]$

dx = (At + Bt^2) dt

Integrating both sides, we get

$\int_{x_1}^{x_2} dx = \int_1^2 \left(At + Bt^2\right) dt$

$\Delta x = x_2 - x_1 = A\int_1^2 t dt + B\int_1^2 t^2 dt$

$= A\left[\frac{t^2}{2}\right]_1^2 + B\left[\frac{t^3}{3}\right]_1^2$

$= \frac{A}{2}\left(2^2 - 1^2\right) + \frac{B}{3}\left(2^3 - 1^3\right)$

∴ Distance travelled between 1s and 2s is

$\Delta x = \frac{A}{2} \times (3) + \frac{B}{3}(7) = \frac{3A}{2} + \frac{7B}{3}$

4.

(d) Velocity of each car is given by

$V_P = \frac{dx_P(t)}{dt} = a + 2bt$

and $V_Q = \frac{dx_Q(t)}{dt} = f - 2t$

It is given that $V_P = V_Q$

a + 2bt = f – 2t

$$t = \frac{f-a}{2(b+1)}$$

5.

(d) Given, the initial velocity of a car, u = 0

The acceleration of a car, a = 5 m/s^2

AT t = 4 s, v = u + at

v = 0 +(5)4 ⟹ v = 20 m/s

Thus, the final velocity of car at t = 4 s is 20 m/s.

At t = 4 s, the ball is dropped out of a window by a person sitting in the car.

The velocity of the ball in the x-direction, v_x =20 m/s (due to the car)

Therefore, in the y-direction, the acceleration is equal to the acceleration due to gravity,

a_y = g = 10 m/s^2

The velocity of the ball in the y-direction,

$v_y = u + a_y t \Rightarrow v_y = 0 + 10 \times 2$

v_y = 20 m/s

Thus, the velocity of the ball in y-direction is 20 m/s.

The net velocity at t = 6s,

$$v = \sqrt{v_x^2 + v_y^2} \Rightarrow v = \sqrt{(20)^2 + (20)^2}$$

v = $20\sqrt{2}$ m/s

Thus, the velocity of the ball at t = 6 s is $20\sqrt{2}$ m/s.

and there is no acceleration in the x-direction, a_x = 0 ms^{-2}

In y-direction, a_y = 10 ms^{-2}

Now, we shall determine the net acceleration

at t = 6s, $a = \sqrt{a_x^2 + a_y^2}$

$$a = \sqrt{(0) + (10)^2} \Rightarrow a = 10 \text{ ms}^{-2}$$

6.

(b) Distance covered nth seconds is s_n.

Distance covers in(n + 1) th seconds is s_{n+1}.

Initial velocity of small block, u = 0

Distance cover in nth seconds,

$$s_n = u + \frac{a}{2}(2n-1)$$

$$s_n = 0 + \frac{a}{2}(2n-1)$$

$$s_n = \frac{a}{2}(2n-1) \quad \ldots (i)$$

Distance cover in (n + 1)th seconds,

$$s_{n+1} = u + \frac{a}{2}[2(n+1)-1]$$

$$s_{n+1} = 0 + \frac{a}{2}[2n+2-1]$$

$$s_{n+1} = \frac{a}{2}(2n+1) \quad \ldots (ii)$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{s_n}{s_{n+1}} = \frac{\frac{a}{2}(2n-1)}{\frac{a}{2}(2n+1)}$$

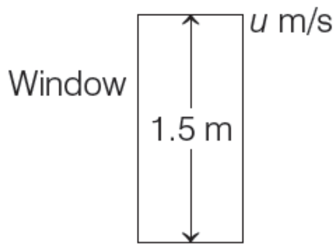
$$\frac{s_n}{s_{n+1}} = \frac{(2n-1)}{(2n+1)}$$

7.

(b) According to question, time taken by the ball to cross the window,

t = 0.1 s

h = 1.5 m



If u be the velocity at the top most point of the window, then from equation of motion,

$$h = ut + \frac{1}{2}gt^2$$

$$1.5 = u \times 0.1 + \frac{1}{2} \times 10 \times (0.1)^2$$

$$1.5 = 0.1u + 0.05$$

$$u = \frac{1.5 - 0.05}{0.1} = \frac{1.45}{0.1}$$

= 14.5 m/s

8.

(c) Given, u = 20 m/s, v = 80 m/s and h= ?

From kinematic equation of motion,

$v^2 = u^2 + 2gh$

$$h = \frac{v^2 - u^2}{2g}$$

$$= \frac{(80)^2 - (20)^2}{2 \times 10} \quad (\because \text{ given, g = 10 m/s}^2)$$

= 300 m

Hence, correct option is (c).

9.

(d) Let h be the height through which the coin is dropped. Then, according to the equation of motion, it is given as

$$h = ut + \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}} \qquad [\because u = 0]$$

$$t \propto \frac{1}{\sqrt{g}}$$

As the elevator is moving uniformly i.e. its velocity is constant, so the acceleration is zero.

∴ Relative acceleration of the lift when it is either moving upward or downward is given as, $g' = g \pm a = g \pm 0 = g$

Hence, the time for the coin to reach the floor will remains same i.e. $t_1 = t_2$.

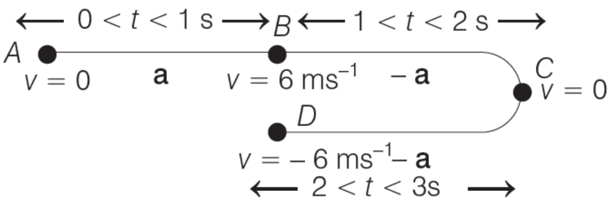
10.

(b) According to the question, For the time duration $0 < t < 1$s,

the velocity increase from 0 to 6 ms^{-1}

As the direction of field has been reversed for, $1 < t < 2$s: the velocity firstly decreases from 6ms^{-1} to 0.

Then, for $2 < t < 3$s; as the field strength is same; the magnitude of acceleration would be same, but velocity increases from 0 to –6 ms^{-1}.



Acceleration of the car

$$|a| = \left|\frac{v-u}{t}\right| = \frac{6-0}{1} = 6 \text{ ms}^{-2}$$

The displacement of the particle is given as

$$s = ut + \frac{1}{2}at^2$$

For t = 0 to t = 1s,

u = 0, a = + 6 m/s^2

$$s_1 = 0 + \frac{1}{2} \times 6 \times (1)^2 = 3 \text{ m}$$

For t = 1 s to t = 2s

u = 6 ms^{-1}, a = –6 ms^{-2}

$$s_2 = 6 \times 1 - \frac{1}{2} \times 6 \times (1)^2 = 6 - 3 = 3\text{m}$$

For t = 2s to t = 3s,

u = 0, a = –6 ms^{-1}

$$s_3 = 0 - \frac{1}{2} \times 6 \times (1)^2 = -3\text{ m}$$

∴ Net displacement, $s = s_1 + s_2 + s_3$

= 3m + 3m – 3m = 3m

Hence, average velocity

$$= \frac{Net\ displacement}{Total\ time} = \frac{3}{3} = 1 \text{ ms}^{-1}$$

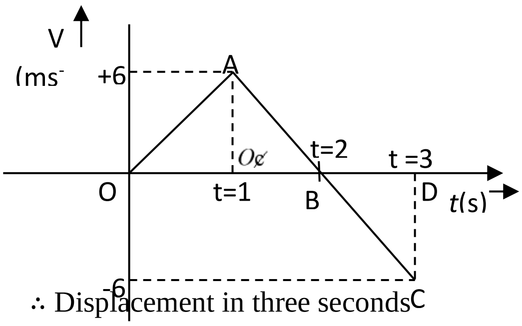
Total distance travelled, d = 9m

Hence, average speed = $\frac{Total\ dis\tan ce}{Total\ time}$

$$= \frac{9}{3} = 3 \text{ ms}^{-1}$$

Alternative Method

Given condition can be represented through graph also as shown below.



∴ Displacement in three seconds

= Area under the graph

= Area of ΔOAO' + Area of ΔAO'B – Area of ΔBCD

$$= \frac{1}{2} \times 1 \times 6 + \frac{1}{2} \times 1 \times 6 - \frac{1}{2} \times 6 \times 1 = 3\text{m}$$

∴ Average velocity = $\frac{3}{3}$ = 1ms^{-1}.

Total distance travelled, d = 9m

∴ Average speed= $\frac{9}{3}$ = 3ms^{-1}

11.

(b) For free fall from a height, u = 0

∴ Distance covered by stone in first 5 s,

$$h_1 = 0 + \frac{1}{2}g(5)^2 = \frac{25}{2}g \qquad \text{... (i)}$$

∴ Distance covered in first 10 s,

$$s_2 = 0 + \frac{1}{2}g(10)^2 = \frac{100}{2}g$$

∴ Distance covered in second 5 s

$$h_2 = s_2 - h_1 = \frac{100}{2}g - \frac{25}{2}g = \frac{72}{2}g \quad \ldots \text{(ii)}$$

Distance covered in first 15 s,

$$s_3 = 0 + \frac{1}{2}g(15)^2 = \frac{225}{2}g$$

∴ Distance covered in last 5 s,

$$h_3 = s_3 - s_2 = \frac{225}{2}g - \frac{100}{2}g = \frac{125}{2}g \quad \ldots \text{(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$h_1 : h_2 : h_3 = \frac{25}{2}g : \frac{75}{2}g : \frac{125}{2}g = 1:3:5$$

$$h_1 = \frac{h_2}{3} = \frac{h_3}{5}$$

Solutions: Expert Test - II

1.

(b) Given, $v = b\sqrt{x}$

or $\frac{dx}{dt} = bx^{1/2}$

or $\int_0^x x^{-1/2}dx = \int_0^t b dt$

or $\frac{x^{1/2}}{1/2} = bt$

or $x = \frac{b^2t^2}{4}$

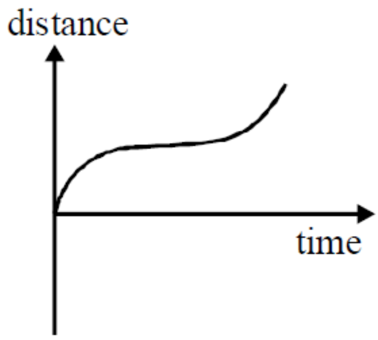
Differentiating w.r.t. time, we get

$\frac{dx}{dt} = \frac{b^2 \times 2t}{4}$ (t = τ)

or $v = \frac{b^2\tau}{2}$

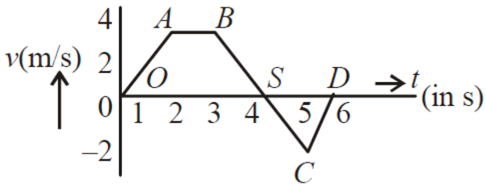
2.

(b) Graphs in option (c) position-time and option (a) velocity-position are corresponding to velocity-time graph option (d) and its distance-time graph is as given below. Hence distance-time graph option (b) is incorrect.



3.

(a)



$$OS = 4 + \frac{1}{3} = \frac{13}{3}$$

$$SD = 2 - \frac{1}{3} = \frac{5}{3}$$

Distance covered by the body = area of v-t graph = area (OABS) + area (SCD)

$$= \frac{1}{2}\left(\frac{13}{3}+1\right)\times 4 + \frac{1}{2}\times\frac{5}{3}\times 2$$

$$= \frac{32}{3} + \frac{5}{3} = \frac{37}{3}m$$

4.

(b) From the third equation of motion

v2 – u2 = 2aS

But, $a = \frac{F}{m}$

∴ $v^2 = u^2 - 2\left(\frac{F}{m}\right)S$

$$v^2 = (1)^2 - (2)\left[\frac{2.5\times10^{-2}}{20\times10^{-3}}\right]\frac{20}{100}$$

$v^2 = 1 - \frac{1}{2} \Rightarrow v = \frac{1}{\sqrt{2}}$ m/s = 0.7 m/s

5.

(b) $x = at + bt^2 - ct^3$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}\left(at + bt^2 + ct^3\right)$

$= a + 2bt - 3ct^2$

Acceleration, $\frac{dv}{dt} = \frac{d}{dt}\left(a + 2bt - 3ct^2\right)$

or $0 = 2b - 3c \times 2t$

$\therefore \; t = \left(\frac{b}{3c}\right)$

and $v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2$

$= \left(a + \frac{b^2}{3c}\right)$

6.

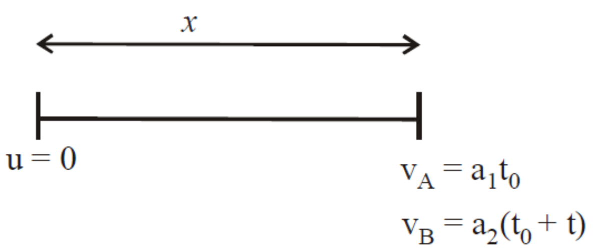
(d) For constant acceleration, there is straight line parallel to t-axis on $\vec{a} - t$.

Inclined straight line on $\vec{v} - t$, and parabola on $\vec{x} - t$.

7.

(c) Let time taken by A to reach finishing point is t_0

$\therefore$ Time taken by B to reach finishing point = $t_0 + t$



$v_A - v_B = v$

$v = a_1 t_0 - a_2(t_0 + t) = (a_1 - a_2)t_0 - a_2 t$ ...(i)

$x_B = x_A = \frac{1}{2}a_1 \; t_0^2 = \frac{1}{2}a_2\left(t_0 + t\right)^2$

$\sqrt{a_1 t_0} = \sqrt{a_2}\left(t_0 + t\right) \Rightarrow \left(\sqrt{a_1} - \sqrt{a_2}\right)t_0 = \sqrt{a_2 t}$

$t_0 = \frac{\sqrt{a_2 t}}{\sqrt{a_1} - \sqrt{a_2}}$

Putting this value of t_0 in equation (i)

$v = \left(a_1 - a_2\right)\frac{\sqrt{a_2 t}}{\sqrt{a_1} - \sqrt{a_2}} = -a_2 t$

$= \left(\sqrt{a_1} + \sqrt{a_2}\right)\sqrt{a_2} t - a_2 t = \sqrt{a_1 a_2} t + a_2 t - a_2 t$

or, $v = \sqrt{a_1 a_2} t$

8.

(c) Using equation, $a = \frac{v - u}{t}$ and $S = ut + \frac{1}{2}at^2$

Distance travelled by car in 15 sec

$= \frac{1}{2}\frac{(45)}{15}(15)^2$

$= \frac{675}{2} m$

Distance travelled by scooter in 15 seconds

= 30 × 15 = 450

(∵ distance = speed × time)

Difference between distance travelled by car and scooter in 15 sec, 450 – 337.5 = 112.5 m

Let car catches scooter in time t;

$\frac{675}{2} + 45(t - 15) = 30t$

337.5 + 45t – 675 = 30t

15t = 337.5

t = 22.5 sec

9.

(a) Let the car turn of the highway at distance 'x' from the point M. So, RM = x

And if speed of car in field is v, then time taken by the car to cover the distance QR = QM – x on the highway,

$t_1 = \frac{QM - x}{2v}$ ... (i)

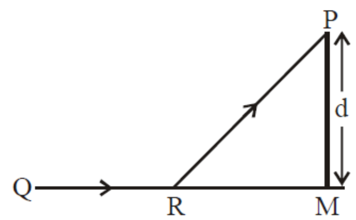
Time taken to travel the distance 'RP' in the field

$t_2 = \frac{\sqrt{d^2 + x^2}}{v}$ ... (ii)

Total time elapsed to move the car from Q to P

$t = t_1 + t_2 = \frac{QM - x}{2v} + \frac{\sqrt{d^2 + x^2}}{v}$

For 't' to be minimum $\frac{dt}{dx} = 0$



$\frac{1}{v}\left[-\frac{1}{2} + \frac{x}{\sqrt{d^2 + x^2}}\right] = 0$

or $x = \dfrac{d}{\sqrt{2^2 - 1}} = \dfrac{d}{\sqrt{3}}$

10.

(c) According to question, object is moving with constant negative acceleration i.e., a = – constant (C)

$\dfrac{vdv}{dx} = -C$

vdv = – Cdx

$\dfrac{v^2}{2} = -Cx + k$

$x = -\dfrac{v^2}{2C} + \dfrac{k}{C}$

Hence, graph (3) represents correctly.

11.

(b) Distance along a line i.e., displacement (s) = t3 (∵ s ∝ t3 given)

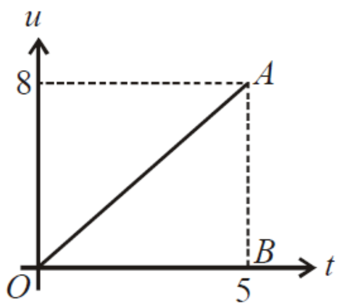
By double differentiation of displacement, we get acceleration,

$V = \dfrac{ds}{dt} = \dfrac{dt^3}{dt} = 3t^2$ and $a = \dfrac{dv}{dt} = \dfrac{d3t^2}{dt} = 6t$

a = 6t or a ∝ t

Hence graph (b) is correct.

12.



Distance travelled = Area of speed – time graph

$= \dfrac{1}{2} \times 5 \times 8$ = 20 m

13.

Distance X varies with time t as

$x^2 = at^2 + 2bt + c$

$2x\dfrac{dx}{dt} = 2at + 2b$

$x\dfrac{dx}{dt} = at + b$

$\dfrac{dx}{dt} = \dfrac{(at+b)}{x}$

$x\dfrac{d^2x}{dt^2} + \left(\dfrac{dx}{dt}\right)^2 = a$

$\dfrac{d^2x}{dt^2} = \dfrac{a - \left(\dfrac{dx}{dt}\right)^2}{x} = \dfrac{a - \left(\dfrac{at+b}{x}\right)^2}{x}$

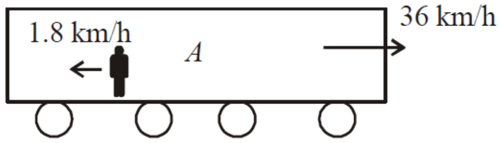
$= \dfrac{ax^2 - (at+b)^2}{x^3} = \dfrac{ac - b^2}{x^3}$

$a \propto x^{-3}$

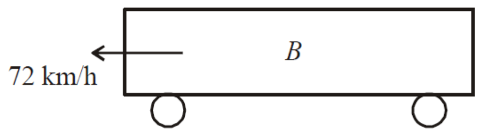
Hence, n = 3

14.

(a) According to question, train A and B are running on parallel tracks in the opposite direction.



V_A = 36 km/h = 10 m/s



V_B = – 72 km/h = –20 m/s

V_{MA} = – 1.8 km/h = – 0.5 m/s

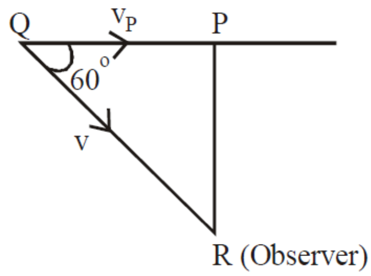
$V_{man, B} = V_{man, A} + V_{A, B}$

$= V_{man, A} + V_A - V_B$ = – 0.5 + 10 – (– 20)

= – 0.5 + 30 = 29.5 m/s.

15.

(d)



Distance, PQ = vp × t

(Distance = speed × time)

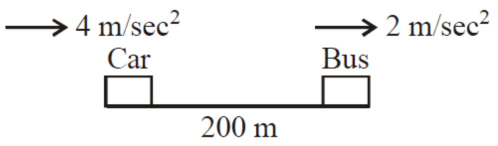
Distance, QR = V.t

$$\cos 60° = \frac{PQ}{QR}$$

$$\frac{1}{2} = \frac{v_p \times t}{V.t} \Rightarrow v_p = \frac{v}{2}$$

16.

(c)



Given, $u_C = u_B = 0$, $a_C = 4$ m/s², $a_B = 2$m/s²

Hence relative acceleration, $a_{CB} = 2$ m/sec²

Now, we know, $s = ut + \frac{1}{2}at^2$

$$200 = \frac{1}{2} \times 2t^2 \qquad \because u = 0$$

Hence, the car will catch up with the bus after time

$t = 10\sqrt{2}$ second

17.

(c) Person's speed walking only is

$$\frac{1}{60}\frac{\text{"escalator"}}{\text{second}}$$

Standing the escalator without walking the speed is

$$\frac{1}{40}\frac{\text{"escalator"}}{\text{second}}$$

Walking with the escalator going, the speed add.

So, the person's speed is

$$\frac{1}{60} + \frac{1}{40} = \frac{15}{120}\frac{\text{"escalator"}}{\text{second}}$$

So, the time to go up escalator $t = \frac{120}{5} = 24$ second.

18.

(c) For upward motion of helicopter;

$v^2 = u_2 + 2gh$

$v_2 = 0 + 2gh$

$v = \sqrt{2gh}$

Now, packet will start moving under gravity.

Let 't' be the time taken by the food packet to reach the ground.

$$s = ut + \frac{1}{2}at^2$$

$$-h = \sqrt{2gh}t - \frac{1}{2}gt^2 - \sqrt{2gh}t - h = 0$$

$$\text{or,} \quad t = \frac{\sqrt{2gh} \pm \sqrt{2gh + 4 \times \frac{g}{2} \times h}}{2 \times \frac{g}{2}}$$

$$\text{or,} \quad t = \sqrt{\frac{2gh}{g}}\left(1+\sqrt{2}\right) \Rightarrow t = \sqrt{\frac{2h}{g}}\left(1+\sqrt{2}\right)$$

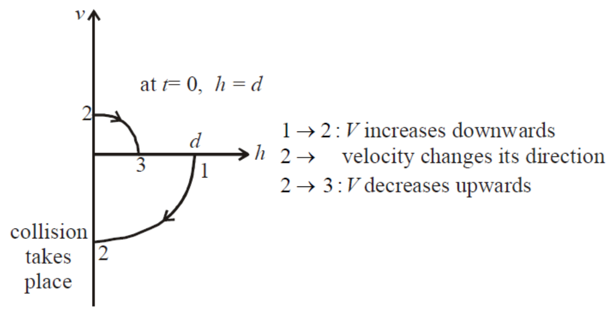
$$\text{or,} \quad t = 3.4\sqrt{\frac{h}{g}}$$

19.

(c) For uniformly accelerated/deaccelerated motion :

$v^2 = u^2 \pm 2gh$

As equation is quadratic, so, v-h graph will be a parabola



Initially velocity is downwards (–ve) and then after collision it reverses its direction with lesser magnitude, i.e. velocity is upwards (+ve).

Note that time t = 0 corresponds to the point on the graph where h = d.

Next time collision takes place at 3.

20.

(a) For a body thrown vertically upward acceleration remains constant (a = – g) and velocity at anytime t is given by V = u – gt

During rise velocity decreases linearly and during fall velocity increases linearly and direction is opposite to each other.

Hence graph (a) correctly depicts velocity versus time.

21.

(b) $y_1 = 10t - 5t^2$; $y_2 = 40t - 5t^2$

For $y_1 = -240$m, t = 8s

$\therefore y_2 - y_1 = 30t$ for $t \leq 8$s.

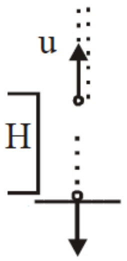
for t > 8s,

$y_2 - y_1 = 240 - 40t - \frac{1}{2}gt^2$

22.

(c) Speed on reaching ground

$v = \sqrt{u^2 + 2gh}$



Now, v = u + at

$\sqrt{u^2 + 2gh} = -u + gt$

Time taken to reach highest point is $t = \frac{u}{g}$,

$t = \frac{u + \sqrt{u^2 + 2gH}}{g} = \frac{nu}{g}$ (from question)

$2gH = n(n-2)u^2$

23.

Let the ball takes time t to reach the ground

Using, $S = ut + \frac{1}{2}gt^2$

$S = 0 \times t + \frac{1}{2}gt^2$

$200 = gt^2$ [∵ 2S = 100m]

$t = \sqrt{\frac{200}{g}}$ ... (i)

In last $\frac{1}{2}s$, body travels a distance of 19 m, so

in $\left(t - \frac{1}{2}\right)$

distance travelled = 81

Now, $\frac{1}{2}g\left(t - \frac{1}{2}\right)^2 = 81$

$\therefore g\left(t - \frac{1}{2}\right)^2 = 81 \times 2$

$\left(t - \frac{1}{2}\right) = \sqrt{\frac{81 \times 2}{g}}$

$\therefore \frac{1}{2} = \frac{1}{\sqrt{g}}\left(\sqrt{200} - \sqrt{81 \times 2}\right)$ from eq. (i)

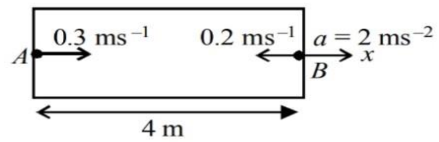
$\sqrt{g} = 2\left(10\sqrt{2} - 9\sqrt{2}\right)$

$\sqrt{g} = 2\sqrt{2}$

∴ g = 8 m/s^2

Solutions: Pro Test - I

1.



For ball A

$u_1 = 0.3$m/s, $a_1 = -2$ms^{-2}, $s_1 = x$, $t_1 = t$

$\therefore s_1 = u_1t_1 + \frac{1}{2}u_1t_1^2$

$x = 0.3t - t^2$ ....(1)

For ball B

$u_2 = 0.2$m/s, $a_2 = 2$ms^{-2}, $s_2 = 4 - x$, $t_2 = t$

$\therefore s_2 = u_2t_2 + \frac{1}{2}u_2t_2^2$

$4 - x = 0.2t - t^2$ ....(2)

Adding eq. (1) and (2)

$4 = 0.5t$ $\therefore t = \frac{4}{0.5} = 8$s.

2.

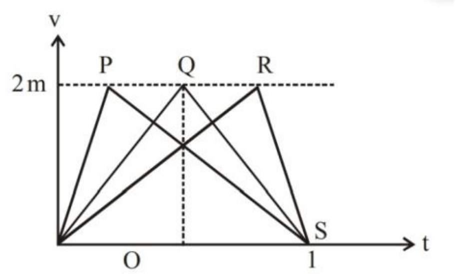
(a, c, d) At t = 0 and t = 1 body is at rest initially α is positive so that the body acquires some velocity. Then α should be negative so that the body comes rest. Hence α cannot remain positive for all time in the interval $0 \le t \le 1$

The journey is depicted in the following v – t graph.

Total time of journey = 1sec

Total displacement = 1m = area under (v – t) graph

$v_{max} = \frac{2s}{t} = \frac{2\times1}{1} = 2$m/s



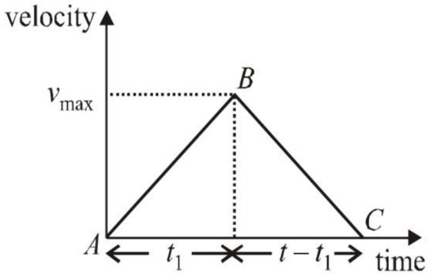
For path OQ, acceleration (α)

$= \frac{change\ in\ velocity}{time} = \frac{2}{1/2} = 4$m/s^2

For path QS is retardation = -4m/s^2

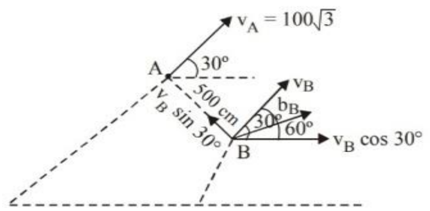
For path OP, α(acceleration) > 4m/s^2
For path PS (acceleration) < - 4m/s^2
For path OP, acceleration α < 4m/s^2
For path RS, retardation α > 4m/s^2
Hence $\alpha \geq 4$ at some point or points in its path.

3.
(1) Let t_1 be the time taken by the car to attain the maximum velocity v_m while it is acceleration.
Using v = u +at_2
$V_m = 0 + \alpha t_1$ or $t_1 = \frac{v_m}{\alpha}$........(1)
Since the total time elapsed is t, the car decelerators for time $t_2 = (t-t_1)$ to come by rest , a = -β and v = 0
$0 = v_m - \beta(t - t_1)$ or $t_1 = t + \frac{v_m}{\beta}$.......(2)
Using (1) and (2) , we get
$\frac{v_m}{\alpha} = t - \frac{v_m}{\beta}$ or $t = v_m(\frac{1}{\alpha} + \frac{1}{\beta})$
Or $v_m = \frac{t\alpha\beta}{(\alpha+\beta)}$ ......(3)



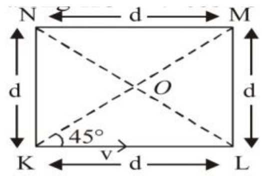
(2) Total distance travelled = area of Δ ABC
$= \frac{1}{2} \times$ base $\times$ altitude $= \frac{1}{2} \times t \times v_{max}$
$= \frac{1}{2} \times t \times \frac{\alpha\beta}{\alpha+\beta} t = \frac{1}{2} (\frac{\alpha\beta}{\alpha+\beta}) t^2$

4.



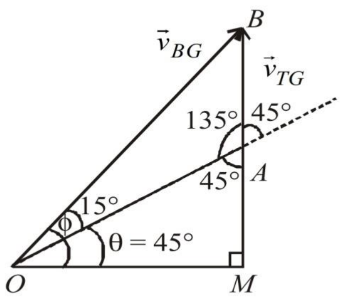
The relative velocity of B with respect to A is perpendicular to the line of motion of A i.e., OA. As the velocity component of A and B along the same line OA.
∴ $v_B \cos 30° = v_A = 100\sqrt{3}$
⤇ $v_B = \frac{v_A}{\cos 30°} = \frac{100\sqrt{3}}{\sqrt{3}/2} = 200$ms^{-1}
∴ $t_0 = \frac{displacement}{velocity} = \frac{500}{v_B \sin 30°} = \frac{500}{200\times\frac{1}{2}} = 5$s

5.
($\frac{d}{v}$) Each person K, L, M, N moves with a uniform speed v such that K always move directly towards L, L directly towards M, M directly N and N directly towards K.
Here on the basis of symmetry we can say that K, L, M, and N will meet at the centre of the square 'O'.
At any instant velocity component along KO = v cos45°



$= \frac{v}{\sqrt{2}}$
Distance KO = d cos45° $= \frac{d}{\sqrt{2}}$
∴ Time taken to meet at 'O' $= \frac{distance}{velocity}$
Or, $t = \frac{d/\sqrt{2}}{v/\sqrt{2}} = \frac{d}{v}$

6.
(a) For the ball B to hit the trolley-A relative velocity of w.r.t . A, v_{BA} should be along OA. So, v_{BA} will make an angle 45° with +(ve) X-axi



(b) Here $\theta = 45^\circ$
∴ $\phi = \frac{4\theta}{3} = \frac{4\times45}{3} = 60^\circ$
With +ve X-axis.
In ΔOMA,
$\theta = 45^\circ \Rightarrow \angle OAM = 45^\circ$
∴ $\angle OAB = 135^\circ$
Also $\angle BOA = 60^\circ - 45^\circ = 15^\circ$
Using sine law in ΔOBA
$\frac{v_B}{sin135^\circ} = \frac{v_T}{sin15^\circ}$
$v_B = \frac{vsin135^\circ{}_T}{sin15^\circ} = \frac{(\sqrt{3}-1)\times0.71}{0.2588} \approx 2$m/s
Hence, speed of ball w.r.t ground, v_B and 2m/s

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BOOK *Development* JOURNEY

2016-19

COLLECTIONS OF QUESTIONS & FRAMING

It was mostly from Foriegn author books and Kota Teachers notes. This collections took 3 years total.

2019

IDEA TO MAKE BEST NOTES FOR TEACHER

When I was in FIITJEE, Chennai, after 3 years of work, my teachers self reference note was completed.

2019-20

DEVELOPED EDUCATION APP FOR PHYSICS

I hired MTouch Labs, and made an app for students as well as teachers so that they can refer questions and theory.

2020

PUBLISHED 1ST TWO E-BOOKS ON AMAZON

It was the version 1.0 of the book. The book got decent sales without marketing in next 6 months.

2021-22

IDEA -BOOK TO HAVE MORE SUB-TOPIC QUESTIONS

When version 1.0 was live, I believe something is missing, So I started working on detailed sub-topic oriented style.

2022

TYPING, FORMATING, DIAGRAMS & REVIEWS

Once all subtopic segregations done, I assigned and hired various freelancers to type and format the book.

2023

REVIEWS FROM VARIOUS TEACHERS

I hired more than 15 teachers from all over india to review my book, finding typing mistakes, answer key corrections.

2024

VERSION 3.0 ZERO TO HERO RELEASED

2023 Year, I published Adaptive Problems Book, which was version 2.0 of my content, once all reviews are done. Then I started working on my final book project version 3.0.

Painful Endless Days & Nights, Tremendous Hardwork of Author & His Team

SATYAM SIR

Physics is a subject of complexity and understanding. To make concepts simpler and practice it in a more systematic manner, the Bihar born genius, Satyam Sir has made his deep efforts to publish this book. He has already helped many students to understand in depth subject concepts in the easiest possible manner. He is gifted with rare ability to simplify concepts and motivate the children how to solve complex problems in minutes. Sir has graduated from IIT Kharagpur in Civil Engineering and has been teaching Physics for JEE Mains and Advanced for more than a decade. The main goal of this book is to enhance problem solving ability in students. Sir is having hope that you would enjoy this journey of learning physics.

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