SATYAM SIR

ADAPTIVE PROBLEMS BOOK IN PHYSICS

PHYSICS FACTOR
Fall in Love with Physics

Crafted for JEE, NEET,Olympiad Exams

HEAT & THERMODYNAMICS

Thermal Prop. of Matter
Thermodynamics
Kinetic Theory of Gases

5

SOLVED QUESTION BANK

ADAPTIVE

PROBLEMS BOOK

IN PHYSICS

Volume 05

Heat and Thermodynamics

for

Board Exams

State Engineering Entrance

NEET Medical Entrance Exam

IIT JEE Mains & Advanced Exams

KVPY & International Physics Olympiad

2nd Edition (Feb 2024)

Disclaimer

In spite of many efforts taken to present this book without errors, still some errors might be possible. The author doesn't take any responsibility for such errors in any form. We request you to kindly give error information to author at satyamquestapps@gmail.com , so that he will rectify those mistakes in his next edition of his book.

Thermal Properties
Thermodynamics
Kinetic Theory

Author’s Message

Physics is a subject to understand nature. How it works? There are certain laws that applies to nature that we study in physics in terms of conceptual theory. Then we apply these theories in day-to-day life or its applications in form of numerical. It is the toughest subject for those who tend to mug up this subject. A subject requires an IQ. However, an IQ is a subjective prospect. To measure, it has numerous parameters. It grows, as we tend to understand the subject. The intension is to create this book to present physics subject as in a most systematic approach to learn in depth knowledge and develop a good problem solving skill. All the chapters presented in this book starts with an adaptive topic wise question bank, that personally I have made especially for other teacher to use in their class. However, it is not limited to do so. These problems are given in order of difficulty level. Further, the chapter gives a plenty of unsolved questions exercise questions to practice, plus last year asked JEE Mains and Advanced Questions along with NCERT questions. This book also contains selected BOARD level problems in every chapter, which gives a good boost in your school and board exams. I hope you would enjoy this journey of learning. Though an enormous of hard work and efforts have been to make this book as error free as possible still I expect feedbacks and content mistakes (if any), so to remove in the next upcoming editions.

Enjoy learning Physics!

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ACKNOWLEDGMENTS

I am thankful to my family members for the support provided throughout this project. Especial thanks to Kasibhatla Suryanarayana Sir, my colleague who helped in proofread this book. I am thankful to my few close friends for helping me editing this book with their great efforts. I am also thankful to all my coaching institute friends for helping me out with several solutions and removing ambiguous questions. I also give credit to some of the writers and publications whose help been taken to produce this book. I also helpful to my college friends for the final proofread of this book. I am grateful for all the efforts you all people made to make my dream true.

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	TOTAL QUESTIONS	**195**	**92**	**25**

Thermal Properties of Matter

Chapter Summary

Calorimetry

A branch of science which deals with the measurement of temperature of a substance is known as calorimetry.

Temperature

Temperature is defined as the degree of hotness or coldness of a body. The natural flow of heat is from higher temperature to lower temperature.

Two bodies are said to be in thermal equilibrium with each other, when no heat flows from one body to the other. That is when both the bodies are at the same temperature.

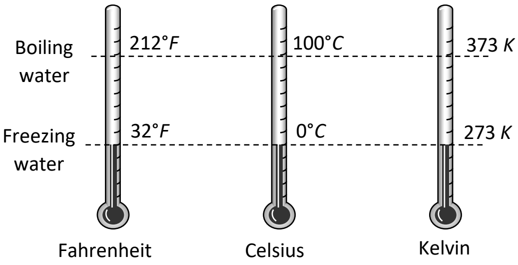
Temperature is one of the seven fundamental quantities with dimension [*K*].

It is a scalar physical quantity with S.I. unit Kelvin.

Normal temperature of human body is 310. 15 K ($37°C = 98.6°F$)

NTP or STP implies 273.15K ($0°C = 32°F$)

Temperature Scales



$$\frac{\text{SR} - MP}{\text{BP} - MP} = \text{Constant for all scales}$$

where,

SR is the Scale Reading

MP is the Melting point of Water

BP is the Boiling point of Water

All these temperatures are related to each other by the following relationship

$$\frac{C-0}{100} = \frac{F-32}{212-32} = \frac{K-273.15}{373.15-273.15}$$

Heat Exchange

The form of energy which is exchanged among various bodies or system on account of temperature difference is defined as heat.

It is a scalar quantity. It's units are *joule, erg, cal, kcal etc.*

1 *calorie* = 4.18 *J*

Specific Heat Capacity (S)

The amount of heat required to raise the temperature of a unit mass by 1°C is called Specific heat capacity.

$$Q = mS\Delta T$$

SI unit is Unit is J/kg°C

Other commonly used unit is Cal/g°C

$S_{ice} = 0.5$ Cal/g°C

$S_{water} = 1.0$ Cal/g°C

$S_{vapor} = 2.0$ Cal/g°C

Heat Capacity (C)

The amount of heat required to raise the temperature by 1°C is called Specific heat capacity.

$$Q = C\Delta T$$

SI unit is Unit is J/°C

Other commonly used unit is Cal/°C

Latent Heat (L)

The amount of heat required to raise the temperature by 1°C is called Specific heat capacity.

$$Q = mL$$

SI unit is Unit is J/kg

Other commonly used unit is Cal/gram

$L_F = L_{\text{ice}} \approx 80\, cal\,/\,gm \approx 60\, kJ\,/\,mol$

$\approx 336\, kilo\ joule\,/\,kg$

$L_V = L_{\text{steam}} \approx 540\, cal\,/\,gm \approx 40.8\, kJ\,/\,mol$

$\approx 2260\, kilo\ joule\,/\,kg$

Principle of Calorimetry

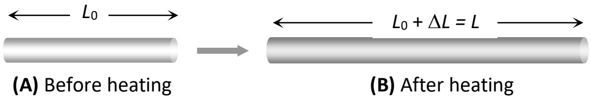
When two bodies (one being solid and other liquid or both being liquid) at different temperatures are mixed, heat will be transferred from body at higher temperature to a body at lower temperature till both acquire same temperature. The body at higher temperature releases heat while body at lower temperature absorbs it, so that

$$Q_{Lost} = Q_{Gain}$$

Thermal Expansion

When matter is heated without any change in it's state, it usually expands. As with rise in temperature the amplitude of vibration and hence energy of atoms increases, hence the average distance between the atoms increases. So the matter as a whole expands.

Linear Expansion: When a solid is heated and it's length increases, then the expansion is called linear expansion.



Change in length $\Delta L = L_0 \alpha \Delta T$

(L_0 = Original length, ΔT = Temperature change)

Final length $L' = L_0(1 + \alpha\Delta T)$

where α Co-efficient of linear expansion

Superficial (Areal) Expansion: When the temperature of a $2D$ object is changed, it's area changes, then the expansion is called superficial expansion.

Change in area is $\Delta A = A_0\beta\Delta T$

(A_0 = Original area, ΔT = Temperature change)

Final area $A = A_0(1 + \beta\Delta T)$

where, β *is the c*o-efficient of superficial expansion

$\beta = 2\alpha$ (only for solids)

Volume or Cubical Expansion: When a solid is heated and it's volume increases, then the expansion is called volume or cubical expansion.

Change in volume is $\Delta V = V_0\gamma\Delta T$

(V_0 = Original volume, ΔT = change in temperature)

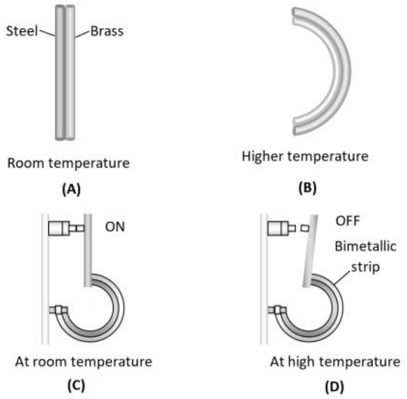
Final volume $V = V_0(1 + \gamma\Delta T)$

γ is the volume co-efficient of expansion

$\gamma = 3\alpha$ (only for solids)

Application of Thermal Expansion in Solids

(1) **Bi-metallic strip:** Two strips of equal lengths but of different materials (different coefficient of linear expansion) when join together, it is called "bi-metallic strip", and can be used in thermostat to break or make electrical contact. This strip has the characteristic property of bending on heating due to unequal linear expansion of the two metal. The strip will bend with metal of greater α on outer side *i.e.* convex side.



(2) **Effect of temperature on the time period of a simple pendulum :** A pendulum clock keeps proper time at temperature θ. If temperature is increased to $\theta'(>\theta)$ then due to linear expansion, length of pendulum and hence its time period will increase.

Fractional change in time period $\dfrac{\Delta T}{T} = \dfrac{1}{2}\alpha\,\Delta\theta$

(i) Due to increment in its time period, a pendulum clock becomes slow in summer and will lose time.

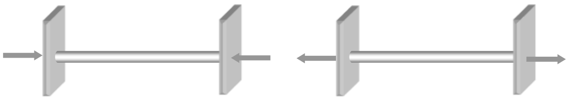
Loss of time in a time period $\Delta T = \dfrac{1}{2}\alpha\,\Delta\theta\,T$

(ii) Time lost by the clock in a day (t = 86400 *sec*)

$$\Delta t = \frac{1}{2}\alpha\,\Delta\theta\,t = \frac{1}{2}\alpha\,\Delta\theta\,(86400) = 43200\alpha\,\Delta\theta\ sec$$

(iii) The clock will lose time *i.e.* will become slow if $\theta' > \theta$ (in summer) and will gain time *i.e.* will become fast if $\theta' < \theta$ (in winter).

(3) **Thermal stress in a rigidly fixed rod :** When a rod whose ends are rigidly fixed such as to prevent expansion or contraction, undergoes a change in temperature, due to thermal expansion or contraction, a compressive or tensile stress is developed in it. Due to this thermal stress the rod will exert a large force on the supports. If the change in temperature of a rod of length L is $\Delta\theta$ then


(A) Heating **(B)** Cooling

Thermal strain $= \dfrac{\Delta L}{L} = \alpha\Delta\theta$

$$\left[\text{As } \alpha = \frac{\Delta L}{L} \times \frac{1}{\Delta\theta}\right]$$

So, Thermal stress $= Y\alpha\Delta\theta$

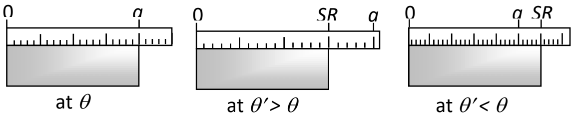
$$\left[\text{As } Y = \frac{\text{stress}}{\text{strain}}\right]$$

Or, Force on the supports $F = YA\alpha\,\Delta\theta$

(4) **Error in scale reading due to expansion or contraction :** If a scale gives correct reading at temperature θ, at temperature $\theta'(>\theta)$ due to linear expansion of scale, the scale will expand and scale reading will be lesser than true value so that,

True value = Scale reading $[1 + \alpha(\theta' - \theta)]$

i.e. $\quad \text{TV} = SR[1 + \alpha\,\Delta\theta]$ with $\Delta\theta = (\theta' - \theta)$



However, if $\theta' < \theta$, due to contraction of scale, scale reading will be more than true value, so true value will be lesser than scale reading and will still be given by above equation with $\Delta\theta = (\theta' - \theta)$ negative.

Variation of Density with Temperature

$$\rho' = \rho(1-\gamma\,\Delta T)$$

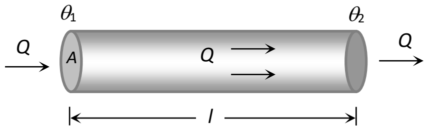
Heat Transfer

Heat energy transfers from a body at higher temperature to a body at lower temperature. The transfer of heat from one body to another may take place by one of the following modes.

Heat Conduction

The process of transmission of heat energy in which the heat is transferred from one particle to other particle without dislocation of the particle from their equilibrium position is called conduction.

Law of Thermal conductivity : Consider a rod of length *l* and area of cross-section *A* whose faces are maintained at temperature θ_1 and θ_2 respectively. The curved surface of rod is kept insulated from surrounding to avoid leakage of heat



$$i_{th} = \frac{dQ}{dt} = -KA\frac{d\theta}{dx}$$

Thermal Conductivity (K) It is the measure of the ability of a substance to conduct heat through it.

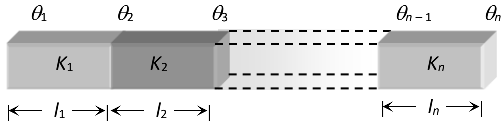
Its Units are: *Cal/cm-sec °C* (in C.G.S.), *kcal/m-sec-K* (in M.K.S.) and *W/m- K* (in S.I.)

Thermal resistance (R_{th}) The thermal resistance of a body is a measure of its opposition to the flow of heat through it.

$$R_{th} = \frac{l}{KA}$$

Combination of Metallic Rods

(1) **Series combination :** Let *n* slabs each of cross-sectional area *A*, lengths $l_1, l_2, l_3, \ldots\ldots l_n$ and conductivities $K_1, K_2, K_3 \ldots\ldots K_n$ respectively be connected in the series



Equivalent thermal Resistance :

$$R = R_1 + R_2 + \ldots\ldots R_n$$

Equivalent thermal conductivity

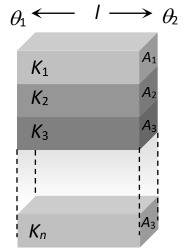
$$\Rightarrow\ K_s = \frac{l_1 + l_2 + \ldots\ldots l_n}{\frac{l_1}{K_1} + \frac{l_2}{K_2} + \ldots\ldots\ldots \frac{l_n}{K_n}}$$

(a) For *n* slabs of equal length

$$K_s = \frac{n}{\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} + \ldots\ldots \frac{1}{K_n}}$$

(b) For two slabs of equal length, $K_s = \frac{2K_1K_2}{K_1 + K_2}$

Parallel Combination: Let *n* slabs each of length *l*, areas $A_1, A_2, A_3, \ldots\ldots A_n$ and thermal conductivities $K_1, K_2, K_3, \ldots\ldots K_n$ are connected in parallel then



Equivalent Resistance :

$$\frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots \frac{1}{R_n}$$

For two slabs $R_s = \frac{R_1R_2}{R_1 + R_2}$

Heat Convection

Mode of transfer of heat by means of migration of material particles of medium is called convection. It is of two types.

Heat Radiation

Radiation is the process by which heat is transmitted from one place to another without heating and transferring the intervening medium.

Properties of Thermal Radiation

1. The energy radiated due to its higher temp is called thermal radiation.

2. Wavelength of thermal radiation (λ_r) ranges from (4×10^{-4} m to 8×10^{-7} m)

3. All objects above 0 Kelvin radiates thermal energy continuously.

4. Thermal Radiation can be transferred via vacuum also.

5. It follows straight line path and obeys laws of refraction & reflection.

6. Velocity of all thermal radiation is the speed of light in vacuum

$V_{\text{th. Rad}} = c = 3 \times 10^8$ m/s

7. Intensity of thermal radiation is $I \propto \frac{1}{r^2}$

8. It doesn't affect medium in which it travels.

Important Terms Related to Thermal Radiation

When thermal radiations (Q) fall on a body, they are partly reflected, partly absorbed and partly transmitted.

Absorptive of surface (a)

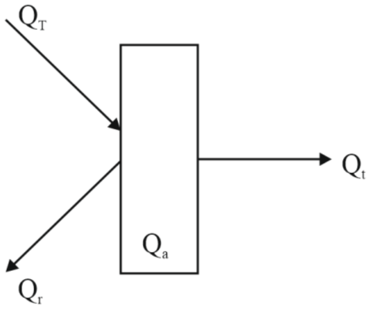
$$a = \frac{Q_{absorbed}}{Q_{total}} = \frac{Q_a}{Q}$$

Reflectance of Surface (r)

$$r = \frac{Q_{reflected}}{Q_{total}} = \frac{Q_r}{Q}$$

Transmittance of surface (t)

$$t = \frac{Q_{transmitted}}{Q_{total}} = \frac{Q_t}{Q}$$



$$a + r + t = 1$$

Different bodies

(i) If $a = t = 0$ and $r = 1 \rightarrow$ body is perfect reflector

(ii) If $r = t = 0$ and $a = 1 \rightarrow$ body is perfectly black body

(iii) If, $a = r = 0$ and $t = 1 \rightarrow$ body is perfect transmitter

(iv) If $t = 0 \Rightarrow r + a = 1$ or $a = 1 - r$ *i.e.* good reflectors are bad absorbers.

Black Body

A body that absorbs all the radiation falling on it is called a blackbody. A perfect black body neither reflects nor transmits any radiation.

A perfect blackbody, absorbing 100% of the radiation falling on it, is only an ideal concept.

Among the materials, lampblack is close to a blackbody. It reflects only about 1% of the radiation falling on it.

When perfectly black body is heated to suitable high temperature, it emits all type of radiations of all possible wavelengths.

Example: sun, the stars

Emissive Power

It is defined as the total amount of thermal energy emitted per unit time, per unit area of the body for all possible wavelengths.

$$E = \frac{Q}{tA}$$

SI Unit : $\frac{Joule}{m^2 \times \sec}$ or $\frac{Watt}{m^2}$

Absorptive Power (a)

Absorptive power of a body is defined as the fraction of the incident radiation that is absorbed by the body.

It is denoted by a.

$$a = \frac{Q_a}{Q_T}$$

It is Unitless & dimension less quantity

As all the radiation incident on a blackbody is absorbed, the absorptive power of a blackbody is unity.

Emissivity (ε) : Emissivity of a body at a given temperature is defined as the ratio of the total emissive power of the body (e) to the total emissive power of a perfect black body (E) at that temperature

$$\varepsilon = \frac{E}{E_B}$$

E = Emission power of any body

E_B = Emission power of black body

(i) For perfectly black body $\varepsilon = 1$

(ii) For highly polished body $\varepsilon = 0$

(iii) But for practical bodies emissivity (ε) lies between zero and one ($0 < \varepsilon < 1$).

Kirchhoff's Law

According to this law the ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

Hence $\frac{e_1}{a_1} = \frac{e_2}{a_2} = \ldots \left(\frac{E}{a}\right)_{\text{Perfectly black body}}$

$$\frac{E_{body}}{a_{body}} = E_{blackbody}$$

But for perfectly black body a = 1 *i.e.* $\frac{e}{a} = E$

Kirchhoff's law tells that if a body has high emissive power, it should also have high absorptive power to have the ratio E/a same.

Similarly, a body having low emissive power should have low absorptive power.

Good emitters are also a good absorber

Stefan's Law

According to it the radiant energy emitted by a perfectly black body per unit area per sec (*i.e.* emissive power of black body) is directly proportional to the fourth power of its absolute temperature,

i.e. $E \propto T^4 \Rightarrow \boldsymbol{E_B = \sigma T^4}$

where σ is a constant called Stefan's constant having value $5.67 \times 10^{-8} W / m^2 K^4$

For any other body,

$$\boldsymbol{E = \varepsilon \sigma T^4}$$

Stefan's Boltzmann's Law

If an ordinary body at temperature T is surrounded by a body at temperature T_0, then this law may be put as

Net Heat Radiated per unit time per unit area

$$\frac{Q_{net}}{tA} = \varepsilon\sigma\left(T^4 - T_0^{\,4}\right)$$

Newton's Law of Cooling

It states that the rate of cooling of the body is directly proportional to the temperature difference between body and the surrounding, provided it to be small. $\Delta T << T_o$

$$\frac{dT}{dt} = -k\left(T - T_0\right)$$

where,

T = Temperature of a body

T_0 = Surrounding temperature

t = Time

k = constant

Average form of Newton's Law of Cooling

$$\left[\frac{T_1 - T_2}{t}\right] = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

Where, T_0 is the surrounding temperature

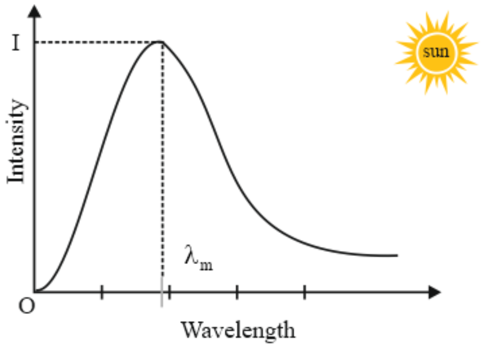
And the body cools down from T_1 to T_2 in time t

Wien's Displacement Law

According to Wien's law the product of wavelength corresponding to maximum intensity of radiation and temperature of body (in Kelvin) is constant, *i.e.*

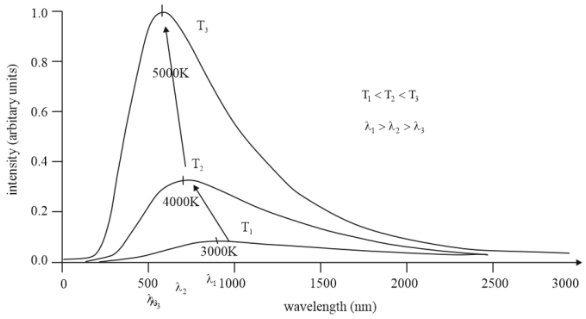
$$\lambda_m T = b = \text{constant}$$

where b is Wien's constant and has value $2.89 \times 10^{-3}\, m\text{-}K$.



As the temperature of the body increases, the wavelength at which the spectral intensity (E_λ) is maximum shifts towards left.

Therefore, it is also called Wien's displacement law.



Beginner **Temperature Scales**

1. Absolute zero (0 K) is that temperature at which

(a) Matter ceases to exist

(b) Ice melts and water freezes

(c) Volume and pressure of a gas becomes zero

(d) None of these

Solution:

The correct answer is C.

Absolute zero is characterised by no atomic motion.

Both Charles' law and the pressure law lead to an extrapolation back to zero volume or pressure which would imply the temperature scale can go no further. For all gases, absolute zero is the same and although clearly the would no longer be a gas there, this is an important implication.

2. Oxygen boils at – 183°C. This temperature is approximately

(a) 215°F **(b) – 297°F**

(c) 329°F **(d) 361°F**

Solution:

The answer is B.

$$\frac{-183 - 0}{100} = \frac{F - 32}{180}$$

$$\Rightarrow F = -329.4 + 32 = -297.4°F$$

3. The temperature on Celsius scale is 25°C. What is the corresponding temperature on the Fahrenheit scale?

(a) 40°F **(b) 77°F**
(c) 50°F **(d) 45°F**

Solution:

The correct answer is B.

The formula to convert Celsius to Fahrenheit is given

by $°F = °C \times \left(\frac{9}{5}\right) + 32$

$F = \left[C \times \left(\frac{9}{5}\right) + 32\right]$

$Given\ that, C = 25$

$F = 25 \times \left(\frac{9}{5}\right) + 32$

$F = 45 + 32$

$F = 77$

Thus, $25°C$ is equivalent to $77°F$

4. If a graph is plotted between temperature of a body on degree Celsius (along y-axis) and degree Fahrenheit [along x-axis] at different temperatures, then the slope of the graph will be:

(a) $\frac{5}{9}$ **(b)** $\frac{9}{5}$
(c) $\frac{3}{5}$ **(d)** $\frac{5}{3}$

Solution:

The relation between degree Celsius and degree Fahrenheit is,

$C = (F - 32)\frac{5}{9}$

C - along y-axis

F - along x-axis

$y = (x - 32)\frac{5}{9}$

$y = \frac{5x}{9} - \frac{32 \times 5}{9}$

y-mx + c .. equation of straight line On comparing we get,

$\text{Slope } = m = \frac{5}{9}$

5. The temperature of a body is raised from 20°C to 45°C. The rise in temperature of the body in the Fahrenheit scale is

(a) 25°F **(b) 77°F**
(c) 45°F **(d) 207°F**

Solution:

As we know,

$F = 32 + \frac{9}{5}C$

$\Delta F = \frac{9}{5}\Delta C$

$\Delta C = 45° - 20° = 25°C$

$\Delta F = \frac{9}{5} \times 25 = 45°F$

6. The readings of a bath on Celsius and Fahrenheit thermometers are in the ratio of 2: 5. The temperature of the bath is:

(a) –26.66°C **(b) 40°C**
(c) 45.71°C **(d) 26.66°C**

Solution:

$\frac{C}{F} = \frac{2}{5}$

C = 0.4 F ……… (i)

Now we know

$C = \frac{5}{9}[F - 32]$

$0.4F = \frac{5}{9}[F - 32]$

$F = 114.28°F$

$C = 45.71°C$

7. Recently, the phenomenon of superconductivity has been observed at 95 K. This temperature is nearly equal to

(a) – 288°F **(b) – 146°F**
(c) – 368°F **(d) +178°F**

Solution:

The answer is A.

Use the formula

$\frac{C - 0}{100} = \frac{K - 273}{100} = \frac{F - 32}{180}$

$\frac{95 - 273}{100} = \frac{F - 32}{180}$

$\Rightarrow F = -320 + 32 = -288°F$

8. The absolute zero temperature in Fahrenheit scale is

(a) – 273°F **(b) – 32°F**
(c) – 460°F **(d) – 132°F**

Solution:

The correct answer is C.

Absolute temperature in Celsius is $-273.15°C$

$F = \frac{9}{5}C + 32$

$F = \frac{9}{5} \times (-273.15) + 32 = -459.67°F$

$F \cong -460°F$

Hence, absolute temperature is in Fahrenheit.

9. The temperature of a substance increases by 27°C. On the Kelvin scale this increase is equal to

(a) 300 K (b) 2.46 K

(c) 27 K (d) 7 K

Solution:

The answer is C.

Temperature increases by 27°C on Kelvin scale increment will be same. Hence value of increase is 27K

10. At what temperature the centigrade (Celsius) and Fahrenheit, readings are the same

(a) – 40° (b) + 40°

(c) 36.6° (d) – 37°

Solution:

The correct answer is A.

Rather than converting one temperature to another (which is not helpful because it assumes you already know the answer), you can set degrees Celsius and degrees Fahrenheit equal to each other using the conversion formula between the two scales:

$$°F = \left(°C \times \frac{9}{5}\right) + 32$$

$$℃ = (℉ - 32) \times \frac{5}{9}$$

It does not matter which equation you use; simply use x instead of degrees Celsius and Fahrenheit.You can solve this problem by solving for x:

$$℃ = \frac{5}{9} \times (℉ - 32)$$

$$x = \frac{5}{9} \times (x - 32)$$

$$x = \left(\frac{5}{9}\right) x - 17.778$$

$$1x - \left(\frac{5}{9}\right) x = -17.778$$

$$0.444x = -17.778$$

$x = -40$ degrees Celsius or Fahrenheit

Working using the other equation, you get the same answer:

$$℉ = \left(℃ \times \frac{9}{5}\right) + 32$$

$$°x - \left(°x \times \frac{9}{5}\right) = 32$$

$$-\frac{4}{5} \times °x = 32$$

$$°x = -32 \times \frac{5}{4}$$

$$x = -40°$$

Expert Temperature Scales

1. On centigrade scale the temperature of a body increases by 30 degrees. The increase in temperature on Fahrenheit scale is

(a) 50° (b) 40°

(c) 30° (d) 54°

Solution:

The correct answer is D

$$F = \frac{9}{5} C + 32$$

So, $F_1 = \frac{9}{5} C_1 + 32$ & $F_2 = \frac{9}{5} C_2 + 32$

$$F_1 - F_2 = \frac{9}{5}(C_1 - C_2) = \frac{9}{5}(30) = 54\text{°F}$$

2. On a new scale of temperature (which is linear) and called the W scale, the freezing and boiling points of water are 39°W and 39°W respectively. What will be the temperature on the new scale, corresponding 39°C on the Celsius scale?

(a) 78°W (b) 117°W

(c) 200°W (d) 139°W

Solution:

In general, whenever we are to go from any known scale to any unknown scale, then we follow the equation

$$\frac{(\text{Temperature on known scale}) - (\text{LFP for known scale})}{(\text{UFP} - \text{LFP})_{\text{known}}} = \frac{(\text{Temperature on unknown scale}) - (\text{LFP for unknown scale})}{(\text{UFP} - \text{LFP})_{\text{unknown}}}$$

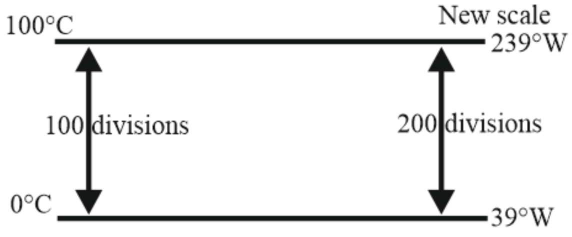
Or, $\frac{39 - 0}{100 - 0} = \frac{t - 39}{239 - 39}$

Or, t = 117°W

Note : LFP → Lower fixed point

UFP → Upper fixed point

Alternative



$\therefore 39°C = 39 \times 2 + 39 = (78 + 39)°W = 117°W$

3. In a new temperature scale, freezing point of

water is given a value 10x and boiling point of water is 90x. Reading of new temperature scale for a temperature equal to 10°C is

(a) 14x **(b) 16x**

(c) 18x **(d) 12x**

Solution:

$$\frac{T_c - 0}{100 - 0} = \frac{T' - 10x}{90x - 10x}$$

$$\frac{T_c}{100} = \frac{T'10x}{80x}$$

$$\frac{T_c}{5} = \frac{T'10x}{4x}$$

$$4T_c = \frac{5T' - 50x}{x}$$

$$4 \times 10x = 5T' - 50x$$

$$T' = 18x$$

4. The temperature of a body on Kelvin scale is found to be x K. When it is measured by Fahrenheit thermometer, it is found to be x°F, then the value of x is

(a) 40 **(b) 313**

(c) 574.25 **(d) 301.25**

Solution:

The answer is C.

Relation between Kelvin and fahrenheit is given by

$$F = \frac{9}{5}(K - 273) + 32$$

$$\therefore X = \frac{9}{5}(X - 273) + 32$$

$$\Rightarrow X = 574.25$$

5. A centigrade and a Fahrenheit thermometer are dipped in boiling water. The water temperature is lowered until the Fahrenheit thermometer registers 140°. What is the fall in temperature as registered by the Centigrade thermometer?

(a) 30° **(b) 40°**

(c) 60° **(d) 80°**

Solution:

The answer is B.

From the formula

$$\Rightarrow \frac{C}{5} = \frac{140-32}{9} = \frac{108}{9}$$

C = 60 ℃

Since, the temperature of boiling water is 100 ℃

So, fall of temperature in ℃ is $100 - 60 = 40$℃

6. If a thermometer reads freezing point of water as 20°C and boiling point as 150°C, then what will the thermometer read when the actual temperature is 60°C?

(a) 98°C **(b) 110°C**

(c) 40°C **(d) 60°C**

Solution:

The answer is A.

We let the unknown degree scale be called $x°$

Now we know that the freezing point = $20°x$

The boiling point = $150°x$

This also means that $20°x = 0°C$ (∴ $ON\ A\ Celsius\ Scale, the\ freezing\ point\ is$ 0°)

And

$150°x = 100°C$ (∴ $on\ a\ Celsius\ Scale$ $100° - 0° = 100°$)

So, if $100°C = 130°x$

Then $1°C = 1.3°x$

To go from C degree to x, we need to use:

$$x = 1.3C + 20$$

$$And\ C = \frac{(x - 20)}{1.3}$$

So if the temperature given is 60℃, then we calculate x as:

$$C = \frac{(x - 20)}{1.3}$$

$$\Rightarrow 60 = \frac{(x - 20)}{1.3}$$

$$\Rightarrow 60 \times 1.3 = x - 20$$

$$\Rightarrow 78 = x - 20$$

$$\Rightarrow x = 98°$$

So $60°C$ will be equal to 98°

Beginner Calorimetry

1. When vapour condenses into liquid

(a) It absorbs heat

(b) It liberates heat

(c) Its temperature increases

(d) Its temperature decreases

Solution:

The answer is B.

It liberates heat.

When vapour condenses to a liquid, heat is released.

2. The SI unit of mechanical equivalent of heat is

(a) joule × calorie **(b) joule/calorie**

(c) calorie × erg **(d) erg/calorie**

Solution:

The correct answer is B.

The mechanical equivalent of heat implies that motion and heat are interchangeable and that a defined amount of work generates the same amount of heat in all situations if the work is completely converted to heat energy. The mathematical expression for the mechanical equivalent of heat is,

$$J=\frac{w}{q}$$

Where

w = The amount of heat required to generate heat

q = Amount of heat

SI unit of mechanical equivalent

The SI unit of mechanical equivalent of heat is Joules/calorie.

3. If specific heat of a substance is infinite, it means

(a) Heat is given out

(b) Heat is taken in

(c) No change in temperature takes place whether heat is taken in or given out

(d) All of the above

Solution:

The answer is C.

Specific heat of a substance at infinity means heat is taken in or given out and no change takes place in the temperature of substance.

4. The quantity of heat required to change unit mass of a substance, from solid state to liquid state, at constant temperature, is known as

(a) Latent heat **(b) Sublimation**

(c) Hoar frost **(d) Latent heat of fusion**

Solution:

The answer is D.

During fusion of solid into liquid some energy is used up to transform the state of matter from solid to liquid. Since it does not increase the kinetic eneygy of particles, there is no increase in the temperature of the material. So, phase transformation takes place at constant temperature.

5. The latent heat of vaporization of a substance is always

(a) Greater than its latent heat of fusion

(b) Greater than its latent heat of sublimation

(c) Equal to its latent heat of sublimation

(d) Less than its latent heat of fusion

Solution:

The correct answer is A.

The energy required to completely separate the molecules, moving from liquid to gas is much greater than if you were just to reduce their separation, solid to liquid. Hence, the reason why the heat of vaporisation is greater than the latent heat of fusion.

6. 4200 J of work is required for

(a) Increasing the temperature of 10 gm of water through 10°C

(b) Increasing the temperature of 100 gm of water through 10°C

(c) Increasing the temperature of 1 kg of water through 10°C

(d) Increasing the temperature of 10 kg of water through 10°C

Solution:

The correct answer is B.

$$4200=m\times 4.2\times \Delta T$$

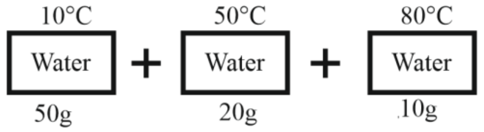
$$1000=m\Delta T$$

Mass should be in gram and temp on ℃ .As per the data in option (B) we will get $m\Delta T\ =\ 1000.$

Increasing the temperature of 100gm of water through 10℃

7. Find the final temperature of the mixture.

T_f =



Solution:

So,

$$\left(T_F-10\right)+20\left(T_F-50\right)+10\left(T_F-80\right)20$$

$$\Rightarrow 80\ T_F-2300=0$$

$$\Rightarrow T_F=\frac{1400}{80}=17.5°C$$

$$T_F=\frac{2300}{80}=\frac{230}{8}$$

8. How much heat energy is gained when 5 kg of water at 20°C is brought to its boiling point? (Specific heat of water = 4.2 kJ kg^{-1}C^{-1})

(a) 1680 kJ **(b) 1700 kJ**

(c) 1720 kJ **(d) 1740 kJ**

Solution:

The correct answer is A.

$$Q=mc\Delta\theta$$

$$=5\times(1000\times 4.2)\times(100-20)$$

$$=1680\times 10^{3}J$$

$$=1680KJ$$

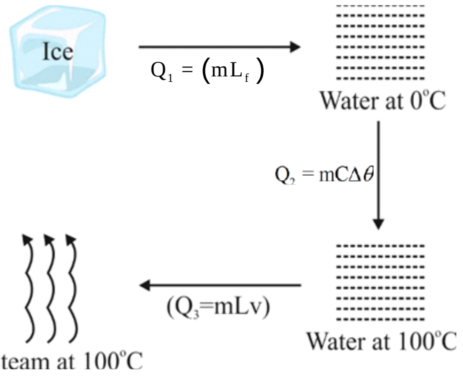
9. Heat required to convert one gram of ice at 0°C into steam at 100°C is (given $L_{fusiom} = 80$ cal/g, $S_w = 1$ cal/g C^{-1} , $L_{steam} = 536$ cal/gm)

(a) 100 calorie (b) 0.01 kilocalorie

(c) 716 calorie (d) 1 kilocalorie

Solution:

The correct answer is C.



$$= Q_1 + Q_2 + Q_3$$

$$= 1 \times 80 + 1 \times 1 \times (100 - 0) + 1 \times 536$$

$$= 716\ cal$$

10. A lead bullet of 10 g travelling at 300 m/s strikes against a block of wood and comes to rest. Assuming 50% of heat is absorbed by the bullet, the increase in its temperature is (Specific heat of lead = 150J/kg K)

(a) 100°C (b) 125°C

(c) 150°C (d) 200°C

Solution:

The correct answer is C.

Since specific heat of lead is given in Joules, hence use $W = Q$ instead of

$$Q = m.c.\Delta\theta$$

$$Q = \frac{1}{2} \times \left(\frac{1}{2}mv^2\right)$$

$$\Delta\theta = \frac{v^2}{4c} = \frac{(300)^2}{4 \times 150} = 150°\text{C}$$

11. What will be the final temperature if 50 g of water at 0°C is added to 250 g of water at 90°C?

(a) 15°C (b) 30°C

(c) 45°C (d) 75°C

Solution:

Correct option is D

Heat lost by hot body = heat gained by cold body

$$250 \times 1 \times [90 - \theta] = 50 \times 1 \times (\theta - 0°)$$

$$5[90 - \theta] = \theta$$

$$450 = 6\theta$$

$$\theta = \frac{450}{6} = 75°\text{C}$$

12. The heat capacity of a body depends on

(a) the heat given

(b) the temperature raised

(c) the mass of the body

(d) the material of the body

Solution:

Correct option is C and D.

Heat capacity of a body depends on mass of the body and type of material it is made up of.

13. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetics energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal (g – °C)(1 cal = 4.2 × 10^7 ergs) close to: (a) 87.5°C

(b) 83.3°C

(c) 119.2°C

(d) 38.4°C

Solution:

(a)According to question, one half of its kinetic energy is converted into heat in the wood.

$$\frac{1}{2}mv^2 \times \frac{1}{2} = ms\Delta T$$

$$\Delta T = \frac{v^2}{4 \times s} = \frac{210 \times 210}{4 \times 4.2 \times 0.3 \times 1000} = 87.5\ °\text{C}$$

14. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be:

(a) 85°C (b) 60°C

(c) 80°C (d) 70°C

Solution:

(c) Heat loss = Heat gain = $mS\Delta\theta$

So, $m_AS_A\Delta\theta_A = m_BS_B\Delta\theta_B$

$100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5S_B$

$$S_A = \frac{3}{4}S_B$$

Now, $100 \times S_A \times (100 - \theta) = 50 \times S_B \times (\theta - 50)$

$$2 \times \left(\frac{3}{4}\right) \times (100 - \theta) = (\theta - 50)$$

$300 - 3\theta = 2\theta - 100$

$400 = 5\theta$

$\theta = 80$ °C

15. In an experiment a sphere of aluminium of mass 0.20 kg is heated upto 150°C. Immediately, it is put into water of volume 150 cc at 27°C kept in a calorimeter of water equivalent to 0.025 kg. Final temperature of the system is 40°C. The specific heat of aluminium is : (take 4.2 Joule = 1 calorie)

(a) 378 J/kg – °C (b) 315J/kg – °C

(c) 476J/kg – °C (d) 434 J/kg – °C

Solution:

(d)According to principle of calorimetry,

$Q_{given} = Q_{used}$

$0.2 \times S \times (150 - 40) = 150 \times 1 \times (40 - 27) + 25 \times (40 - 27)$

$0.2 \times S \times 110 = 150 \times 13 + 25 \times 13$

Specific heat of aluminium

$$S = \frac{13 \times 25 \times 7}{0.2 \times 110}$$

= 434 J/kg-°C

Expert Calorimetry

1. An electric heater supplies 1.8 kW of power in the form of heat to a tank of water. How long will it take to heat the 200 kg of water in the tank from 10 to 70°C? Assume heat losses to the surroundings to be negligible.

(a) 1.75 h (b) 7.75 h

(b) 4.75 h (d) 5.75 h

Solution:

Correct option is B.

The heat added to tank $\Delta Q = power \times time$

$\Delta Q = 1.8 \times 10^3 \times t(J) - - - -(i)$

The heat absorbed in water

$$\Delta Q = mc\Delta Q = mc\Delta T = 200 \times 10^3 \times 1 \times 60$$
$$= 12 \times 10^6 cal$$

$= 12 \times 10^{66} \times 4.2J$

Hence,

$$t = \frac{12 \times 10^6 \times 4.2}{1.8 \times 10^3} = 2.78 \times 10^4 s = 7.75h$$

2. The water equivalent of a copper calorimeter is 4.5 gm.

If the specific heat of copper is .09 cal/gm/°C

(a) mass of the calorimeter is .5 kg

(b) thermal capacity of the calorimeter is 4.5 cal/°C

(c) Heat required to raise the temperature of the calorimeter by 8°C will be 36 cal

(d) heat required to melt 15 gm of ice placed in the calorimeter will be 1200 cal

Solution:

B thermal capacity of the calorimeter is $4.5\ cal°C^{-1}$

C heat required to raise the temperature of the calorimeter by 8°C will be 36 cal.

D heat required to melt 15 gm of ice placed in the calorimeter will be 1200 cal

$$W = \left(\frac{ms}{S_w}\right) \Rightarrow 4.5 = \frac{m \times 0.09}{1}$$

$$\therefore m = \frac{4.5}{0.09} = 50g = 0.05\ kg$$

(b) Thermal capacity $= ms$

$= 4.5 \times 1\ cal°C^{-1} = 4.5\ cal°C^{-1}$

(c) $Q = ms\Delta T = 4.5 \times 8\ cal = 36\ cal$

(d) $Q = mL = 15 \times 80 = 1200\ cal$

3. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by (Given: room temperature = 30° C, specific heat of copper=0.1 cal/gm °C

(a)1250°C (b) 825°C

(c) 800°C (d) 885°C

Solution:

(d) According to principle of calorimetry,

Heat lost = Heat gain

$100 \times 0.1(T - 75) = 100 \times 0.1 \times 45 + 170 \times 1 \times 45$

$10T - 750 = 450 + 7650 = 8100$

$T - 75 = 810 = 885$ °C

4. The quantities of heat required to raise the temperature of two solid copper spheres of radii r_1 and r_2 ($r_1 = 1.5r_2$) through 1K are in the ratio

(a) $\frac{9}{4}$ **(b)** $\frac{3}{2}$

(c) $\frac{5}{3}$ **(d)** $\frac{27}{8}$

Solution:

(d) Since, heat required, $Q = mc\Delta T$

$$= \left(\frac{4}{3}\pi r^3 \cdot \rho\right)c\Delta T$$

$[\because m = V_{sphere}\,\rho]$

Since, π, ρ, c and T are constants.

$$Q \propto r^3 \text{ or } \frac{Q_1}{Q_2} = \frac{r_1^3}{r_2^3}$$

$$= \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1.5r_2}{r_2}\right)^3 = \frac{27}{8}$$

Hence, correct option is (d).

5. Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is at 100°C, while the other one is at 0°C. If the two bodies are brought into contact, then assuming no heat loss, the final common temperature is

(a) 50°C

(b) more than 50°C

(c) less than 50°C but greater than 0°C

(d) 0°C

Solution:

(b) Heat lost by Ist body = heat gained by IInd body. Body at 100°C temperature has greater heat capacity than body at 0°C so final temperature will be closer to 100°C. So, $T_c > 50°$ C.

6. 50 gm of copper is heated to increase its temperature by 10°C. If the same quantity of heat is given to 10 gm of water , then the rise in its temperature is (Specific heat of copper = 420 Joule-kg^{-1} °C^{-1})

(a) 5°C **(b) 6°C**

(c) 7°C **(d) 8°C**

Solution:

The answer is A.

Heat absorbed by copper = heat absorbed by water

$$Heat\ absorbed = \text{mass} \times \text{specific heat} \times \text{temperature difference}$$

$$\Rightarrow 50 \times 0.42 \times 10 = 10 \times 4.2 \times \Delta T$$

$$\Rightarrow \Delta T = 5℃$$

Pro Calorimetry

1. Two liquids A and B are at 32°C and 24°C. When mixed in equal masses the temperature of the mixture is found to be 28°C. Their specific heats are in the ratio of

(a) 3 : 2 **(b) 2 : 3**

(c) 1 : 1 **(d) 4 : 3**

Solution:

The answer is C.

Let specific heat of A and B be C_a and C_b respectively.

Also there masses are m and final temperature $t = 28℃$

Heat released by A = heat absorbed by B

$$\Rightarrow mC_a(32 - t) = mC_b(t - 24)$$

$$\Rightarrow \frac{C_a}{C_b} = \frac{4}{4} = 1$$

Thus required ratio is 1 : 1

2. A liquid of mass m and specific heat c is heated to a temperature 2T. Another liquid of mass m/2 and specific heat 2c is heated to a temperature T. If these two liquids are mixed, the resulting temperature of the mixture is

(a) (2/3)T **(b) (8/5)T**

(c) (3/5)T **(d) (3/2)T**

Solution:

The answer is D.

$$\theta_{min} = \frac{m_1c_1\theta_1 + m_2c_2\theta_2}{m_1c_1 + m_2c_2}$$

$$= \frac{m \times c \times 2T + \frac{m}{2}(2c)T}{m.c + \frac{m}{2}(2c)} = \frac{3}{2}T$$

3. How many grams of a liquid of relative specific heat 0.2 at a temperature 40°C must be mixed with 100 gm of a liquid of relative specific heat of 0.5 at a temperature 20°C, so that the final temperature of the mixture becomes 32°C

(a) 175 g **(b) 300 g**

(c) 295 g **(d) 375 g**

Solution:

The correct answer is D.

$$Temperature\ of\ mixture\ \theta = \frac{m_1c_1\theta_1 + m_2c_2\theta_2}{m_1c_1 + m_2\theta_2}$$

$$\Rightarrow 32 = \frac{m_1 \times 0.2 \times 40 + 100 \times 0.5 \times 20}{m_1 \times 0.2 + 100 \times 0.5}$$

$$\Rightarrow m_1 = 375\ gm$$

4. A vessel contains 110 g of water. The heat capacity of the vessel is equal to 10 g of water. The initial temperature of water in vessel is 10°C. If 220 g of hot water at 70°C is poured into the vessel, then the final temperature of mixture (neglecting radiation loss) will be

(a) 70°C **(b) 80°C**

(c) 60°C **(d) 50°C**

Solution:

The correct answer is D

$Let\ the\ final\ temperature\ of\ water\ be\ \theta$

$Heat\ taken = Heat\ given$

Heat $Q = mc\Delta\theta$

$$110 \times 1(\theta - 10) + 10(\theta - 10) = 220 \times 1(70 - \theta)$$

$\Rightarrow \theta = 48.8°\text{C} = 50°\text{C}$

5. A liquid of mass M and specific heat S is at a temperature 2t. If another liquid of thermal capacity 1.5 times that of the first liquid and at a temperature of $\frac{t}{3}$ is added to it, then the resultant temperature will be

(a) $\frac{4}{3}t$ **(b) t**

(c) $\frac{t}{2}$ **(d)** $\frac{2}{3}t$

Solution:

The correct answer is B.

Heat lost by one liquid = Heat gained by another liquid

$$m_1.C_1.\Delta T_1 = m_2.C_2.\Delta T_2$$

$$m(s)(2t - T) = m(1.5S)\left(T - \frac{t}{3}\right)$$

$$2t - T = 3\frac{T}{2} - \frac{t}{2}$$

$$5\frac{T}{2} = 5\frac{t}{2}$$

$$T = t$$

So, the resultant temperature is t.

6. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 J kg^{-1} K^{-1})

(a) 458 J kg^{-1} K^{-1} **(b) 1232 J kg^{-1} K^{-1}**

(c) 916 J kg^{-1} K^{-1} **(d) 654 J kg^{-1} K^{-1}**

Solution:

(c) Let specific heat of unknown metal be ‘s’

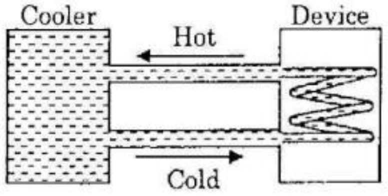
According to principal of calorimetry, Heat lost = Heat gain

$m \times s\Delta\theta = m_1 s_{\text{brass}} (\Delta\theta_1) + m_2\, s_{\text{water}}\, \Delta\theta_2$

$192 \times S \times (100 - 21.5) = 128 \times 394 \times (21.5 - 8.4) + 240 \times 4200 \times (21.5 - 8.4)$

S = 916 Jkg^{-1}k^{-1}

7. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is (Specific heat of water is 4.2 kj kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3}



(a) 1600 **(b) 2067**

(c) 2533 **(d) 3933**

Solution:

$$\text{P}_{\text{heater}} - \text{P}_{\text{cooler}} = \frac{mc\Delta T}{t} = \frac{V\text{p}c\Delta T}{t}$$

$$\therefore (3000 - \text{P}_{\text{cooler}}) = \frac{0.12\times1000\times4.2\times10^3\times20}{3\times60\times60}$$

$$\therefore \text{P}_{\text{cooler}} = 2067\text{W}$$

8. A current carrying wire heats a metal rod. The wire provides a constant power P to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) is the metal rod changes with time (t) as $\boldsymbol{T(t) = T_0(1 + \beta t^{1/4})}$ Where β is a constant with appropriate dimensions while T$_0$ is a constant with dimensions of temperature. The heat capacity of metal is:

(a) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$ **(b)** $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$

(c) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ **(d)** $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

Solution:

(d) Power,

$$P = \frac{dQ}{dt} = \frac{d}{dt}(mc)T = (H)\frac{dT}{dt}$$

$$= (H)\frac{d}{dt}\left[T_0\left(1 + \beta t^{\frac{1}{4}}\right)\right]$$

$$P = (H)T_0$$

$$= \frac{\beta t^{-\frac{3}{4}}}{4} \text{ where } H = \text{heat capacity}$$

$$\therefore (H) = \frac{4\text{P}t^{\frac{3}{4}}}{T_0\beta} \quad \text{... (i)}$$

$$\text{But } t^{\frac{1}{4}} = \frac{T(t) - T_0}{\beta T_0}$$

$$\therefore t^{\frac{3}{4}} = \frac{[T(t) - T_0]^3}{\beta^3 T_0^3} \quad \text{... (ii)}$$

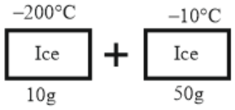
From eq. (i) & (ii),

$$(H) = \frac{4P}{T_0\beta}\frac{[T(t) - T_0]^3}{\beta^3 T_0^3}$$
$$= \frac{4P[T(t) - T_0]^3}{\beta^4 T_0^4}$$

Beginner **State Change Mixing**

1. Find the final temperature of the mixture

T_f =



Solution:

$(t_F + 200) + (t_F + 10)$

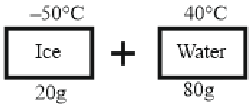
$. \Rightarrow 60t_F \times 2500 = 0$

$\Rightarrow t_F = \frac{-2500}{60} = -41.66°C$

$6t_F + 200 + 50 = 0$

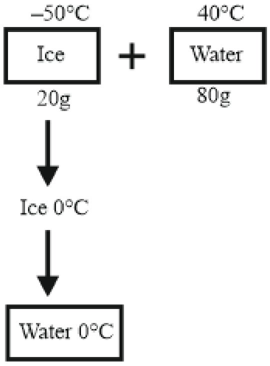
$dt_F = -250$

2. Find the final temperature of the mixture



$T_F = ?$

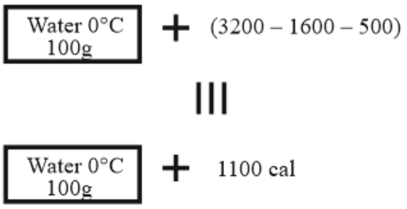
Solution:



$Q_1 = 20 \times \frac{1}{2} \times 50 = 500\text{cal}$

$Q_2 = 20 \times 80 = 1600\text{cal}$

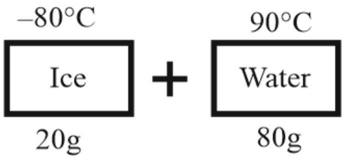
$Q = 80 \times 1 \times 40 = 3200\text{ cal}$



$1100 = 100 \times 1 \times x$

$x = 11°C$

3. Find the final temperature of the mixture



T_f =

Solution:

$Q_1 = 20 \times \frac{1}{2} \times 80° = 800$

Ice 0°C

$Q_2 = 20 \times 80 = 1600$

Water 0°C

$= (1600 \times 800)$

$= T_f =$

$Q = 80 \times 1 \times 90° = 7200°$

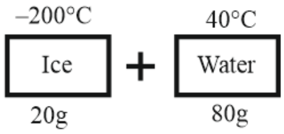
Water 0°C + 7200°

$\therefore Q = mx\Delta t$

$\Rightarrow 4800 = 100 \times 1 \times x$

$\Rightarrow x = 48°C$

4. Find the final temperature of the mixture



T_f =

Solution:

$Q_1 = 20 \times \frac{1}{2} \times 200 = 200\text{ cal}$

Ice 0°C

$Q_2 = 1600\text{ cal}$

Water 0°C

$Q_3 = 3200\text{ cal}$

Water 100g 0°C + (3200 – 3600)

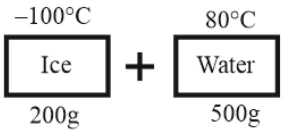
100g water0°C = 400 cal

$(x)80 = 400$

x = 5gram ice

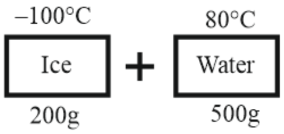
$T_f = 0°C$

5. Find the final temperature of the mixture.



$T_f =$

Solution:



$Q_1 = 10,000 \text{ cal}$

Ice 0°C

$Q_2 = 16,000 \text{ cal}$

Water 0°C

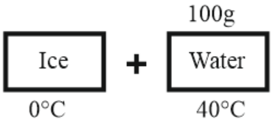
$Q_3 = 40,000 \text{ cal}$

Water 0°C

700 gram water 0°C + 14000 cal

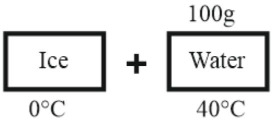
$700 \times 1 \times x = 14000, x = 20°C$

6. Find the final temperature of the mixture.



T_f =

Solution:



T_f = 0°C

Water

Q = 8000

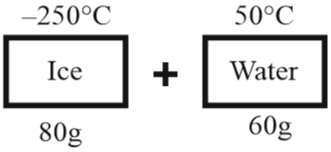
Water

Q = 4000 cal

200 g water – 4000

$4000 = 80x, x = 50g$

7. Find the final temperature of the mixture



T_f =

Solution:

–250°C ice 80g

$\downarrow$

$Q = 80 \times \frac{1}{2} \times 250$

$= 10,000 \text{ cal}$

$\downarrow$

0°C ice 80g

$\downarrow$

$Q = 80 \times 80 = 6400 \text{ cal}$

$\downarrow$

0°C water 80 g

50°C water 60g

$\downarrow$

$Q = 60 \times 1 \times 50$

$= 3000 \text{ cal}$

$\downarrow$

50°C water60g

Water 0°C 140g + 3000 – (10,000 +6400)

140g = –13,400 cal

Check :-

$140 \times 80 = 11200$

$13400 - 11200 = 2200 \text{ cal}$

All water will become ice

Ice 0°C 140 g – 2200 cal

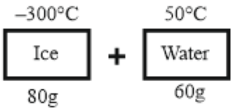
$= 140 \times \frac{1}{2} \times x = 2200$

$70x = 2200$

$x = \frac{220}{7} = 31.43$

$T_f = -31.43°C$

8. Find the final temperature of the mixture



Solution:

–300°C ice 80g

$\downarrow$

$Q = 80 \times \frac{1}{2} \times 300$

$= 12,000 \text{ cal}$

$\downarrow$

0°C ice 80g

$\downarrow$

$Q = 80 \times 80 = 6400 \text{ cal}$

$\downarrow$

0°C water 60g

$\downarrow$

$Q = 60 \times 1 \times 50$

$= 3000 \text{ cal}$

$\downarrow$

50°C water 60g

Calculation :

Water 0°C 140g + 3000 – (12000+6400)

Check :

$140 \times 80 = 11200$

$13400 - 11200 = 4200$ cal

All water will become ice

Ice 0°C 140 g – 4200 cal

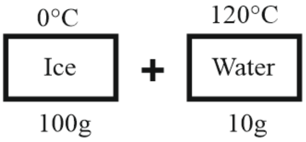
$= 140 \times \frac{1}{2} \times x = 4200$

$70x = 4200$

$x = \frac{420}{7} = 60$

$T_f = -60°C$

9. Find the final temperature of the mixture



$T_f = ?$

Solution:

Vapour 120°C 10g

↓

$Q = 10 \times 2 \times 20 = 400$ cal

↓

Vapour 100°C 10g

↓

$Q = 540 \times 10 = 5400$ cal

↓

water 100°C 10g

↓

$Q = 10 \times 1 \times 100 = 1000$ cal

↓

water 0°C 10g

Calculation

Water at 0°C 110 g + 1000 + 5400 +400

= 6800 cal

Check :

$Q = 110 \times 1 \times 100$

If cant go to 100°

Lets assume, $T_f = x$

$110 \times 1 \times x = 6800$

$x = \frac{6800}{110}$

$= \frac{680}{11} = 61.8°C$

10. 300 gm of water at 25°C is added to 100 gm of ice at 0°C. The final temperature of the mixture is

(a) $-\frac{5}{3}°C$ **(b)** $-\frac{5}{2}°C$

(c) – 5°C **(d) 0°C**

Solution:

The correct answer is D.

We know that latent heat of fusion of ice is $79.9\ Cal\ per\ gram.$

Let final temperature be T.

Then

$m_1 s\Delta T = m_2 L$

$300 \times 1 \times (25 - T) = 100 \times 75$

$(25 - T) = \frac{100 \times 75}{300}$

$(25 - T) = 25$

$T = 0$℃

After that total energy left = 4.7 × 100

Total mass of water = $400\ g$

Amount of water again converted into ice

$m = \frac{470}{79.7}$

$m = 5.9\ g$

Thus whole mass is converted into water at 0℃ and about $5.9g$ water is again converted into ice whose temperature is also 0℃.

After achieving the temperature of 0℃ , latent heat of fusion is required firstly for conversion of water into ice than further lowering of temperature is possible. So final temperature will be 0℃.

11. 540 g of ice at 0°C is mixed with 540 g of water at 80°C. The final temperature of the mixture is (Latent heat of fusion of ice = 336Jg^{-1}, specific heat of water = 4.2Jg^{-1} ^{0}C^{-1})

(a) 0°C **(b) 40°C**

(c) 80°C **(d) Less than 0°C**

Solution:

The correct answer is A.

Let the final temperature be T,

$mL + mc\Delta T = heat\ gaine$d by ice

= heat lost by water.

$540 \times 336 + 540 \times 4.2 \times (T - 0)$

$= 540 \times 4.2 \times (80 - T)$

$\Rightarrow T = 0$℃

12. 80 gm of water at 30°C is poured on a large

block of ice at 0°C. The mass of ice that melts is

(a) 30 gm **(b) 80 gm**

(c) 1600 gm **(d) 150 gm**

Solution:

The correct answer is A.

A/C to thermodynamics,

Heat flow from higher temperature body to lower temperature body

Heat loss by water = Heat gain by ice

$Given, mass\ of\ water = 80gm$

$$Specific\ heat\ of\ water = \frac{\frac{1Cal}{gm}}{°C}$$

$\therefore Heat\ loss\ by\ water = ms\Delta T$

$= 80 \times 1 \times (30 - 0)$

$= 80 \times 1 \times 30 = 2400cal$

Heat gain by ice = ML_f

$$L_f\ is\ latent\ heat\ of\ fusion = \frac{80Cal}{gm}$$

M is the mass of melted ice

$Now, 2400 = 80M$

$M = 30gm$

Hence, 30gm of ice is melted.

13. One kilogram of ice at 0°C is mixed with one kilogram of water at 80°C. The final temperature of the mixture is (Take: specific heat of water $= 4200\, J\, kg^{-1}\, K^{-1}$**, latent heat of ice = 336 kJ kg^{-1})**

(a) 40°C **(b) 60°C**

(c) 0°C **(d) 50°C**

Solution:

The correct answer is C.

$It\ is\ known\ that\ for\ water\ an$d ice mixing $\theta_{mix} = \frac{m_W\theta_W - \frac{m_iL_i}{c_W}}{m_i + m_W}$

$$\theta_{mix} = \frac{m_W\theta_W - \frac{m_iL_i}{c_W}}{m_i + m_W}$$

$$\therefore m_i = m_w \Rightarrow \theta_{mix} = \frac{\theta_W - \frac{L_i}{c_W}}{2} = \frac{80 - \frac{336}{4.2}}{2}$$

$$= 0°C$$

14. A block of mass 2.5 kg is heated to temperature of 500°C and placed on a large ice block. What is the maximum amount of ice that can melt (approx). Specific heat for the body = 0.1 Cal/gm°C.

(a) 1 kg **(b) 1.5 kg**

(c) 2 kg **(d) 2.5 kg**

Solution:

Correct option is B.

$(Converts\ units\ to\ SI)\ 0.1\frac{cal}{gm}$

$= 0.1 \times 1000\frac{cal}{kg}$

$Maximum\ heat\ released\ by\ block, Q$

$= 2.5 \times 0.1 \times 10^3 \times (500 - 0)cal$

Now heat released Q will melt some ice

$Q = mL = m \times 80000$

So,$m = 1.56\ kg$

15. 1 kg of ice at – 10°C is mixed with 4.4 kg of water at 30°C. The final temperature of mixture is: (specific heat of ice is 2100 J/kg/k)

(a) 2.3°C **(b) 4.4°C**

(c) 5.3°C **(d) 8.7°C**

Solution:

Correct option is D.

$Heat\ release$d by water to reach 0°C is Q_1

$= mc\Delta T = 4.4 \times 4200 \times 30 = 554400\, J$

$Heat\ absorbe$d by ice to melt is Q_2

$= 2100 \times 1 \times 10 + 1 \times 336 \times 10^3$

$= 357000\, J$

$extra\ heat, Q_1 - Q_2 = 197400\, J$

$(m_1 + m_2)C\Delta T = 197400$

$$\Rightarrow (T - 0) = \frac{197400}{\left((4.4 + 1) \times 4200\right)}$$

So,

$T = 8.7°C$

16. Steam at 100°C is added slowly to 1400 gm of water at 16°C until the temperature of water is raised to 80°C. The mass of steam required to do this is (L$_V$ = 540 cal/gm):

(a) 160 gm **(b) 125 mg**

(c) 250 gm **(d) 320 gm**

Solution:

Correct option is A.

He$at\ supplie$d to water is Q_1

$= 1400 \times (80 - 16)$

Now this heat supplied to water will be released by condensation of vapour to liquid.

Also,

$Q_1 = m\left(L_{vap} + 1 \times (100 - 80)\right)$

So, m = 160 gm.

17. M grams of steam at 100°C is mixed with 200

g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is _____________.

Solution:

Using the principal of calorimetry

$M_{ice}\,L_f + m_{ice}\,(40 - 0)\,C_w = m_{stream}\,L_v + m_{stream}\,(100 - 40)\,C_w$

$M(540) + M \times 1 \times (100 - 40) = 200 \times 80 + 200 \times 1 \times 40$

$600\,M = 24000$

$M = 40g$

18. Ice at –20°C is added to 50 g of water at 40°C, When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to (Specific heat of water = 4.2J/g/°C Specific heat of Ice = 2.1 J/g/°C Heat of fusion of water at 0°C = 334J/g)

(a) 50g
(b) 100 g
(c) 60 g
(d) 40 g

Solution:

(d) Let m gram of ice is added.

From principal of calorimeter

Heat gained (be ice) = heat lost (by water)

$\therefore 20 \times 2.1 \times m + (m - 20) \times 334 = 50 \times 4.2 \times 40$

$376\,m = 8400 + 6680$

$m = 40.1$

19. A piece of ice falls from a height h so that it melts completely. Only one-quarter of the heat produced is absorbed by the ice and all energy of ice gets converted into heat during its fall. The value of his [Latent heat of ice is 3.4×10^5 J/kg and g = 10N/kg]

(a) 544 km
(b) 136 km
(c) 68 km
(d) 34 km

Solution:

(b) According to question as conservation of energy, energy gained by the ice during its fall from height his given by

E = mgh

As given, only one quarter of its energy is absorbed by the ice.

So, $\dfrac{mgh}{4} = mL_f \Rightarrow h = \dfrac{mL_f \times 4}{mg}$

$= \dfrac{L_f \times 4}{g} = \dfrac{3.4 \times 10^5 \times 4}{10} = 13.6 \times 10^4 = 136000$ m

= 136 km

Expert State Change Mixing

1. Pure water super cooled to –15°C is contained in a thermally insulated flask. Small amount of ice is thrown into the flask. The fraction of water frozen into ice is:

(a) 3/35
(b) 6/35
(c) 6/29
(d) 2/35

Solution:

Correct option is B.

$$Heat\ required,\ (M \times 1 \times 15) = (m \times 80) + (m \times 0.5 \times 15)$$

$$\therefore m \times 87.5 = M \times 15$$

$$\therefore \frac{m}{M} = \frac{15}{87.5} = \frac{6}{35}$$

$Where\ \frac{m}{M}\ is\ the\ fraction\ of\ water\ frozen\ into\ ice.$

2. A volume of 120 ml of drink (half alcohol + half water by mass) originally at a temperature of 25°C is cooled by adding 20 gm ice at 0°C. If all the ice melts, find the final temperature of the drink. (density of drink = 0.833 gm/cc, specific heat of alcohol = 0.6 cal/gm/°C)

Solution:

$volume\ of\ \text{drink} = 120\ \text{ml}$

$\text{density of drink}\ \ d = 0.833\,\dfrac{\text{gm}}{\text{cc}}$

$$d = \frac{2d_1d_2}{d_1 + d_2}$$

$d_1 = \text{density of alcohol}$

$d_2 = \text{density of water} = \dfrac{1\text{gm}}{\text{cc}}$

$$0.833 = \frac{2d_1}{d_1 + 1}$$

$$\frac{1}{d_1} = \frac{1 - 0.416}{0.416} = \frac{0.6}{0.4} = \frac{3}{2}$$

$$d_1 = \frac{2}{3}$$

Heat lost by drink = Heat gained by ice

Heat lost by (alcohol + water) = Heat gained by ice

$$[V_1d_1S_1 + S_2V_2d_2]\Delta\theta = m_{ice}L_{ice} + m_{ice} \times S_w \times \Delta\theta$$

$$\left[60\times\frac{2}{3}\times 0.6+1\times 60\times 1\right][25-0]$$

$$= 20[80]+20(\theta-0)$$

$(24+60)(25)=1600+200$

On simplifying , $\theta \cong 4°$

3. A block of ice with mass m falls into a lake. After impact, a mass of ice m/5 melts. Both the block of ice and the lake have a temperature of 0°C. If L represents the heat of fusion, the minimum distance the ice fell before striking the surface is

(a) $\frac{L}{5g}$ **(b)** $\frac{5L}{g}$

(c) $\frac{gL}{5m}$ **(d)** $\frac{mL}{5g}$

Solution:

Correct option is A.

Let the height from where the ice block falls to the lake is h.

Change in potential energy $P.E.=\ mgh$

Where g is the acceleration due to gravity.

The PE loss on reaching the lake is converted to KE of the ice block and is converted into heat on impact.

Amount of heat required to melt mass of $\frac{m}{5}$ *is* $\frac{m}{5}L$

Where L is the latent heat of fusion.

Equating both, we get

$$mgh=\frac{m}{5}L$$

$$\therefore h=\frac{L}{5g}$$

4. Some steam at 100°C is passed into 1.1 kg of water contained in a calorimeter of water equivalent 0.02 kg at 15°C so that the temperature of the calorimeter and its contents rises to 80°C. What is the mass of steam condensing. (in kg)

(a) 0.130 **(b) 0.065**

(c) 0.260 **(d) 0.135**

Solution:

Correct option is A.

Let M be the mass of steam condensed.

Amount of heat lost by the steam due to the latent heat of condensation and due to cooling to $80^{o}C$.

$=M\times 540+m(100-80)$

Amount of heat absorbed by the water and calorimeter system

$=1100\times 1\times(80-15)+20\times 1\times(80-15)$

From conservation of heat energy,

$\Rightarrow M\times 540+M(100-80)$

$=1100\times 1\times(80-15)+20\times 1\times(80-15)$

$\Rightarrow M=130g=0.130kg$

5. The specific heat of water = 4200 Jkg^{-1} K^{-1} and the latent heat of ice = 3.4 × 10^{5} J kg^{-1}. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams):

(a) 61.7 **(b) 63.8**

(c) 69.3 **(d) 64.6**

Solution:

(a) Here ice melts due to water.

Let the amount of ice melts = m_{ice}

$m_w s_w \Delta\theta = m_{ice}L_{ice}$

$$\therefore\ m_{ice}=\frac{m_w s_w \Delta\theta}{L_{ice}}$$

$$=\frac{0.2\times 4200\times 25}{3.4\times 10^{5}}=0.0617\text{ kg}=61.7\text{ g}$$

6. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1} °C^{-1})

(a) 2 **(b) 4**

(c) 3.2 **(d) 2.6**

Solution:

(a) Heat given by water = $m_wC_w(T_{mix}-T_w)$

$=200\times 1\times(31-25)$

Heat taken by steam = $m\,L_{steam}+m\,C_w(T_s-T_{mix})$

$=m\times 540+m(1)\times(100-31)$

$=m\times 540+m(1)\times(69)$

From the principal of calorimeter,

Heat lost = Heat gained

$\therefore (200)(31-25)=m\times 540+m(1)(69)$

$1200=m(609)$

$m\approx 2.$

7. When M$_1$ gram of ice at –10°C (Specific heat = 0.5 cal g^{-1}°C^{-1}) is added to M$_2$ gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is:

(a) $\frac{50M_2}{M_1}-5$ **(b)** $\frac{5M_1}{M_2}-50$

(c) $\frac{50M_2}{M_1}$ **(d)** $\frac{5M_2}{M_1}-5$

Solution:

(a) $M_1C_{ice} \times (10) + M_1L = M_2C_{\omega}$ (50)

or $M_1 \times 0.5) \times 10 + M_1L = M_2 \times 1 \times 50$

$$L=\frac{50M_2}{M_1}-5$$

Pro State Change Mixing

---

1. A bullet moving with a uniform velocity v, stops suddenly after hitting the target and the whole mass melts. Mass of bullet is m, its specific heat is S, initial temperature is 25°C, melting point of bullet is 475°C and its latent heat is L. Then v is given by

(a) $mL = mS(475-25)+\frac{1}{2}\cdot\frac{mv^2}{J}$

(b) $mS(475-25)+mL=\frac{mv^2}{2J}$

(c) $mS(475-25)+mL=\frac{mv^2}{J}$

(d) $mS(475-25)-mL=\frac{mv^2}{2J}$

Solution:

The correct answer is B.

Firstly the temperature of bullet rises up to melting point, then it melts. Hence according to W = JQ

$$\Rightarrow \frac{1}{2}mv^2 = J[m.c.\Delta\theta + mL]$$

$$= J[m\,s(475-25)+mL]$$

$$\Rightarrow ms(475-25)+mL=\frac{mv^2}{2J}$$

2. Some water at 0°C is placed in a large insulated enclosure (vessel). The water vapour formed is pumped out continuously. What fraction of the water will ultimately freeze, if the latent heat vaporization is seven times the latent heat of fusion?

(a) 7/8 **(b) 8/7**

(c) 3/8 **(d) 5/8**

Solution:

The correct answer is A.

Let f = fraction which freeze m_s, = mass of water

$L_v = 7L_f$ (Given)

Mass of water freezes = m_f

Heat lost by freezing water = m_fL_f

Heat gained by vapour = $m(1-f)L_v$

Using principle of calorimetry

$$mfL_f = m(1-f)L_v$$

$$\frac{1}{7}L_vf = L_v - f\,L_v$$

$$f\,\frac{8}{7}\,L_v = L_v$$

$$f = \frac{7}{8}$$

3. How should 1 kg of water at 5°C be divided into two parts so that if one part turned into ice at 0°C, it would release enough heat to vaporize the other part? Latent heat of steam = 540 cal/gm and latent heat of ice = 80 cal/gm.

Solution:

*Let the mass be divi*ded into x grams(*for ice*) *and*(1000 − x)*grams*(*for vapour*).

*Heat release*d by x grams of water

$$= x \times 1 \times 5 + x \times 80$$

*Heat absorbe*d by (1000 − x)*grams of water*

$$= (1000-x)\times 1 \times 95 + (1000-x)\times 540$$

Assuming that the conversion of the other part takes place at 100℃

$$85x = 95(1000-x)+540(1000-x)$$

$$or\ x = 882\ g$$

Thus, the mass is to be divided into 882 g for conversion into ice and 118 conversion into vapour.

4. An aluminium container of mass 100 gm contains 200 gm of ice at – 20°C. Heat is added to the system at the rate of 100 cal/s. The temperature of the system after 4 minutes will be (specific heat of ice = 0.5 and L = 80 cal/gm, specific heat of Al = 0.2 cal/gm/°C)

(a) 40.5°C **(b) 25.5°C**

(c) 30.3°C **(d) 35.0°C**

Solution:

Correct option is B.

container contains $m_1 = 100g$ *of Al and* $m_2 = 200g$ *of ice*

In 4 min *times heat gives to system*, $Q = rate \times time$

$$Q = 100 \times 4 \times 60 = 24000\ cal$$

Assume the whole ice melts to water (thereby all mass of ice m_2 gains latent heat) and change in temperature be $\Delta T, T \rightarrow final\ temp$

Thus from heat transfer law,

$Q = m_1S_1\Delta T + m_2S_2(20) + m_2L + m_2(1)T(water\ phase)[ice\ consume\ heat\ in -20°C\ to\ 0°C\ only]$

$$= 24000 = 100 \times 0.2(T + 20) + 200 \times 0.5 \times 20 + 200 \times 80 + 200T$$

$$= 24000 = 20(T + 20) + 2000 + 16000 + 200T$$

$= 24000 = 18400 + 220T$

$= 5600 = 220T$

$$T = \frac{560}{22} = 25.45°C$$

5. 10 gm of ice at 0°C is kept in a calorimeter of water equivalent 10 gm. How much heat should be supplied to the apparatus to evaporate the water thus formed? (Neglect loss of heat) (Latent heat of fusion of ice 336J/gm , specific heat of water is 4.2J/gm , latent heat of vapourisation of water is 2260J/gm)

(a) 6200 cal **(b) 7200 cal**

(c) 13600 cal **(d) 8200 cal**

Solution:

Correct option is D.

$heat\ supplied$ in $L_{fusion} + Mc\Delta T + ML_{vap}$

$Q_1 = 10 \times 336 + 10 \times 4.2 \times 100 + 10 \times 2260$

$Q_1 = 30160\ J\ or\ 7200\ cal$

$$Heat\ for\ calorimeter\ \ Q_2 = 10 \times 1 \times 100 = 1000\ cal$$

$Q = Q_1 + Q_2 = 8200\ cal$

6. Heat is being supplied at a constant rate to a sphere of ice which is melting at the rate of 0.1 gm/sec. It melts completely in 100 sec. The rate of rise of temperature thereafter will be (Assume no loss of heat.)

(a) 0.8 °C/sec **(b) 5.4 °C/sec**

(c) 3.6 °C/sec **(d) will change with time**

Solution:

Correct option is A.

Rate of melting of ice is 0.1 $gm\ sec - 1$

$total\ mass\ of\ ice\ m_{ice} = 0.1 \times 100$

$= 10\ gms\ since\ all\ the\ ice\ melts\ \ in\ 100\ sec.$

$total\ amount\ of\ heat\ supplied$ to melt 10 gms of ice Q = 10×80 cal = 800 cal

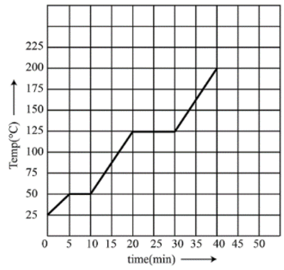
After all the ice melts, the water temp will rise.

$rate\ at\ which\ heat\ is\ supplied\ \frac{dQ}{dt}$

$$= \frac{\frac{dQ}{dt}}{m_{water} \times s_{water}} = \frac{8}{10 \times 1}$$

$= 0.8°C\ sec^{-1}$

7. The graph shown in the figure represent change in the temperature of 5 kg of a substance as it absorbs heat at a constant rate of 42 kJ min^{-1}. The latent heat of vaporization of the substance is:



(a) 630 kJ kg^{-1} **(b) 126 kJ kg^{-1}**

(c) 84 kJ kg^{-1} **(d) 12.6 kJ kg^{-1}**

Solution:

Correct option is C.

First mass will melt and then vaporize, we can see there is a constant temperature line at $T = 125°C$, that is its vaporization temperature, and heat supplied during that duration is latent heat of vaporization.

$Heat\ absorbed$ is $42 \times (30 - 20) = 5 \times L$

So,

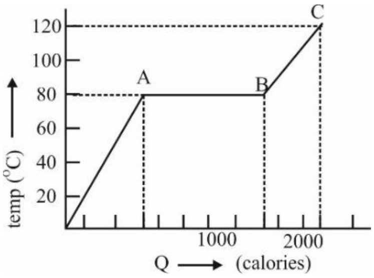
$$L = 84\frac{kJ}{kg}$$

8. A substance is in the solid form at 0°C. The amount of heat added to this substance and its temperature are plotted in the following graph. If the relative specific heat capacity of the solid substance is 0.5, find from the graph

(a) the mass of the substance;

(b) the specific latent heat of the melting process

(c) the specific heat of the substance in the liquid state.



Solution:

i. The mass of the substance

$\theta = ms\Delta T$

$1000 = m \times 0.5 \times 80$

$m = 25\ g$

$\theta = ms\Delta T$

$800 = m \times 0.5 \times 80$

$m = 20\ g$m

Or, $m = 0.02kg$

ii. Latent Heat $= \frac{Heat\ added\ from\ A\ to\ B}{m} = \frac{1600-800}{20}$

$= 40\frac{calories}{gm}$

$= \frac{40000cal}{kg}$

iii. The specific heat of substance in the liquid state

$\theta = mS\Delta T$

$from\ B \rightarrow C\ (liqui$d state)

$600 = 20 \times S \times (120 - 80)$

$S = .75\frac{calories}{gm}c$

$S = \frac{750cal}{kgK}$

9. A vessel containing 100 gm water at 0°C is suspended in the middle of a room. In 15 minutes the temperature of the water rises by 2°C. When an equal amount of ice is placed in the vessel, it melts in 10 hours. Calculate the specific heat of fusion of ice.

Solution:

Let the power delivered by the surrounding be P

$Q = P \times t$

$but\ \ Q = mc\Delta T$

Thus,

$P = \frac{mc\Delta T}{t}\quad ------①$

Where $t\ =\ time\ , T\ =\ temperature$

Now when a mass of ice melt heat that need to be supplied = mL

Where $L\ =\ latent\ heat\ of\ fusion$

$Thus\ we\ have\ mL = P \times 10 \times 60 \times 60$

But

$P = \frac{mc2}{15 \times 60}$

Thus,

$L = \frac{4.2 \times 2 \times 10 \times 60}{15} = 336J\ gm^{-1}$

10. Ice at 0°C is added to 200 g of water initially at 70°C in a vacuum flask. When 50 g of ice has been added and has all melted the temperature of the flask and contents is 40°C. When a further 80g of ice has been added and has all melted, the temperature of the whole is 10°C. Calculate the specific latent heat of fusion of ice.[Take S_w =1 cal /gm °C.]

(a) 3.51 ×10^5 J/ kg (b) 1.2 ×10^5 J/ kg

(c) 2.4 ×10^5 J/ kg (d) 3.0 ×10^5 J/ kg

Solution:

Correct option is A

After 50gm of ice melted completely, the mass of water is the mixture becomes 250gm and will at

40^oC

When 80gm of ice a added,

$250 \times 1 \times (40 - 10) = Heat\ lost\ by\ water$

$= Heat\ gaine$d by ice

$250 \times 30 = 80(L) + 80 \times 1 \times (10 - 0)$

$8L = 670$

$L = \frac{670}{8} = 83.75\frac{cal}{gm}$

$L = 3.51 \times \frac{10^5J}{kg}$

11. An experiment takes 10 minutes to raise the temperature of water in a container from 0°C to 100°C and another 55 minutes to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of water to be 1 cal/g °C, the heat of vapourization according to this experiment will come out to be :

(a) 560 cal/g **(b) 550 cal/ g**

(c) 540 cal/g **(d) 530 cal/ g**

Solution:

(b) As Pt = mCΔT

So, P × 10 × 60 = mC 100 ... (i)

and P × 55 × 60 = mL ... (ii)

Dividing equation (i) by (ii) we get

$\frac{10}{55} = \frac{C \times 100}{L}$

∴ L = 550 cal./g.

12. A liquid at 30°C is poured very slowly into

Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5gm of the liquid completely evaporates. After pouring another 80gm of the liquid to its specific heat will be __________________°C. [Neglect the heat exchange with surrounding]

Solution:

(270°C) Let C be the specific heat capacity of liquid and L be the latent heat of vaporization.

From principal of calorimetry,

Heat = Loss

$m_C S_C\,\Delta\mathrm{T} = mC\,\Delta T + mL$

Or

$m_C S_C\,(110 - 80) = 5\mathrm{C}\,(80 - 30) + 5\mathrm{L} \ldots\ldots\ldots$ (i)

Where, m_C = mass of calorimeter

S_C = specific heat of calorimeter

Again, when 80g liquid is poured and equilibrium temperature is 50°C

$m_C S_C\,(80 - 50) = 80C(50 - 30) \ldots\ldots\ldots\ldots$ (ii)

From eq. (i) & (ii)

1600 C = 250C + 5L

$\therefore \frac{L}{C} = \frac{1350}{5} = 270°\mathrm{C}$

Beginner **Thermal Expansion**

1. A bar of iron is 10 cm at 20°C. At 19°C it will be (α of iron = 11 × 10^{-6}/°C)

(a) 11 × 10^{-6} cm longer

(b) 11 × 10^{-6} cm shorter

(c) 11 × 10^{-5} cm shorter

(d) 11 × 10^{-5} cm longer

Solution:

The correct answer is C.

Generally, when we are reducing the temperature of the iron bar it will become shorter than before

$change\ in\ length\ l_2 = \alpha(l_1)(t_2 - t_1)$

$= (11 \times 10^{-6})(10)(19 - 20)$

$= 11 \times 10^{-6}(10)(-1) = 11 \times 10^{-5}\ cm$

2. Density of substance at 0°C is 10 g/cc and at 100°C, its density is 9.7 g/cc. The coefficient of linear expansion of the substance will be (in / °C)

(a) 10^2 **(b) 10^{-2}**

(c) 10^{-3} **(d) 10^{-4}**

Solution:

The correct answer is D.

$\rho_0 = 10\frac{g}{cc}$

$\rho_{100} = \frac{9.7g}{cc}, \alpha = ?$

From

$\gamma = \frac{\rho_{100} - \rho_0}{\rho_{100} \times (\Delta T)} = \frac{10 - 9.7}{10 \times 100} = 3 \times 10^{-4}$

Coefficient of linear expansion.

$\alpha = \frac{\gamma}{3} = \frac{3 \times 10^{-4}}{3} = 1 \times 10^{-4}°\mathrm{C}^{-1}.$

3. Coefficient of real expansion of mercury is 0.18 × 10^{-3}/°C. If the density of mercury at 0°C is 13.6 g/cc then its density at 473K is

(a) 13.11 g/cc **(b) 26.22 g/cc**

(c) 52.11 g/cc **(d) None of these**

Solution:

The correct answer is A.

Co-efficient of volume expansion of density of mg $is\ r_0$

Density at

We know $\rho = \frac{\rho_0}{(1 + r_0 T)}$

Where,

$\rho \rightarrow$ density at any temp T other than 0°C.

$\rho_0 \rightarrow$ density at 0°C.

$r_0 \rightarrow co - efficient\ of\ volume\ of\ expansion.$

$T \rightarrow temp\ at\ which$ density is to be calculated.

$\rho = \frac{13.6}{1 + 0.18 \times 200}$ {Temp given is 473K ,in °C. It is 200°C.}

$\rho = \frac{13.6}{1.036}$

$\rho = 13.11\frac{g}{cc}$

4. The length of a metallic rod is 5m at 0°C and becomes 5.01 m, on heating upto 100°C. The linear expansion of the metal will be

(a) 2.33 × 10^{-5} /°C **(b) 6.0 × 10^{-5} /°C**

(c) 4.0 × 10^{-5} /°C **(d) 2.0 × 10^{-5} /°C**

Solution:

The correct answer is D.

$\Delta L \propto L_O \Delta T$

$= \alpha\ L_O \Delta T$

$\mathrm{A} = \frac{\Delta L}{L_O \times \Delta T} = \frac{0.01}{5 \times 100} = 2 \times \frac{10^{-5}}{°\mathrm{C}}$

5. A pendulum clock runs faster by 5 seconds per day at 20°C and goes slow by 10 second per day at 35°C. It shows the correct time at a temperature of

(a) 27.5°C **(b) 25°C**

(c) 30°C **(d) 33°C**

Solution: (b)

$$\frac{1}{2}\alpha(35)-T\times 86400=10 \text{ ……….. } (1)$$

$$\frac{1}{2}\alpha(T-)20\times 86400=5 \text{ ………… } (2)$$

Solving (1) and (2) : T = 25°C

6. The value of coefficient of volume expansion of glycerin is $5\times10^{-4}K^{-1}$. The fractional change in the density of glycerin for a rise of 40°C in its temperature is -

(a) 0.015 **(b) 0.020**
(c) 0.025 **(d) 0.010**

Solution:

$$\rho_T=\rho_0(1-\gamma\Delta T)$$

$$\frac{\rho_T}{\rho_0}-(1-\gamma\Delta T)$$

$$1-\frac{\rho_T}{\rho_0}=\gamma\Delta T$$

$$\frac{\rho_0-\rho_T}{\rho_0}=\gamma\Delta T$$

$$=5\times10^{-4}\times 40=0.020$$

7. A copper rod at 88 cm and an aluminium rod of an unknown length have an equal increase in their lengths independent of an increase in temperature The length of the aluminium rod is.

$\left(\alpha_{Cu}=1.7\times10^{-5}K^{-1} \text{ and } \alpha_{Al}=2.2\times10^{-5}K^{-1}\right)$

(a) 68 cm **(b) 6.8 cm**
(c) 113.9 cm **(d) 88 cm**

Solution:

When the temperature is increased by ΔT, their final lengths,

$$L_{Cu}=L_1[1+\alpha_1\Delta T]$$

$$L_{Al}=L_2[1+\alpha_2\Delta T]$$

As change in lengths are equal, $L_{Cu}-L_1=L_{Al}-L_2$

$$\Rightarrow \alpha_1L_1\Delta T=\alpha_2L_2\Delta T$$

$$\Rightarrow 1.7\times10^{-5}\times0.88$$

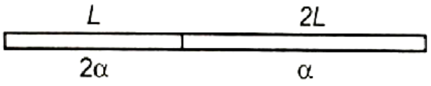
$$=2.2\times10^{-5}\times L_2$$

$$\Rightarrow L_2=\frac{1.7\times10^{-5}\times0.88}{2.2\times10^{-5}}$$

$$L_2=0.88\text{m}$$

$$=68\text{m}$$

8. Two uniform rods of length L and 2L and thermal coefficient of linear expansion 2α and α **respectively, are connected as shown in the figure. The equivalent coefficient of linear expansion is:**



(a) $\frac{3\alpha}{4}$ **(b)** $\frac{4\alpha}{3}$

(c) $\frac{2\alpha}{3}$ **(d)** $\frac{5\alpha}{3}$

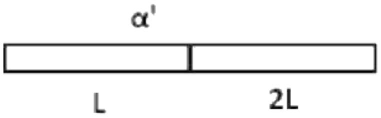
Solution: Hint : $\frac{\Delta L}{L}=\alpha\Delta T$

Step 1: Write the equation of thermal expansion for individual rods.

$$L_1'=L(1+2\alpha\Delta T)$$

$$L_2'=2L(1+\alpha\Delta T)$$

Step 2: Write the equation for the combined rod.



$$\Rightarrow L_2'=3L(1+\alpha'\Delta T)$$

$$\Rightarrow L_2'=L_1'+L_2'$$

$$\Rightarrow 3L(1+\alpha'\Delta T)$$

$$=L(1+2\alpha\Delta T)+2L(1+\alpha\Delta T)$$

$$\Rightarrow \alpha'=\frac{4\alpha}{3}$$

Hence, option (b) is the correct answer

9. The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:

$$R=R_0[1+\alpha(T-T_0)]$$

The resistance is $101.6\,\Omega$ **at the triple-point of water 273.16 K, and** $165.5\,\Omega$ **at the normal melting point of lead (600.5 K). What is the temperature when the resistance is** $123.4\,\Omega$

(a) 384.8 K **(b) 486.8 K**
(c) 287.61 K **(d) 185.61 K**

Solution:

Hint: Use the concept of thermal expansion.

Step 1: Find the coefficient of thermal expansion.

$R_0=101.6\,\Omega, T=273.16\text{ K}$

Case (i) :

$R_1=165.5\,\Omega, T=600.5\text{ K}$

$R_2=123.4\Omega, T_2=?$

$$165.5=101.6\left[1+\alpha\left(600.5-273.16\right)\right]$$

$$\alpha=\frac{63.9}{101.6\times327.34}$$

Step 2: Find the new temperature.

Use the formula again,

$$123.4=101.6\left[1+\frac{63.9}{101.6\times327.34}\left(T_2-273.16\right)\right]$$

$$T_2=\frac{\left(123.4-101.6\right)}{63.9}\times327.34+273.16$$

$$=384.83K$$

.

10. A steel tape 1 m long is correctly calibrated for a temperature of 27°C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45°C. What is the actual length of the steel rod on that day? (Coefficient of linear expansion of steel = $=1.20\times10^{-5}K^{-1}$)

(a) 62.485 cm **(b) 60.762 cm**

(c) 65.935 cm **(d) 63.013 cm**

Solution:

Hint: Apply the concept of thermal expansion.

Step 1: Find the length of the tape at 45°C.

Let I be the actual length of the steel tape and I' be the length at 45°C. Given:

Length of the steel tape at the temperature T = 27°C, 1 = 1 m = 100 cm

Length of the steel rod at the temperature T_1 = 45 °C, l_1 = 63 cm

Coefficient of linear expansion, $\alpha=1.2\times10^{-5}K$

$$1'-1=1\alpha\left(T_1-T\right)$$

$$1'=100\left(1+1.2\times10^{-5}\times\left(45-27\right)\right)$$

= 100.0216 cm

Step 2: Find the actual length of the rod.

Hence the actual length of the steel rod measured by steel tape at 45° is given by,

$$1_2=\frac{100.0216}{100}\times63=63.0136\text{ cm}$$

Hence, option (d) is the correct answer.

11. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C? (Co-efficient of linear expansion of copper $=1.70\times10^{-5}K^{-1}$)

(a) 0.0144 cm **(b) 0.0234 cm**

(c) 0.0123 cm **(d) 0.0111 cm**

Solution: Hint: Apply the concept of thermal expansion.

Initial temperature, T_1 = 27.0°C

Diameter of the hole at T_1 = d_1 = 4.24 cm

Final temperature, T_2 = 227° C

Diameter of the hole at T_2 = D_2

Step 1: Use the formula of areal expansion.

Coefficient of linear expansion, $\alpha_{Cu}=1.7\times10^{-5}K$

$$\frac{\Delta A}{A}=\beta\Delta T \qquad (\text{where } \beta=2\alpha)$$

$$\frac{\Delta A}{A}=\frac{\frac{\pi d_2^2}{4}-\frac{\pi d_1^2}{4}}{\frac{\pi d_1^2}{4}}=\frac{d_2^2-d_1^2}{d_1^2}$$

$$\Rightarrow\frac{d_2^2-d_1^2}{d_1^2}=2\alpha\Delta T$$

Substituting d_1 and α, we get:

d_2 = 4.2544 cm

Change in diameter

$$=d_2-d_1=4.2544-4.24=0.0144\text{cm}$$

12. A uniform copper rod of length 50 cm and diameter 3.0 mm is kept on a frictionless horizontal surface at 20 °C. The coefficient of linear expansion of copper is $2.0\times10^{-5}K^{-1}$ and Young's modulus is $1.2\times10^{11}N/m^2$. The copper is heated to 100°C, then the tension developed in the copper rod is:

(a) 12×10^3N **(b)** 36×10^3N

(c) 18×10^3N **(d) Zero**

Solution:

Since both ends of the rod are free to expand, there will be no tension developed in the rod.

13. A metal rod of silver at 0°C is heated to 100°C. It's length is increased by 0.19 cm. Coefficient of cubical expansion of the silver rod is (Assume length 1 m)

(a) 5.7 × 10^{-5}/°C **(b) 0.63 × 10^{-5}/°C**

(c) 1.9 × 10^{-5}/°C **(d) 16.1 × 10^{-5}/°C**

Solution:

The correct answer is A.

$$coefficient\ of\ linear\ expansion\ is\ \alpha=\frac{\Delta l}{l\Delta t}$$

$$=\frac{0.0019}{100}=1.9\times10^{-5}$$

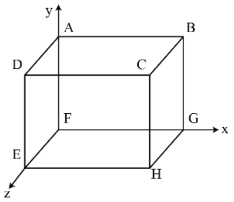
$$coefficient\ of\ volume\ expansion\ is\ 3\alpha$$

$$=5.7\times\frac{10^{-5}}{°C}$$

14. A cuboid ABCDEFGH is anisotropic with α_x = 1 × 10^{-5}/°C, α_y = 2 × 10^{-5} /°C, α_z = 3 × 10^{-5} /°C. Coefficient of superficial expansion of faces can be

(a) β_{ABCD} = 5 × 10^{-5} /°C

(b) $\beta_{BCGH} = 4 \times 10^{-5}$ /°C
(c) $\beta_{CDEH} = 3 \times 10^{-5}$ /°C
(d) $\beta_{EFGH} = 2 \times 10^{-5}$ /°C



Solution:

The correct answer is C.

$Coefficient\ of\ superficial\ expansion\ of\ face\ ABCD =$
$\alpha_x + \alpha_z = 4 \times \frac{10^{-5}}{°C}$
$Coefficient\ of\ superficial\ expansion\ of\ face\ BCGH =$
$\alpha_y + \alpha_z = 5 \times \frac{10^{-5}}{°C}$
$Coefficient\ of\ superficial\ expansion\ of\ face\ CDEH =$
$\alpha_x + \alpha_y = 3 \times \frac{10^{-5}}{°C}$
$Coefficient\ of\ superficial\ expansion\ of\ face\ EFGH =$
$\alpha_x + \alpha_z = 4 \times \frac{10^{-5}}{°C}$

15. If the length of a cylinder on heating increases by 2%, the area of its base will increase by

(a) 0.5% **(b) 2%**
(c) 1% **(d) 4%**

Solution:

The correct answer is D. If length is increases by 2 % then it radius will also increases by 2%.

$$\frac{\Delta A}{A} = 2\frac{\Delta L}{L}$$

$$\Rightarrow \frac{\Delta A}{A} = 2 \times 2 = 4\%$$

16. If percentage change in length is 1% with change in temperature of a cube, then what is percentage change in its area and its volume?

(a) 2% , 3% **(b)4%, 5%**
(C)3% , 6% **(D) 3% , 2%**

Solution:

The correct answer is A.

Percentage change in length will change in temperature = $\% \, l$

$$\frac{\Delta l}{l} \times 100 = \alpha \Delta\theta \times 100 = 1\%$$

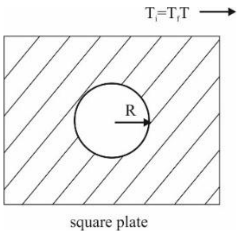
Change in area

$$\Rightarrow \% \, A = \frac{\Delta A}{A} \times 100 = \beta \Delta\theta \ \times 100$$
$$\Rightarrow 2\alpha\Delta\theta \times 100$$
$$\% \, A = 2 \, \%$$

Change in volume

$$\% \, V = \frac{\Delta V}{V} \times 100 = \gamma \Delta\theta \times 100$$
$$= 3\alpha\Delta\theta \times 100$$
$$\% \, V = 3 \, \%$$

17. A given plate is heated so that its temperature is increased. What will happen to its area & radius R?



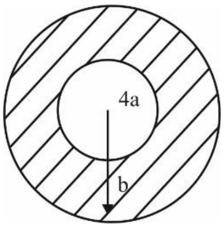
(a) a ↑ R ↓ **(b) a ↑ R ↑**
(c) a ↓ R ↓ **(d) a ↓ R ↑**

Solution:

The answer is B.

On increasing temperature, the avg. separation between any two atoms increases. As every linear dimension increases upon increasing the temperature. Therefore, area & radius will increases.

18. A given circular plate is heated so that its temperature is increased. Compare its both radius.



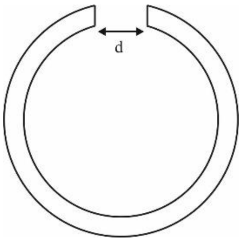
(a) a ↑ b ↓ **(b) a ↑ b ↑**
(c) a ↓ b ↓ **(d) a ↓ b ↑**

Solution:

The answer is B.

On increasing temperature, the avg. separation between any two atoms increases. As every linear dimension increases upon increasing the temperature. Therefore, outer & inner dimension will increases.

19. A body is heated as shown in the diagram. What will happen to d?



(a) d ↑ **(b) d ↓**

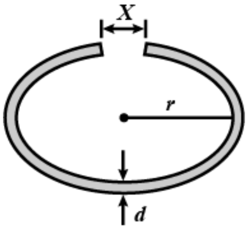
(c) d = Remains same **(d) None**

Solution:

The answer is A.

On increasing temperature, the avg. separation between any two atoms increases. As every linear dimension increases upon increasing the temperature. Therefore, dimension will increases.

20. A cylindrical metal rod of length L_0 is shaped into a ring with a small gap as shown. On heating the system



(a) x decreases, r and d increase

(b) x and r increase, d decreases

(c) x, r and d all increase

(d) Data insufficient to arrive at a conclusion

Solution:

The correct answer is C.

On heating the system: x, r and d all increases, since the expansion of isotropic solids is similar to true photographic enlargement.

21. An open vessel is filled completely with oil which has same coefficient of volume expansion as that of the vessel. On heating both oil and vessel,

(a) the vessel can contain more volume and more mass of oil

(b) the vessel can contain same volume and same mass of oil

(c) the vessel can contain same volume but more mass of oil

(d) the vessel can contain more volume but same mass of oil

Solution:

Correct option is D.

Since the coefficient of expansion of both oil and vessel is same, as the temperature increases, volume of both oil and vessel increase same amount without any overflow of oil, hence more volume of oil is now being contained. However, the vessel is always completely filled, never empty by any fraction, hence the mass of oil containable remains the same.

22. A thin copper wire of length L increase in length by 1% when heated from temperature T_1 to T_2. What is the percentage change in area when a thin copper plate having dimensions 2L × L is heated from T_1 to T_2 ?

(a) 1% **(b) 2%**

(c) 3% **(d) 4%**

Solution:

Correct option is B.

Since, Area expansion = 2 × $linear\ expansion$

Therefore, surface area will increase by 2%

23. If two rods of length L and 2L having coefficients of linear expansion α and 2α respectively are connected so that total length becomes 3L, the average coefficient of linear expansion of the composition rod equals:

(a) $\frac{3}{2}\alpha$ **(b)** $\frac{5}{2}\alpha$

(c) $\frac{5}{3}\alpha$ **(d) none of these**

Solution:

Correct option is C.

$Initial\ length\ of\ combination\ is\ L + 2L$
$= 3L$

$Increment\ in\ first\ rod$ is $L\alpha\Delta t$

$Increment\ in\ second$ rod is $(2L)(2\alpha)\Delta t$
$= 4L\alpha\Delta t$

$So\ increment\ in\ combination\ is\ sum\ of\ both\ so, 5L\alpha\Delta t$

Initial length of combination was 3L so coefficient for combination is

$$\frac{change}{initial\ length\ \times change\ in\ temperature}$$

$$= \frac{5L\alpha\Delta t}{3L\Delta t} = \frac{5\alpha}{3}$$

24. The coefficient of volume expansion of mercury is 20 times the coefficient of linear expansion of glass. Find the volume of mercury that must be poured into a glass vessel of volume V so that the volume above mercury may remain constant at all temperature.

Solution:

$$\gamma_{mercury} = 20\alpha_{glass} = \frac{20}{3}\gamma_{glass}$$

Let the Volume of mercury is $V_{mercury}$

Since the volume above mercury remains same,

$$\gamma_{mercury}V_{mercury} = \gamma_{glass}V_{glass}$$

$$\Rightarrow \frac{20}{3}\gamma_{glass}V_{mercury} = \gamma_{glass}V$$

$$\Rightarrow V_{mercury} = \frac{3V}{20}$$

25. Due to thermal expansion with rise in temperature

(a) metallic scale reading becomes lesser than true value.

(b) pendulum clock becomes fast.

(c) a floating body sinks a little more.

(d) the weight of a body in a liquid increase (neglect expansion of body).

Solution:

Correct option is A

a. On increasing temperature, the distance between scales of metallic scale will increase and hence the measured value will be lower than actual value.

b. On heating, since l increases, time period will increase and clock will become slower.

c. With rise in temperature upthrust decreases. As weight

remains same, hence, a floating body sinks a little less.

d. Since due to thermal expansion, volume of the body will increase and hence the volume of water displaced by the body will also increase. Thus, force of buoyancy on the body will also increase and hence its apparent weight will decrease.

26. A metallic circular disc having a circular hole at its center rotates about an axis passing through its center and perpendicular to its plane. When the disc is heated:

(a) Its speed will increase

(b) Its speed will decrease

(c) Its diameter will increase

(d) Its moment of inertia will increase

Solution:

Correct options are B, C & D.

When the disc with central hole is heated, diameter of hole as well as outer diameter of disc both increases. As a result of this, mass of the disc will be distributed more away from its axis which means that moment of inertia will increase on heating. Now according to the law of conservation of angular momentum as $I\omega = constant$. So, ω will decrease.

27. When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%.

(a) Percentage rise in the area of a face is 0.4 %

(b) Percentage rise in the thickness is 0.4 %

(c) Percentage rise in the volume is 0.6 %

(d) Coefficient of linear expansion of copper is 0.25×10^{-4} C^{o-1}.

Solution:

Correct options are A, C and D.

Since,

$$Area\ expansion = 2 \times linear\ expansion$$

$$Volume\ expansion = 3 \times linear\ expansion$$

Therefore, area increases by 0.4% and volume increases by 0.6%

Now

$$\frac{\Delta D}{D} = 0.002 = \alpha\Delta\theta$$

Therefore

$$\alpha = \frac{0.002}{80}$$

$$\alpha = \frac{20}{80} \times 10^{-4}$$

$$\alpha = 0.25 \times 10^{-4}$$

28. When the temperature of a metal wire is increased from 0°C to 10°C, its length increased by 0.02%. The percentage change in its mass density will be closest to :

(a) 0.06 **(b) 2.3**

(c) 0.008 **(d) 0.8**

Solution:

(a) Change in length of the metal wire (Δl) when its temperature is changed by ΔT is given by

$\Delta l = l\alpha\Delta T$

Here, α = Coefficient of linear expansion

Here, Δl = 0.02%, ΔT = 10°C

$$\therefore\ \alpha = \frac{\Delta l}{l\Delta T} = \frac{0.02}{100\times 10}$$

$\alpha = 2\times 10^{-5}$

Volume coefficient of expansion,

$\gamma = 3\alpha = 6\times 10^{-5}$

$$\because \rho = \frac{M}{V}$$

$$\frac{\Delta V}{V}\times 100 = \gamma\Delta T = \left(6\times 10^{-5}\times 10\times 100\right)$$

$= 6\times 10^{-2}$

Volume increase by 0.06% therefore, density decreases by 0.06%.

29. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4:3, then the value of T is :

(a) 230°C **(b) 270°C**

(c) 200°C **(d) 250°C**

Solution:

(a) Change in length in both rods are same i.e.

$\Delta l_1 = \Delta l_2$

$l\alpha_1\Delta\theta_1 = l\alpha_2\Delta\theta_2$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta\theta_2}{\Delta\theta_1} \qquad \left[\because \frac{\alpha_1}{\alpha_2} = \frac{4}{3}\right]$$

$$\frac{4}{3} = \frac{\theta - 30}{180 - 30}$$

$\theta = 230\ ^\circ C$

30. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is:

(a) $\frac{F}{A\alpha\Delta T}$ **(b)** $\frac{F}{A\alpha\left(\Delta T - 273\right)}$

(c) $\frac{F}{2A\alpha\Delta T}$ **(d)** $\frac{2F}{A\alpha\Delta T}$

Solution:

(a) Young's modulus Y = $\frac{stress}{strain} = \frac{F/A}{A\left(\Delta l/l\right)}$

Using, coefficient of linear expansion,

$$\alpha = \frac{\Delta l}{l\Delta T} \Rightarrow \frac{\Delta l}{l} = \alpha\Delta T$$

$$\therefore\ Y = \frac{F}{A\left(\alpha\Delta T\right)}$$

Expert **Thermal Expansion**

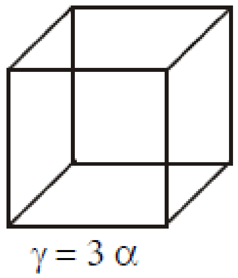
1. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and *α* is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:

(a) $\frac{3\alpha}{PK}$ **(b) 3PKα**

(c) $\frac{P}{3\alpha K}$ **(d)** $\frac{P}{\alpha K}$

Solution:

(c) As we know, Bulk modulus



$$K = \frac{\Delta P}{\left(\frac{-\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = \frac{P}{K}$$

$V = V_0\left(1 + \gamma\Delta t\right)$

$$\frac{\Delta V}{V_0} = \gamma\Delta t$$

$$\therefore\ \frac{P}{K} = \gamma\Delta t \Rightarrow \Delta t = \frac{P}{\gamma K} = \frac{P}{3\alpha K}$$

2. A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that its temperature increases by ΔT. The net change in its length is zero. Let l be the length of the rod, A its area

of cross-section, Y its Young's modulus, and α its coefficient of linear expansion. Then, F is equal to :

(a) $l^2 Y\alpha\, \Delta T$ (b) $lAY\alpha\, \Delta T$

(c) $AY\alpha\, \Delta T$ (b) $\frac{AY}{\alpha\Delta T}$

Solution:

(c) Due to thermal exp., change in length

$$(\Delta l) = l\alpha\Delta T \qquad \text{... (i)}$$

Young's modulus (Y) = $\frac{Normal\ stress}{Longitudinal\ strain}$

$$Y = \frac{F/A}{\Delta l / l} \Rightarrow \frac{\Delta l}{l} = \frac{F}{AY}$$

$$\Delta l = \frac{Fl}{AY}$$

From eqn (i), $\frac{Fl}{AY} = l\alpha\Delta T$

3. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}\ °C^{-1}$ and $\gamma_{(mercury)} = 1.5 \times 10^{-4}\ °C^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to _____________.

Solution:

Volume capacity of beaker, $V_0 = 500$ cc

$V_b = V_0 + V_0\,\gamma_{beaker}\,\Delta T$

When beaker is partially filled with V_m volume of mercury,

$V_b^1 = V_m + V_m\gamma_m\Delta T$

Unfilled volume ($V_b - V_b'$) = $V_0 - V_m$

For the volume to remain constant at all temp

Ex of liquid = Exp of solid

$V_0\gamma_{beaker} = V_m\gamma_M$

$\therefore V_m = \frac{V_0\gamma_{beaker}}{\gamma_M}$

or, $V_m = \frac{500 \times 6 \times 10^{-6}}{1.5 \times 10^{-4}}$ = 20 cc.

4. A non-isotropic solid metal cube has coefficients of linear expansion as: 5×10^{-5}/°C along the x-axis and 5×10^{-6}/ °C along the y and the z-axis. If the coefficient of volume expansion of the solid is C × 10^{-6}/°C then the value of C is __________.

Solution:

Volume, V = lbh

$$\therefore \gamma = \frac{\Delta V}{V} = \frac{\Delta l}{l} + \frac{\Delta b}{b} + \frac{\Delta h}{h}$$

(γ = coefficient of volume expansion)

$\gamma = 5 \times 10^{-5} + 5 \times 10^{-6} + 5 \times 10^{-6}$

$= 60 \times 10^{-6}$/°C

∴ Value of C = 60.00

5. A copper rod of 88 cm and an aluminum rod of unknown length have their increase in length independent of increase in temperature. The length of aluminium rod is ($\alpha_{Cu} = 1.7 \times 10^{-5}\ K^{-1}$, $\alpha_{Al} = 2.2 \times 10^{-5}\ K^{-1}$)

(a) 113.9 cm (b) 88 cm

(c) 68 cm (d) 6.8 cm

Solution:

(c) Due to change in temperature, the thermal strain produced in a rod of length L is given by

$$\frac{\Delta L}{L} = \alpha\Delta T$$

ΔL = L α ΔT

Where l = original length of rod and α = coefficient of liner expansion of solid rod

As the change in length(Δl) of the given two rods of copper and aluminium are independent of temperature change, i.e. ΔT is same for both copper and aluminium.

$L_{Cu}\,\alpha_{Cu} = L_{Al}\alpha_{Al}$ … (i)

Here, $\alpha_{Cu} = 1.7 \times 10^{-5}\ K^{-1}$

$\alpha_{Al} = 2.2 \times 10^{-5}\ K^{-1}$

and L_{Cu} = 88 cm

Substituting the given values in Eq. (i), we get

$$L_{Al} = \frac{L_{Cu}\alpha_{Cu}}{\alpha_{Al}} = \frac{1.7 \times 10^{-5} \times 88}{2.2 \times 10^{-5}} \simeq 68 \text{ cm}$$

6. Two different wires having lengths L_1 and L_2 and respective temperature coefficient of linear expansion α_1 and α_2 are joined end-to-end. Then the effective temperature coefficient of linear expansion is:

(a) $\frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$ **(b)** $2\sqrt{\alpha_1\alpha_2}$

(c) $\frac{\alpha_1 + \alpha_2}{2}$ **(d)** $4\frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\frac{L_2 L_1}{(L_2 + L_1)^2}$

Solution:

(a) Let L_1' and L_2' be the lengths of the wire when temperature is changed by ΔT°C.

At T°C,

$L_{eq} = L_1 + L_2$

At T = Δ°C

$L'_{eq} = L'_1 + L'_2$

$\therefore\ L_{eq}\left(1+\alpha_{eq}\Delta T\right) = L_1\left(1+\alpha_1\Delta T\right) + L_2\left(1+\alpha_2\Delta T\right)$

$\left[\because L' = L\left(1+\alpha\Delta T\right)\right]$

$\left(L_1+L_2\right)\left(1+\alpha_{eq}\Delta T\right) = L_1 + L_2 + L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$

$$\alpha_{eq} = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$$

7. Coefficient of linear expansion of brass and steel rods are α₁ and α₂ Lengths of brass and steel rods are l_1 and l_2 respectively. If ($l_2 – 1_1$) is maintained same at all temperatures, which one of the following relations holds good?

(a) $\alpha_1 l_2^2 = \alpha_2 l_1^2$ **(b)** $\alpha_1^2 l_2 = \alpha_2^2 l_1$

(c) $\alpha_1 l_1 = \alpha_2 l_2$ **(d)** $\alpha_1 l_2 = \alpha_2 l_1$

Solution:

(c) According to question,

Coefficient of linear expression of brass = α_1

Coefficient of linear expression of steel = α_2

Length of brass and steel rods are l_1 and l_2 respectively.

As given difference increase in length $\left(l'_2 - l'_1\right)$ is same for all temperature.

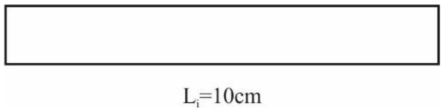
So, $l'_2 - l'_1 = l_2 - l_1$

$l_2\left(1+\alpha_2\Delta t\right) - l_1\left(1+\alpha_1\Delta t\right) = l_2 - l_1$

$l_2\alpha_2 = l_1\alpha_1$

Pro **Thermal Expansion**

1. A body of initial length 10 cm with thermal coefficient α = 2T² + 3T is heated from 20°C → 30°C. Find the final length of the body.



(a)134166.67cm **(b)132166.67cm**

(c)134167.76cm **(d)134176.667cm**

Solution:

The answer is D

$$\boldsymbol{\Delta l = \int l\alpha\Delta \mathrm{T}}$$

$$\Delta l = l\int_{20}^{30}(2T^2 + 3T)dT$$

$$= 10\left[\frac{2T^3}{3}\right]_{20}^{30} + 10\left[\frac{3T^2}{2}\right]_{20}^{30}$$

$$\Delta l = \frac{20}{3}[30^3 - 20^3] + \frac{10}{2}\times 3[30^2 - 20^2]$$

$$\Delta l = \frac{20\times 10^3}{3}[27-8] + \frac{10\times 3}{2}[900-400]$$

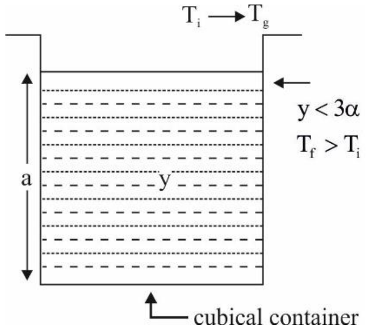
$$\Delta l = \frac{10^4}{3}\times 38 + \frac{30}{2}(500)$$

$$\Delta l = 126666.667 + 7500$$

$$l_2 = (10 + 134166.667\ )cm$$

$$l_2\ =\ 134176.667cm$$

2. A cubical container of side length a is filled with water with volume thermal coefficient γ. The linear thermal coefficient of container is α. If the container is heated from T_i to T_f. Will the water overflow? If No, Find the final height. Given $3\alpha\ >\ \gamma$



(a) $\boldsymbol{h = \frac{a(1+\gamma\Delta T)}{(1+2\alpha\Delta T)}}$ **(b)** $\boldsymbol{h = \frac{a(1-\gamma\Delta T)}{(1+2\alpha\Delta T)}}$

(c) $\boldsymbol{h = \frac{a(1+\gamma\Delta T)}{(1+\alpha\Delta T)}}$ **(d)** $\boldsymbol{h = \frac{a(1-\gamma\Delta T)}{(1+\alpha\Delta T)}}$

Solution:

The answer is A

No, it will not overflow as $3\alpha > \gamma$

with increase in temperature

$\left(V_f\right)_{water} = a^3(1+\gamma T)$

Let say find height = h

$\left(A_f\right)_{container} h = \left(V_f\right)_{water}$

$a^2(1 + 2\alpha\Delta T)h = a^3(1+\gamma\Delta T)$

$$h = \frac{a(1+\gamma\Delta T)}{(1+2\alpha\Delta T)}$$

3. A vertical column 50 cm long at 50°C balances another column of same liquid 60 cm long at 100°C. The coefficient of absolute expansion of the liquid is

(a) 0.005/°C **(b) 0.0005/°C**

(c) 0.002/°C **(d) 0.0002/°C**

Solution:

The answer is A.

$Given\ vertical\ column\ of\ liquid\ h_1 = 50\ cm$

$Temperature\ T_1 = 50℃$

Given another vertical column of same liquid

$h_2 = 60\ cm$

$T_2 = 100℃$

$Let\ the$ density of gives liquid at STP $= \rho_0$

If both vertical columns balances each other then their pressure should be equal

$P = \rho g H$

$\rho =$ density

$g = acceleration$ due to gravity

$H = height$

$$\rho_1 g h_1 = \rho_2 g h_2 \Rightarrow \frac{\rho_1}{\rho_2} = \frac{h_2}{h_1} \ldots (1)$$

Let r be coefficient of expansion of liquid

$$\Rightarrow \rho_1 = \frac{\rho_0}{(1 + rT_1)} \qquad \rho_2 = \frac{\rho_0}{(1 + rT_2)}$$

Putting this in (1)

$$\frac{\rho_1}{\rho_2} = \frac{\frac{\rho_0}{(1 + rT_1)}}{\rho_0(1 + rT_2)} = \frac{h_2}{h_1} = \frac{60}{50}$$

$$\Rightarrow \frac{1 + rT_2}{1 + rT_1} = \frac{6}{5} \Rightarrow 5 + 5rT_2 = 6 + 6rT_1$$

$$\Rightarrow 5rT_2 - 6rT_1 = 1$$

$$= 5 \times r \times 100 - 6 \times r \times 50 = 1$$

$$= 200r = 1 \Rightarrow r = \frac{0.005}{°\mathrm{C}}$$

Hence coefficient of absolute expansion of liquid is $\frac{0.005}{°\mathrm{C}}$

4. The apparent coefficient of expansion of a liquid when heated in a copper vessel is C and when heated in a silver vessel is S. If A is the linear coefficient of expansion of copper, then the linear coefficient of expansion of silver is

(a) $\frac{C + S - 3A}{3}$ **(b)** $\frac{C + 3A - S}{3}$

(c) $\frac{S + 3A - C}{3}$ **(d)** $\frac{C + S + 3A}{3}$

Solution:

The correct answer is B

$C = \gamma - 3A$

$S = \gamma - 3\alpha_{Ag}$

$C + 3A = S + 3\alpha_{Ag}$

$$\alpha_{Ag} = \frac{C - S + 3A}{3}$$

5. The real coefficient of volume expansion of glycerine is 0.000597 per°C and linear coefficient of expansion of glass is 0.000009 per°C. Then the apparent volume coefficient of expansion of glycerine is

(a) 0.000558 per°C (b) 0.00057 per°C

(c) 0.00027 per°C (d) 0.00066 per°C

Solution:

The correct answer is B

$Coefficient\ of\ volume\ expansion\ of\ glycerine$

$= 0.000597 per°\mathrm{C}$

$Coefficient\ of\ volume\ expansion\ of\ glass$

$= 3 \times 0.000009 = 0.000027 per°\mathrm{C}$

Apparent coefficient of volume expansion

$= 0.000597 - 0.000027 = 0.00057 per°\mathrm{C}$

6. The volume of a metal sphere increases by 0.24% when its temperature is raised by 40°C. The coefficient of linear expansion of the metal is ……..…… °C^{-1}

(a) 2 × 10^{-5} (b) 6 × 10^{-5}

(c) 2.1 × 10^{-5} (d) 1.2 × 10^{-5}

Solution:

The correct answer is A.

The change in volume is given by;

$V_T = V_0(1 + \gamma \Delta T)$

$$\Rightarrow \frac{V_T - V_0}{V_0} = (\gamma \Delta T)$$

$$\frac{0.24}{100} = \gamma \times 40°\mathrm{C}$$

$$\gamma = \frac{O.24}{100 \times 40} = 6 \times 10^{-5} °\mathrm{C}^{-1}$$

Therefore, Coefficient of linear expansion,

$$\alpha = \frac{\gamma}{3} = \frac{6 \times 10^{-5}}{3} = 2 \times 10^{-5} °\mathrm{C}^{-1}$$

7. A clock with a metallic pendulum is 5 second fast each day at a temperature of 15°C and 10 seconds slow each day at a temperature of 30°C . Find coefficient of linear expansion for the metal.

(a)$\mathbf{2.31 \times 10^{-2}} °\mathrm{C}^{-1}$ **(b)**$\mathbf{2.31 \times 10^{-6}} °\mathrm{C}^{-1}$

(c) $\mathbf{2.31 \times 10^{-5}} °\mathrm{C}^{-1}$ **(d)**$\mathbf{2.31 \times 10^{-4}} °\mathrm{C}^{-1}$

Solution:

The correct answer is C

Fractional increment in time with Temperature change

$$\frac{\Delta t}{t} = \frac{1}{2}\alpha \Delta T \qquad \boldsymbol{eq(1)}$$

Where

$\frac{\Delta t}{t} = fractional\ increment$

$\alpha = coefficient\ of\ linear\ expansion$

$\Delta T = change\ in\ temperature$

Assume @ $T = T_0$, there is no gain and loss in time

Case 1:

$$\frac{\Delta t}{t} = gain\ 5\ \sec per\ \text{day}$$

$$= \frac{5}{24 \times 60 \times 60}$$

$\Delta T = T_0 - 15$

Substitute values back in **eq(1)**

$$\frac{5}{24 \times 60 \times 60} = \frac{1}{2} \times \alpha \times (T_0 - 15) \qquad \boldsymbol{eq(2)}$$

Case 2:

$$\frac{\Delta t}{t} = loss\ 10\ \sec per\ \text{day}$$

$$= -\frac{10}{24 \times 60 \times 60}$$

$\Delta T = T_0 - 30$

Substitute values back in eqn(1)

$$-\frac{10}{24 \times 60 \times 60} = \frac{1}{2} \times \alpha \times (T_0 - 30) \qquad eq(3)$$

$$\frac{eq(3)}{eq(2)}$$

$$-2 = \frac{T_0 - 30}{T_0 - 15}$$

$T_0 - 30 = -2 \times (T_0 - 15)$

$T_0 - 30 = -2T_0 + 30$

$3T_0 = 60$

$T_0 = 20°\text{C}$

Substitute this value of T_0 in eq(2)

$$\frac{5}{24 \times 60 \times 60} = \frac{1}{2} \times \alpha \times (20 - 15)$$

$$\frac{5}{24 \times 60 \times 60} = \frac{1}{2} \times \alpha \times 5$$

Solve for α

$\alpha = 2.31 \times 10^{-5} °\text{C}^{-1}$

8. Find the time during which a layer of ice of thickness 2.0 cm on the surface of a pond will have its thickness increased by 2 mm when the temperature conductivity of ice = 5 × 10^{-3} cal cm^{-1} s^{-1} $(°C)^{-1}$, density of ice at 0°C = 0.91 g cm^{-3} and latent heat of fusion = 80 cal g^{-1}

(a) 6 min 5 s **(b) 2 min 6 s**

(c) 13 min 20 s **(d) 3 min 5 s**

Solution:

The correct answer is C.

Let t = time required to increase the thickness of ice by 1mm = 0.1cm

Mass of ice required to be formed is:

$m = Volume \times Density$

Let A = Area of upper surface

$Volume = Area \times Thickness$

$Volume = A \times 0.1$

$m = (A \times 0.1) \times 1$

$m = 0.1\ A\ gm \to 1$

Now, heat must flow from lower surface to the upper surface of ice and finally into atmosphere

O = heat that flows out of pond into atmosphere

λ = Latent heat of ice

m = Mass of ice

k = co – efficient of thermal conductivity

A = Cross – sectional Area

t = time

x = Density between hot and cold surface

θ_1 = temperature of hot surface

θ_2 = temperature of cold surface

$\therefore\ \theta = mL$

$\theta = 0.1 \times A \times 80\ (Using\ equation\ 1)$

$\theta = 8\ A\ Cal \to 2$

$$\theta = \frac{KA(\theta_1 - \theta_2)t}{x}$$

Using equation 2

But

$$8A = \frac{KA(\theta_1 - \theta_2)t}{x}$$

$$t = \frac{8x}{K(\theta_1 - \theta_2)}$$

Now, $x = 10cm$

$K = 0.005\ cal|cm|\Delta|°\text{C}$

$\theta_1 - \theta_2 = 0 - (-10) = 10°\text{C}$

$$t = \frac{8 \times 5}{0.005 \times 10} = 800\ Sec$$

T= 13 min 20 s

9. A body cools from 60°C to 50°C in 10 minutes. If the room temperature is 25°C and assuming Newton's law of cooling to hold good, find the temperature of the body at the end of the next 10 minutes.

(a) 42.85°C **(b) 12.85°C**

(c) 20.85°C **(d) 52.85°C**

Solution:

The correct answer is A.

According to Newton's law f cooling

$$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1 + \theta_2}{2} - \theta_0\right]$$

$$\therefore\ \frac{60 - 50}{10} = K\left[\frac{60 + 50}{2} - 25\right] \ldots(1)$$

In Second case, if T be the final temperature after 10 minutes.

Then

$$\frac{50-\theta}{10}=K\left[\frac{50+\theta}{3}-25\right] \quad \dots.(2)$$

Dividing (1) and (2)

$$\frac{10}{50-\theta}=\frac{30}{\frac{\theta}{2}} \Rightarrow 10\theta = 60\times 50-60\theta$$

$$70\theta = 60\times 50$$

$$\theta=\frac{60\times 50}{70}=\frac{300}{7}=42.85℃$$

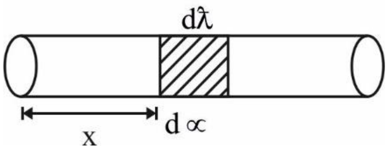
10. A rod of length 2m at 0°C and having expansion coefficient α = (3x + 2) × 10^{-6} °C^{-1} where x is the distance (in cm) from one end of rod. The length of rod at 20°C is:

(a) 2.124 m **(b) 3.24 m**

(c) 2.0120 m **(d) 3.124 m**

Solution:

Correct option is C.



$$\alpha=(3x+2)\times 10^{-6} \quad , \qquad d\theta = 20°$$

$$\Delta L=\int \alpha\, dx\, d\theta$$

$$=10^{-6}\times 20\int_0^l (3x+2)dx$$

$$=10^{-6}\left[\frac{3x^2}{2}+2x\right]_0^l$$

$$=10^{-6}\left[\frac{3l^2}{2}+2l\right]\times 20\, , l=200\ cm$$

$$=10^{-6}\left[\frac{3}{2}(200)^2+2(200)\right]\times 20$$

$$=10^{-5}\times 2\left[\frac{3}{2}x\times 4\times 10^4+400\right]$$

$$=2\times 10^{-5}[6\times 10^4+400]$$

$$=2[0.6+0.004]$$

$$=1.208\ cm$$

$$l_2-l_1=0.01208\ m$$

$$l_2=2.0120\ m$$

11. The volume of the bulb of a mercury thermometer at 0°C is V_0 and cross section of the capillary is A_0. The coefficient of linear expansion of glass is a_g per °C and the cubical expansion of mercury γ_m per °C. If the mercury just fills the bulb at 0°C, what is the length of mercury column in capillary at T°C.

(a) $\frac{V_0T(\gamma_m+3a_g)}{A_0(1+2a_gT)}$ **(b)** $\frac{V_0T(\gamma_m-3a_g)}{A_0(1+2a_gT)}$

(c) $\frac{V_0T(\gamma_m+2a_g)}{A_0(1+3a_gT)}$ **(d)** $\frac{V_0T(\gamma_m-2a_g)}{A_0(1+3a_gT)}$

Solution:

Correct option is B.

Expansion of mercury = $V_0\gamma_m T \quad (in\ the\ volume)$

Expansion in the volume of bulb $=V_0(3a_gT)$

Now after this expansion the mercury fills both the bulb & the column. Hence the volume that goes in the column is $V_0(\gamma_m-3a_g)T\, , i.e\ (V_0\gamma_mT-V_03a_gT)$

Hence, $hA_0(1+2a_gT)=V_0(\gamma_m-3a_g)T$

$$h=\frac{V_0(\gamma_m-3a_g)T}{A_0(1+2a_gT)}$$

12. A metal ball immersed in Alcohol weights W_1 at 0°C and W_2 at 50°C. The coefficient of cubical expansion of the metal $(\gamma)_m$ is less than that of alcohol $(\gamma)_{Al}$. Assuming that density of metal is large compared to that of alcohol, it can be shown that

(a) $W_1 > W_2$ **(b) $W_1 = W_2$**

(c) $W_1 < W_2$ **(d) any of (a), (b) or (c)**

Solution:

Correct option is C.

Since, it is given that the coefficient of cubical expansion of alcohol is more than that of the metal.

This statement means that when we raise the temperature, the volume of alcohol increase at a faster rate than as compared to the increase in volume of metal ball.

Or in other words, the density of alcohol decreases at a faster rate as compared to metal ball. As a result, the metal ball experiences a lesser buoyant force as the temperature is raised. Hence, the weight at increased temperature would be more.

Hence, answer should be $W_1 < W_2$

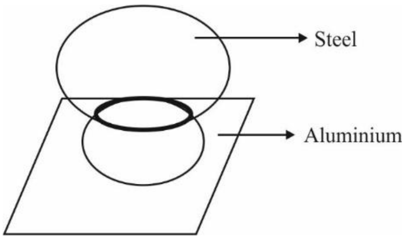
13. In aluminium sheet there is a hole of diameter 2m and is horizontally mounted on a stand. Onto this hole an iron sphere of radius 2.004 m is resting. Initial temperature

of this system is 25° C. Find at what temperature, the iron sphere will fall down through the hole in sheet. The coefficients of linear expansion for aluminium and iron are 2.4×10^{-5} and 1.2×10^{-5} respectively.

(a) 192°C **(b) 142°C**

(c) 145°C **(d) 8°C**

Solution:



As value of coefficient of linear expansion for aluminium is more than that for iron, it expends faster then iron. So at some higher temperature when diameter of hole will exactly become equal to that of iron sphere, the sphere will pass through the hole. Let it happen at some higher temperature T. Thus we have at this temperature T

$$2\left[1+\alpha_{Al}(T-25)\right]=2.004\left[1+\alpha_{\text{iron}}(T-25)\right]2\alpha_{Al}(T-25)=0.004+$$
$$2.004\alpha_{\text{iron}}(T-25)$$

or $T=\left(\frac{0.004}{2\alpha_{Al}-2.004\alpha_{\text{iron}}}+25\right){}^{\circ}\mathrm{C}$

or $T=\frac{0.004}{2\times2.4\times10^{-5}-2.004\times1.2\times10^{-5}}+25$

or $T=191.7^{\circ}\mathrm{C}$

14. An iron ball has a diameter of 6 cm and is 0.010 mm too large to pass through a hole in a brass plate when the ball and plate are at a temperature of 30°C. At what temperature, the same for ball and plate, will the ball just pass through the hole?

(a) 23.8°C **(b) 13.8°C**

(c) 53.8°C **(d) 83.8°C**

Solution:

Correct option is C.

Since the linear expansion is given by the expression $\Delta l = l\alpha\Delta T$

Hence, in the given situation, since both iron ball and brass plate will expand.

Hence,

$d(\alpha_{Brass} - \alpha_{Iron})\Delta T = 0.01\ mm$

$60(\alpha_{Brass} - \alpha_{Iron})\Delta T = 0.01$

$60(19-12)10^{-6}(T-30) = 0.01$

$$T = 53.8℃$$

15. A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape, the real length of the given piece of wood must be:

(a) 25 cm **(b) <25 cm**

(c) >25 cm **(d) can not say**

Solution:

Correct option is B

Since temperature is decreased, length of the steel tape will decrease.

So, whatever we will measure will show more than the actual length.

So the real length of wood must be less than $25cm$.

16. A rod of length 20 cm is made of metal. It expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal B having the same length expands by 0.045 cm for the same change in temperature, a third rod of the same length is composed of two parts one of metal A and the other of metal B. Thus rod expand by 0.06 cm. for the same change in temperature. The portion made of metal A has the length:

(a) 20 cm **(b) 10 cm**

(c) 15 cm **(d) 18 cm**

Solution:

Correct option is B.

Let l_1 be the length of potion of metal A and l_2 be the length of potion of metal B.

Thus $l_1 + l_2 = 20cm$

For the given change in temperature, change in length ∝ initial length of rod

Hence

$$\Delta l_1 = 0.075\frac{l_1}{20}$$

And

$$\Delta l_2 = 0.045\frac{l_2}{20}$$

Thus

$\Delta l_1 + \Delta l_2 = 0.060$

Solving for l_1 gives

$l_1 = 10cm$

17. A sphere of diameter 7 cm and mass 266.5 gm floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature 35°C. If the density of a liquid at 0°C is 1.527 gm/cc, then neglecting the expansion of the sphere, the

coefficient of cubical expansion of the liquid is f:

(a) 8.486 × 10^{-4} per °C **(b) 8.486 × 10^{-5} per °C**

(c) 8.486 × 10^{-6} per °C **(d) 8.486 × 10^{-3} per °C**

Solution:

Correct option is A

$let\ \rho_t =$ density of liquid at t°C and ρ_0

$=$ density of liquid at 0°C

As the sphere just begins to sink at $t = 35$°C, weight of sphere = upward thrust

Or

$$\frac{4}{3}\pi r^3 \rho_t g = 266.5g$$

Or

$$\rho_t = \frac{266.5 \times 3}{4 \times 3.14 \times (3.5)^2} = \frac{1.485g}{cc}$$

Now

$$\rho_0 = \rho_t[1 + \gamma t]$$

Or

$$1.527 = 1.485[1 + \gamma \times 35]$$

Or

$$\gamma = 8.486 \times \frac{10^{-4}}{°\text{C}}$$

18. A glass flask contains some mercury at room temperature. It is found that at different temperatures the volume of air inside the flask remains the same. If the volume of mercury in the flask is 300 cm^3, then volume of the flask is (given that coefficient of volume expansion of mercury and coefficient of linear expansion of glass are 1.8 × 10^{-4} (°C)$^{-1}$ and 9 × 10^{-6} (°C)$^{-1}$ respectively)

(a) 4500 cm^3 **(b) 450 cm^3**

(c) 2000 cm^3 **(d) 6000 cm**

Solution:

Correct option is C.

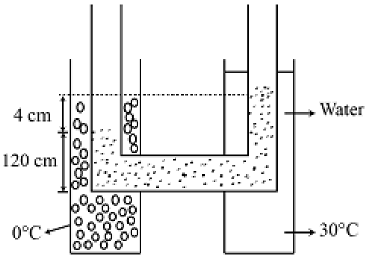
Since the volume of air inside the flask remains same,

$$\gamma_{mercury}V_{mercury} = \gamma_{flask}V_{flask}$$

$$\Rightarrow 1.8 \times 10^{-4} \times 300 = 3 \times (9 \times 10^{-6})V_{flask}$$

$$V_{flask} = 2000cc$$

19. Two vertical glass tubes filled with a liquid are connected by a capillary tube as shown in the figure. The tube on the left is put in an ice bath at 0°C while the tube on the right is kept at 30°C in a water bath. The difference in the levels of the liquid in the two tubes is 4 cm while the height of the liquid column at 0°C is 120 cm. The coefficient of volume expansion of liquid is (Ignore expansion of glass tube)



(a) 22 × 10^{-4} /°C **(b) 1.1 × 10^{-4} /°C**

(c) 11 × 10^{-4} /°C **(d) 2.2 × 10^{-4} /°C**

Solution:

Correct option is C

Pressure at the bottom of both the tubes must be same.

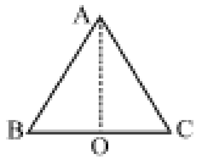
$$h_1\rho_1 g = h_2\rho_2 g$$

$$\Rightarrow h_1\rho_1 = h_2\left(\frac{\rho_1}{1 + \gamma t}\right)$$

$$\Rightarrow a + \gamma t = \frac{124}{120}$$

$$\gamma = 11.11 \times \frac{10^{-4}}{°\text{C}}$$

20. Three aluminium rods of equal length form an equilateral triangle ABC. Taking O (midpoint of rod BC) as the origin. Find the increase in Y-coordinate of center of mass per unit change in temperature of the system. Assume the length of the each rod is 2m, and $\alpha_{al} = 4\sqrt{3} \times 10^{-6} / °C$



Solution:

Distance of O from the centre of mass = Radius of inscribed circle in the triangle

$$= \frac{\sqrt{3}}{6}L = R$$

Thus, change in this radius per unit change in temperature = Rα

$$= \frac{\sqrt{3}}{6}L \times 4\sqrt{3} \times \frac{10^{-6}}{°\text{C}} = 2 \times \frac{10^{-6}m}{°\text{C}}$$

21. A clock is calibrated at a temperature of

20°C. Assume that the pendulum is a thin brass rod of negligible mass with a heavy bob attached to the end ($\alpha_{brass} = 19 \times 10^{-6}/K$).

(a) on a hot day at 30°C the clock gains 8.2 sec

(b) on a hot day at 30°C the clock loses 8.2 sec

(c) on a cool day at 30°C the clock gains 8.2 sec

(d) on a cool day at 30°C the clock gains 8.2 sec

Solution:

For pendulum clock $T \propto \sqrt{l} \propto l^{\frac{1}{2}}$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

$$\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta$$

$$\Delta T = loss\ or\ gain\ of\ time = \frac{1}{2} \times \alpha \times \Delta\theta \times T$$

On a hot day (at high temperature),

There is loss

$Loss\ of\ time\ per$ day ΔT

$$= \frac{1}{2}\alpha\Delta\theta(86400)sec$$

$$\Delta T = \frac{1}{2} \times 19 \times 10^{-6} \times (30 - 20)(86400)$$

$$= 9.5 \times 10^{-6} \times 10 \times 86400$$

$$= 8.208\,\frac{sec}{\text{day}}.$$

22. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to: (Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2, respectively; g = 10 ms^{-2})

(a) 9 kg (b) 0.5 kg

(c) 1.5 kg (d) 6 kg

Solution:

Load = Stress × $Area$

Mg = $\frac{\Delta L\,Y}{L}$A

and $A = \pi r^2 = \pi\left(10^{-3}\right)^2 = \pi \times 10^{-6}$

$$L\alpha\Delta T = \frac{FL}{AY}$$

Or $0.2 \times 10^{-5} \times 20 = \dfrac{F \times 0.2}{\left(\pi \times 10^{-6}\right) \times 10^{11}}$

∴ F = 20π N

∴ $m = \frac{f}{g} = 2\pi$ = 6.28 kg

23. On a linear temperature scale Y, water freezes at – 160° Y and boils at – 50°Y. On this Y scale, a temperature of 340K would be read as : (water freezes at 273 K and boils at 373 K)

(a) – 73.7° Y (b) – 233.7° Y

(c) – 86.3°Y (d) – 106.3° Y

Solution:

(c) $\dfrac{\text{Re}\,ading\ on\ any\ scale - LFP}{UFP - LFP}$

= constant for all scales

$$\frac{340 - 273}{373 - 273} = \frac{°Y - (-160)}{-50 - (-160)}$$

$$\frac{67}{100} = \frac{y + 160}{110}$$

∴ Y = – 86.3° Y

Beginner Heat Transfer - Conduction

1. The coefficient of thermal conductivity depends upon

(a) Temperature difference of two surfaces

(b) Area of the plate

(c) Thickness of the plate

(d) Material of the plate

Solution:

The correct answer is D.

Thermal conductivity of any object depends solely on the material of the object.

It is an intrinsic property and is independent on dimensions/mass/etc of the body.

For example, all iron rods will have same thermal conductivity.

All copper rods will have same thermal conductivity.

But, thermal conductivity of iron will be different from that of copper rod.

2. The dimensions of thermal resistance are

(a) $M^{-1}L^{-2}T^{3}K$ **(b)** $ML^{2}T^{-2}K^{-1}$

(c) $ML^{2}T^{-3}K$ **(d)** $ML^{2}T^{-2}K^{-2}$

Solution:

The correct answer is A.

Thermal resistance is a heat property and a measurement of a temperature difference by which an object or material resists a heat flow (heat per unit time or thermal resistance).

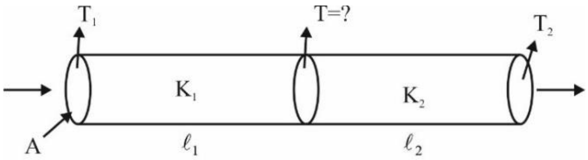
$$Thermal\ Resistance = \frac{Thermal\ \text{difference}}{Thermal\ current}$$

$$= \frac{\Delta T}{Rate\ flow\ of\ heat}$$

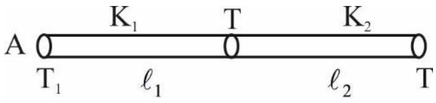
$$\Rightarrow \frac{\Delta T}{\frac{\Delta Q}{\Delta t}} = \frac{[K]}{[ML^2T^{-3}]} = [M^{-1}L^{-2}T^3K]$$

The dimensional formula for thermal resistance is $[M^{-1}L^{-2}T^3K]$

3. Find the final temperature at junction.



Solution:

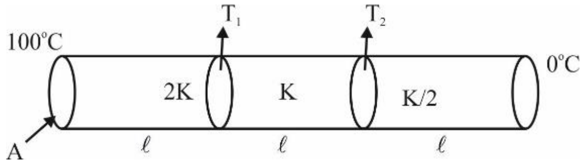


$$\left(\frac{Q_1}{t}\right) = \left(\frac{Q_2}{t}\right)$$

$$\frac{K_1A(T_1-T)}{l_1} = \frac{K_2A(T-T_2)}{l_2}$$

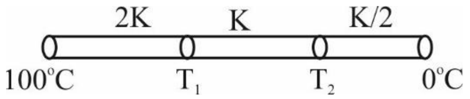
$$T = \frac{K_1T_1l_2 + K_2l_1T_2}{K_1l_2 + K_2l_1}$$

4. Find the junction temperature T₁ & T₂.



Solution:

The answer is 85.7°C & 57.1 °C.



$$\frac{1}{K_e} = \frac{1}{2k} + \frac{1}{k} + \frac{2}{k}$$

$$K_e = \frac{2K}{7}$$

At steady state

$$2K(100 - T_1) = K(T_1 - T_2) = \frac{K}{2}(T_2)$$

$$= \frac{2K}{7}(100)$$

$$2K(100 - T_1) = \frac{2K}{7}(100)$$

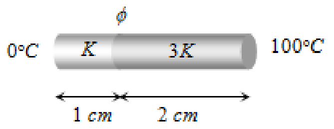
$$100 - T_1 = \frac{100}{7}$$

$$T_1 = 100 - \frac{100}{7} = \frac{600}{7} = 85.7℃$$

$$\frac{K}{2}T_2 = \frac{2K}{7}(100)$$

$$T_2 = \frac{400}{7} = 57.1℃$$

5. Two bars of thermal conductivities K and 3K and lengths 1cm and 2 cm respectively have equal cross sectional area, they are joined lengths wise as shown in the figure. If the temperature at the ends of this composite bar is 0°C and 100°C respectively (see figure), then the temperature ∅ of the interface is



(a) 50°C **(b)** $\frac{100}{3}\,^oC$

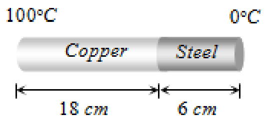
(c) 60°C **(d)** $\frac{200}{3}\,^oC$

Solution:

The correct answer is C.

$$Temperature\ of\ interface\ is\ \theta = \frac{K_1\theta_1l_1+K_2\theta_2l_2}{K_1l_1+K_2l_2} = \frac{K\times0\times1+3K\times100\times1}{K\times2+3K\times1} = \frac{300K}{5K} = 60℃$$

6. The coefficient of thermal conductivity of copper is nine times that of steel. In the composite cylindrical bar shown in the figure. What will be the temperature at the junction of copper and steel



(a) 75°C **(b) 67°C**

(c) 33°C **(d) 25°C**

Solution:

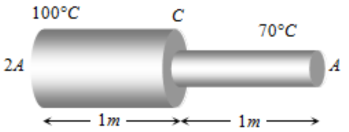
The correct answer is A.

Heat current along the rod will be same

$$\frac{\text{dq}}{\text{dt}} = \frac{9kA(100-T)}{18} = \frac{kA(T-0)}{6}$$

$$T = 75℃$$

7. A metal rod of length 2m has cross sectional areas 2A and A as shown in figure. The ends are maintained at temperatures 100°C and 70°C. The temperature at middle point C is



(a) 80°C **(b) 85°C**
(c) 90°C **(d) 95°C**

Solution:

The correct answer is C.

Here, $K_1 = K_2, l_1 = l_2 = 1m, A_1 = 2A_2 = 2A$

Where, $A_2 = A$

Also, $T_1 = 100\,℃$ and $T_2 = 70℃$

Using

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\Delta T)}{1}$$

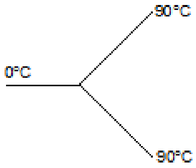
Let temperature at C be T, then

$$\left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CA}$$

$$\therefore \frac{K2A(100-T)}{1} = \frac{KA(T-70)}{1}$$

$$200 - 2T = T - 70 \Rightarrow T = 90℃$$

8. Three rods made of the same material are having the same cross-section have been joined as shown in. Each rod is of the same length. The left and right ends are kept at 0°C and 90°C respectively. The temperature of the junction of the three rods



(a) 45°C **(b) 60°C**
(c) 30°C **(d) 20°C**

Solution:

The answer is B.

Let θ°C be the temperature at B. Let Q be the heat flowing per second from A to B on account of temperature difference by conductivity.

$$\therefore Q = \frac{KA(90-\theta)}{1} \quad \ldots.(1)$$

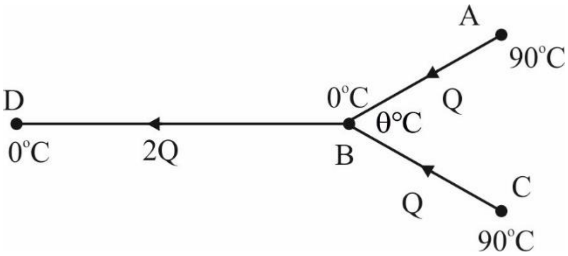
Where k = thermal conductivity of the rod, A = Area of cross section of the rod, l = length of the rod. By symmetry the same will be the case for heat flow from C to B.

∴ The heat flowing per second from B to D will be

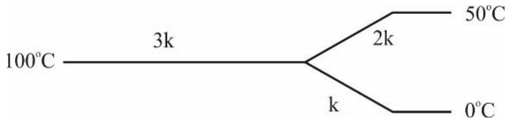
$$2Q = \frac{KA(\theta-0)}{1} \quad \ldots.(2)$$

Dividing Eq. (2) by Eq. (1)

$$2 = \frac{\theta}{90-\theta} \Rightarrow \theta = 60℃$$



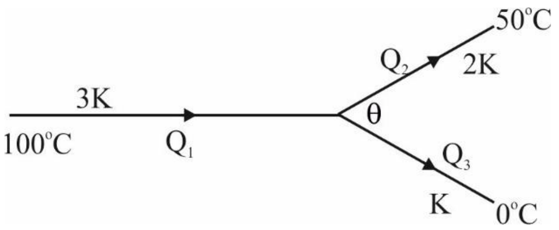
9. Find the temperature at junction.



(a) 55.7°C **(b)58.7°C**
(c) 66.7°C **(d) 59.7°C**

Solution:

The correct answer is C.



$$\frac{Q_1}{t} = \frac{Q_2}{t} = \frac{Q_3}{t}$$

$$3K\big(100-\theta\big) = 2K(\theta-50) + K(\theta - {}^\circ 0)\big)$$

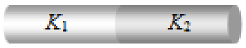
$$3(100-\theta) = 2(\theta-50) + \theta$$

$$300 - 3\theta = 2\theta - 100 + \theta$$

$$6\theta = 400$$

$$\theta = \frac{200}{3} = 66.7℃$$

10. If two metallic rods of equal thicknesses and thermal conductivities K₁ and K₂ are put together face to face and a common junction is constructed, then the equivalent thermal conductivity of this rod will be



(a) $\frac{K_1K_2}{K_1+K_2}$ **(b)** $\frac{2K_1K_2}{K_1+K_2}$

(c) $\frac{(K_1^2+K_2^2)^{3/2}}{K_1K_2}$ **(d)** $\frac{(K_1^2+K_2^2)^{3/2}}{2K_1K_2}$

Solution:

The correct answer is B.

We know that

$$The\ equivalent\ resistance\ R = R_1 + R_2$$

$R_1 = \frac{1}{k_1}\frac{l_1}{A_1}$ and $R_2 = \frac{1}{k_2}\frac{l_2}{A_2}$

But here $L_1 = L_2 = L$ and A_1 and A_2. Then,

$$R_{eq} = \frac{2l}{k_{eq}A} = \frac{l}{k_1 A} + \frac{l}{k_2 A}$$

Where L_1 & L_2 are rod 1 & rod 2 thickness, K_1 & K_2 are coefficient thermal conductivity of rod 1 & rod 2 respectively.

$$K = \frac{2K_1K_2}{K_1 + K_2}$$

11. The ratio of thermal conductivity of two rods of different material is 5 : 4. The two rods of same area of cross-section and same thermal resistance will have the lengths in the ratio

(a) 4 : 5 **(b) 9 : 1**

(c) 1 : 9 **(d) 5 : 4**

Solution:

The correct answer is D.

Given

$$A_1 = A_2\ and\ \frac{K_1}{K_2} = \frac{5}{4}$$

$$\therefore R_1 = R_2 \Rightarrow \frac{l_1}{K_1A_1} = \frac{l_2}{K_2A_2} \Rightarrow \frac{l_1}{l_2} = \frac{K_1}{K_2} = \frac{5}{4}$$

12. The temperature gradient in a rod of 0.5 m long is 80°C /m. If the temperature of hotter end of the rod is 30°C then the temperature of the cooler end is

(a) 40°C **(b) –10°C**

(c) 10°C **(d) 0°C**

Solution:

The correct answer is B.

$$\frac{30 - \theta}{0.5} = 80 \Rightarrow 30 - \theta = 40 \Rightarrow \theta = -10℃$$

13. The heat is flowing through two cylindrical rods of same material. The diameters of the rods are in the ratio 1:2 and their lengths are in the ratio 2:1. If the temperature difference between their ends is the same, the ratio of rate of flow of heat through them will be

(a) 1:1 **(b) 2:1**

(c) 1:4 **(d) 1:8**

Solution:

The correct answer is D.

Diameter are in ratio 1:2, so areas are in ratio 1:4

Lengths are in ratio 2 ∶ 1

Hence, A/L ratio is 1 ∶ 8

Since other factors are constant, ratio of heat flow is 1 ∶ 8

$$Q = \frac{KA(\theta_1 - \theta_2)t}{l}$$

14. Consider a compound slab consisting of two different materials having equal thickness and thermal conductivities K and 2K respectively.

The equivalent thermal conductivity of the slab is

(a) $\sqrt{2K}$ **(b) 3 K**

(c) $\frac{4}{3}K$ **(d)** $\frac{2}{3}K$

Solution:

The correct answer is C .

$$K_{eq} = \frac{2K_1K_2}{K_1 + K_2} = \frac{2.K.2K}{K + 2K} = \frac{4}{3}K$$

15. Two rods having thermal conductivity in the ratio of 5 : 3 having equal lengths and equal cross-sectional area are joined by face to face. If the temperature of the free end of the first rod is 100°C and free end of the second rod is 20°C. Then temperature of the junction is

(a) 70° C **(b) 50° C**

(c) 50° C **(d) 90° C**

Solution:

The correct answer is A.

Let the thermal conductivity of first rod be $5K$

Let the thermal conductivity of second rod be $3K$

Now, sum of heat current flowing through the junction = 0

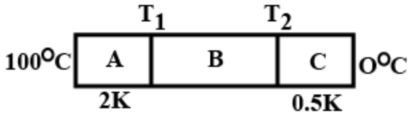
$$\frac{5KA(100 - T)}{x} + \frac{3KA(20 - T)}{x} = 0$$

$$500 - 5T + 60 - 3T = 0$$

$$8T = 560$$

$$\therefore T = 70℃$$

16. Three identical rods A,B and C of equal lengths and equal diameters are joined is series as shown in following fig. Their thermal conductivities are 2K,K and K/2 respectively. Calculate the temperature at two junction points.



(a) 85.7, 57.1°C **(b) 80.85, 50.3°C**

(c) 77.33, 48.3°C **(d) 75.8,49.3°C**

Solution:

Correct option is A

Governing equation:

$$Q = KA\frac{\Delta T}{l}$$

Heat flowing through all the rods is the same, and all rods have same lengths and cross sectional areas.

$$i.e.\frac{Q\times l}{A} = K\Delta T = constant;$$

$$i.e.\,0.5K(T_2 - 0) = K(T_1 - T_2)$$
$$= 2K(100 - T_1)$$

Combining any two of those equations and solving we get $T_1 = 85.7$ and $T_2 = 57.1$

17. A wall has two layer A and B each made of different material, both the layers have the same thickness. The thermal conductivity of the material A is twice that of B. Under thermal equilibrium the temperature difference across the wall B is 36°C. The temperature difference across the wall A is

(a) 6°C **(b) 12°C**
(c) 18°C **(d) 72°C**

Solution:

Correct option is B.

Considered two walls ‘A’ and ‘B’ of thickness ‘t’ each, and thermal conductivity of ‘2K’ and ‘ K’ respectively.

$The\ temperature\ at\ the\ left\ of\ A = T_a$
$The\ temperature\ at\ the\ right\ of\ B = T_b$
$The\ temperature\ at\ the\ junction = T$

Heat flow is a constant in steady state:

$$Q = KA\frac{\mathrm{dT}}{\mathrm{dx}} = constant$$

Equating heat flow for both walls we get,

$$2KA\frac{T - T_a}{t - 0} = KA\frac{T_b - T}{2t - t}$$

$which\ simplifies\ to : 3T = T_b + 2T_a$

Also given that the temperature difference between the walls is 36℃

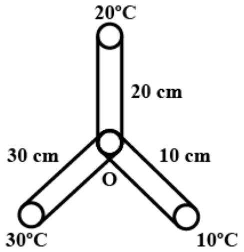
$$T_b - T_a = 36$$

$Combining\ the\ two\ equation\ in\ T, T_a.T_b\ and$ eliminating T_b

We get

$$T - T_a = 12℃$$

18. Three rods made of the same material and having same cross-sectional area but different lengths 10cm, 20 cm and 30 cm are joined as shown. The temperature of the joint is:

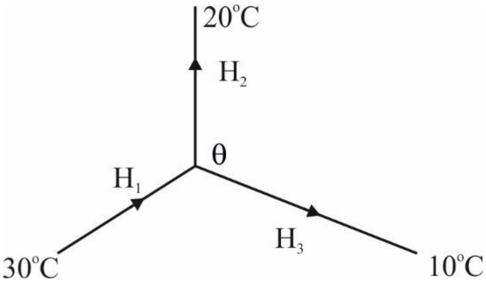


(a) 20°C **(b) 23.7°C**
(c) 16.4°C **(d) 18.2°C**

Solution:

Correct option is C.

$Let\ \theta\ be\ the\ temperature\ of\ junction\ and\ H_1, H_2\ and\ H_3\ the\ heat\ currents. Then$



$$H_1 = H_2 + H_3$$

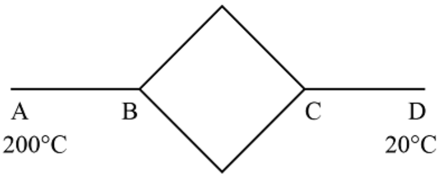
Or

$$\frac{30 - \theta}{\left(\frac{30}{KA}\right)} = \frac{\theta - 20}{\left(\frac{20}{KA}\right)} + \frac{\theta - 10}{\left(\frac{10}{KA}\right)}$$

$$or\ 2(30 - \theta) = 3(\theta - 20) + 3(2\theta - 20)$$

$$or\ \theta = 16.36℃ \approx 16.4℃$$

19. Six identical conducting rods are joined as shown in figure. Points A and D are maintained at temperature of 200°C and 20°C respectively. The temperature of junction B will be:



(a) 120°C **(b) 100°C**
(c) 140°C **(d) 80°C**

Solution:

Correct option is C.

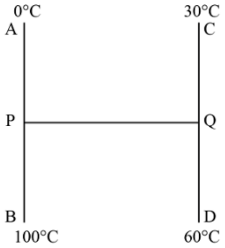
$K_{BC} = k(wheat\ stone\ bridge)$

$$\frac{kA(200 - t_B)}{1} = \frac{kA(t_B - t_C)}{1} = \frac{kA(t_C - 20)}{1}$$

$$t_B = 140℃\ \&\ t_C = 80℃$$

20. Three identical rods AB, CD and PQ are

joined as shown. P and Q are mid points of AB and CD respectively. Ends A, B, C and D are maintained at 0°C, 100°C, 30°C and 60°C respectively. The direction of heat flow in PQ is



(a) from P to Q
(b) from Q to P
(c) heat does not flow in PQ
(d) data not sufficient

Solution:

Correct option is A.

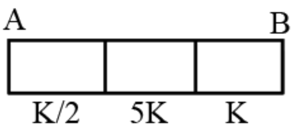
Both the rods AB and CD are in steady state.

i.e the temperature gradient is a constant and the temperature drops linearly with distance.

$$Temperature\ at\ mid\text{point of AB} = \frac{T_A + T_B}{2}$$
$$= 50°\text{C} = T_P$$
$$Temperature\ at\ mid\text{point of CD} = \frac{T_C + T_D}{2}$$
$$= 45°\text{C} = T_Q$$

Since temperature at P is more than that at Q, heat flows from P to Q.

21. A composite rod made of three rods of equal length and cross-section as shown in the fig. The thermal conductivities of the materials of the rods are K/2, 5K and K respectively. The end A and end B are at constant temperatures. All heat entering the face A goes out of the end B there being no loss of heat from the sides of the bar. The effective thermal conductivity of the bar is



(a) 15K/16 **(b) 6K/13**
(c) 5K/16 **(d) 2K/13**

Solution:

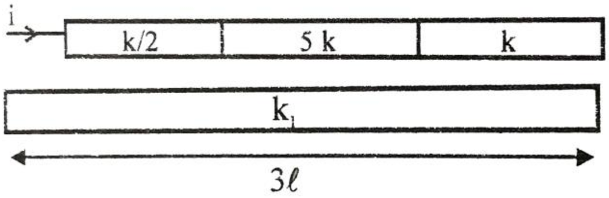
Correct option is A.

By given concept

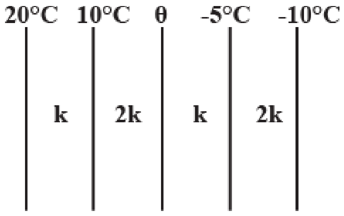
$$\frac{3l}{K} = \frac{l}{\frac{k}{2}} + \frac{l}{5k} + \frac{l}{k}$$

$$\frac{3l}{K} = \frac{2l}{k} + \frac{l}{5k} + \frac{l}{k} = \frac{10l + l + 5l}{5k} = \frac{16l}{5k}$$

$$K = \frac{15k}{16}$$



22. The figure shows the face and interface temperature of a composite slab containing of four layers of two materials having identical thickness. Under steady state condition, find the value of temperature θ.



K = thermal conductivity

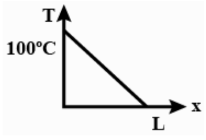
Solution:

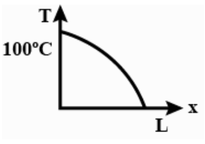
$$Since, \frac{Q}{t} = KA\frac{\text{dT}}{\text{dx}}$$

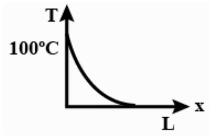
In steady state, same heat will pass

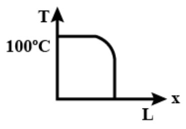
$$2K\frac{(10-\theta)}{\Delta x} = K\frac{(\theta-(-5))}{\Delta x}$$
$$\therefore 3\theta = 15$$
$$\therefore \theta = 5°\text{C}$$

23. A rod of length L and uniform cross sectional area has varying thermal conductivity which changes linearly from 2K at end A to K at the other end B. The ends A and B of the rod are maintained at constant temperature 100°C and 0°C, respectively. At steady state, the graph of temperature: T = T(x) where x = distance from end A will be

(a)


(b)


(c)


(d)


Solution:

Correct option is B.

$$\frac{\text{dQ}}{\text{dt}} = KA\left(\frac{\text{dT}}{\text{dx}}\right) = constant\ at\ stea\text{dy state}$$

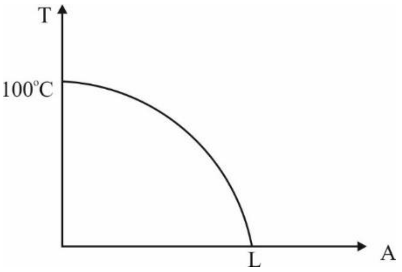
$$\frac{dT}{dx} \propto \frac{1}{K}$$

But K decreases linearly from A to B

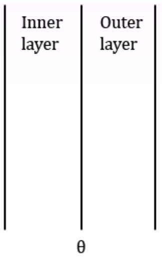
At

$x = 0\,, \quad T = 100℃\,, \quad K_1 = 2K$

$x = 2\,, \quad T = 0℃\,, \quad K_2 = K$



24. The temperature drop through a two-layer furnace wall is 900 ^{0}C. each layer is of equal area of cross section. Which of the following actions will result in lowering the temperature θ of the interface?



(a) by increasing the thermal conductivity of outer layer

(b) by increasing the thermal conductivity of inner layer

(c) by increasing thickness of outer layer

(d) by increasing thickness of inner layer

Solution:

Correct options are A & D.

H = rate of heat flow

$$= \frac{900}{\frac{l_1}{K_1 A} + \frac{l_0}{K_0 A}}$$

$$Now, 1000 - \theta = \frac{H l_i}{K_i A}$$

Or

$$\theta = 1000 - \left[\frac{900}{\frac{l_1}{K_1 A} + \frac{l_0}{K_0 A}} \frac{l_1}{K_1 A}\right]$$

$$= 1000 - \frac{900}{1 + \frac{l_0}{K_1 A}\frac{K}{l_1}}$$

Now, we can see that θ can be decreased by increasing thermal conductivity of outer layer (K_o) and thickness of inner layer (l_1).

25. One end of a conducting rod is maintained at temperature 50°C and at the other end, ice is melting at 0°C. The rate of melting of ice is doubled if:

(a) the temperature is made 200°C and the area of cross-section of the rod is doubled

(b) the temperature is made 100°C and length of rod is made four times

(c) area of cross-section of rod is halved and length is doubled

(d) the temperature is made 100°C and the area of cross-section of rod and length both are doubled.

Solution:

Correct option is D.

$$Rate\ of\ heat\ transfer\ Q = KA\frac{\Delta T}{l}$$

Initially

$$Q = KA\frac{50}{l}$$

Option (a) : $Q_2 = 8Q$

Option (b) : $Q_2 = \frac{Q}{2}$

Option (c) : $Q_2 = \frac{Q}{4}$

Option (d) : $Q_2 = Q$

Expert Heat Transfer - Conduction

1. Two identical conducting rods are first connected independently to two vessels, one containing water at 100ºC and the other containing ice at 0ºC. In the second case, the rods are joined end to end connected to the same vessels. Let m_1 and m_2 g/s be the rate of melting of ice in the two cases respectively, the ratio $\frac{m_1}{m_2}$ is.

(a) 1 : 2

(b) 2 : 1

(c) 4 : 1

(d) 1 : 4

Solution:

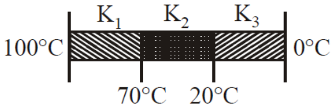
$$dQ = \left(\frac{KA\Delta T}{\ell}\right)$$

$$= \frac{(dmL)}{Qt}$$

$$m_1 = \frac{dm}{dt} = \left(\frac{KA\Delta T}{2\ell}\right)$$

$$m_2 = \left(\frac{k}{2}\right)\frac{A\Delta T}{L(2\ell)}$$

2. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is:



(a) $K_1 : K_3 = 2 : 3$, $K_1 < K_3 = 2 : 5$
(b) $K_1 < K_2 < K_3$
(C) $K_1 : K_2 = 5 : 2$, $K_1 : K_3 = 3 : 5$
(d) $K_1 > K_2 > K_3$

Solution:

(a) As the rods are identical, so they have same length (l) and area of cross-section (A). They are connected in series. So, heat current will be same for all rods.

$$\text{Heat current} = \left(\frac{\Delta Q}{\Delta t}\right)_{AB} = \left(\frac{\Delta Q}{\Delta t}\right)_{BC} = \left(\frac{\Delta Q}{\Delta t}\right)_{CD}$$

$$\frac{(100-70)K_1A}{l} = \frac{(70-20)K_2A}{l} = \frac{(20-0)K_3A}{l}$$

$K_1(100 - 70) = K_2(70 - 20) = K_3(20 - 0)$

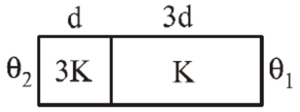
$K_1(30) = K_2(50) = K_3(20)$

$$\frac{K_1}{10} = \frac{K_2}{6} = \frac{K_3}{15}$$

$K_1 : K_2 : K_3 = 10 : 6 : 15$

$K_1 : K_3 = 2 : 3.$

3. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and 3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is:

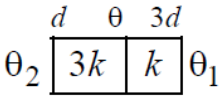


(a) $\frac{\theta_1}{10} + \frac{9\theta_2}{10}$ **(b)** $\frac{\theta_2 + \theta_1}{2}$

(c) $\frac{\theta_1}{6} + \frac{5\theta_2}{6}$ **(d)** $\frac{\theta_1}{3} + \frac{2\theta_2}{3}$

Solution:

(a) $H_1 = H_2$



$$\text{or } (3k)A\left(\frac{\theta_2 - \theta}{d}\right) = kA\left(\frac{\theta - \theta_1}{3d}\right)$$

$$\text{or } \theta = \left(\frac{\theta_1 + 9\theta_2}{10}\right)$$

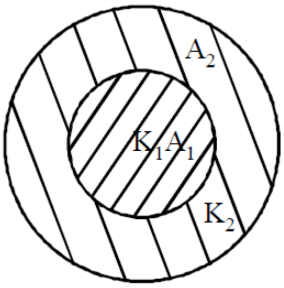
4. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is:

(a) $\frac{K_1 + K_2}{2}$ **(b) $K_1 + K_2$**

(c) $\frac{2K_1 + 3K_2}{5}$ **(d)** $\frac{K_1 + 3K_2}{4}$

Solution:

(d) Effective thermal conductivity of system



$$K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

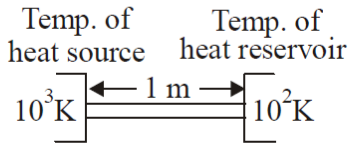
$$= \frac{K_1\pi R^2 + K_2\left[\pi(2R)^2 - \pi R^2\right]}{\pi(2R)^2}$$

$$= \frac{K_1(\pi R^2) + K_2(3\pi R^2)}{4\pi R^2} = \frac{K_1 + 3K_2}{4}$$

5. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 WK^{-1} m^{-1}, the energy flux through it in the steady state is:

(a) 90 Wm^{-2} **(b) 120 Wm^{-2}**
(c) 65 Wm^{-2} **(d) 200 Wm^{-2}**

Solution:(a)



$$\left(\frac{dQ}{dt}\right)=\frac{kA\Delta T}{l}$$

Energy flux, $\frac{1}{A}\left(\frac{dQ}{dt}\right)=\frac{k\Delta T}{l}$

$$=\frac{(0.1)(900)}{1}=90\ \text{W/m}^2$$

6. A deep rectangular pond of surface area A, containing water (density = ρ, specific heat capacity = s), is located in a region where the outside air temperature is a steady value at the –26°C. The thickness of the frozen ice layer in this pond, at a certain instant is x. Taking the thermal conductivity of ice as K, and its specific latent heat of fusion as L, the rate of increase of the thickness of ice layer, at this instant would be given by

(a) 26K/ρ r(L–4s) **(b) $26K/(\rho x^2 - L)$**

(c) 26K/(ρxL) **(d) 26K/ρr(L + 4s)**

Solution:

(c) If area of cross-section of a surface is not uniform or if the steady state condition is not reached, the heat flow equation can be applied to a thin layer of material perpendicular to direction of heat flow.

The rate of heat flow by conduction for growth of ice is given by,

$$\frac{d\theta}{dt}=\frac{KA(\theta_0-\theta_1)}{x}$$

where, $d\theta = \rho A dx L$, $\theta_0 = 0$ and $\theta_1 = -\theta$

Given, $\theta_0 = 0°C$, $\theta_1 = -26°C$

The rate of increase of thickness can be calculated from Eq.

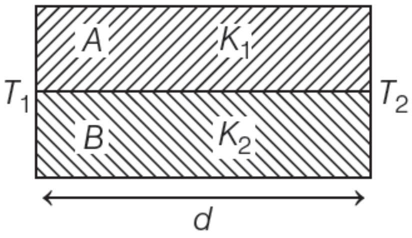
$$\frac{d\theta}{dt}=\frac{KA(\theta_0-\theta_1)}{x}$$

$$\frac{\rho A dx L}{dt}=\frac{KA(\theta_0-\theta_1)}{x}$$

$$\frac{dx}{dt}=\frac{KA(\theta_0-\theta_1)}{\rho A x L}$$

$$=\frac{K[0-(-26)]}{\rho x L}=\frac{26K}{\rho x L}$$

7. Two rods A and B of different materials are welded together as shown in figure. Their thermal conductivities are K_1 and K_2. The thermal conductivity of the composite rod will be



(a) $\frac{K_1+K_2}{2}$ **(b)** $\frac{3(K_1+K_2)}{2}$

(c) $K_1 + K_2$ **(d) $2(K_1 + K_2)$**

Solution:

(a) In parallel arrangement of n rods Equivalent thermal conductivity is given by

$$K_{eq}=\frac{K_1A_1+K_2A_2+\ldots+K_nA_n}{A_1+A_2+\ldots+A_n}$$

If rods are of same area, then

$$K_{eq}=\frac{K_1+K_2+\ldots+K_n}{n}$$

Now, in the question, it is not given that rods are of same area. But we can judge that from given diagram.

∴ Equivalent thermal conductivity of the system of two rods

$$K_{eq}=\frac{K_1+K_2}{2}$$

8. Two rectangular blocks A and B of different metals have same length and same area of cross-section. They are kept in such a way that their cross-sectional area touch each other. The temperature at one end of A is 100°C and that of B at the other end is 0°C. If the ratio of their thermal conductivity is 1 : 3, then under steady state, the temperature of the junction in contact will be

(a) 25°C **(b) 50°C**

(c) 75°C **(d) 100°C**

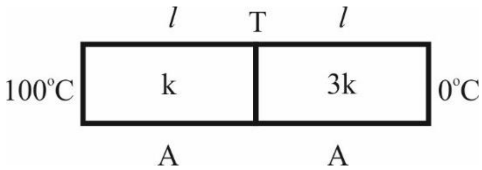
Solution:

The answer is A.

We know that $H_1 = H_2$

$$\Rightarrow \frac{KA(100-T)}{1}=\frac{3KA(T-0)}{1}$$

$$\Rightarrow 100-T=3T \Rightarrow T=25℃$$



9. A slab consists of two parallel layers of copper and brass of the same thickness and having thermal conductivities in the ratio 1 : 4. If the free face of brass is at 100℃ and that of copper at 0℃ , the temperature of interface is

(a) 80°C **(b) 20°C**

(c) 60°C **(d) 40°C**

Solution:

The answer is A.

As we know that conductive heat flow is given as

$$Q = \frac{KA\Delta T}{L}$$

and it is given that relation between thermal conductivity are as follow,

$K_{brass} = 4K_{copper}$

So, let an intermediate temperature T will line at inside the slab.

Therefore, we equate the rate of heat flow from copper surface and brass surface, as follow

$K_{brass}(100 - T) = K_{copper}(T - 0)$

$4(100 - T) = T - 0$

$T = 80℃$

Pro Heat Transfer - Conduction

1. Wires A and B have identical lengths and have circular cross-sections. The radius of A is twice the radius of B i.e. $r_A = 2r_B$. For a given temperature difference between the two ends, both wires conduct heat at the same rate. The relation between the thermal conductivities is given by

(a) $K_A = 4K_B$ **(b) $K_A = 2K_B$**

(c) $K_A = K_B/2$ **(d) $K_A = K_B/4$**

Solution:

The answer is D.

$$\frac{Q}{t} = \frac{KA\Delta\theta}{1} \to \frac{K_A}{K_B} = \frac{A_B}{A_A} = \left(\frac{r_B}{r_A}\right)^2 = \frac{1}{4} \Rightarrow K_A = \frac{K_B}{4}$$

2. A heat flux of 4000 J/s is to be passed through a copper rod of length 10 cm and area of cross-section 100 cm^2. The thermal conductivity of copper is 400 W/m °C. The two ends of this rod must be kept at a temperature difference of

(a) 1°C **(b) 10°C**

(c) 100°C **(d) 1000°C**

Solution:

The correct answer is C.

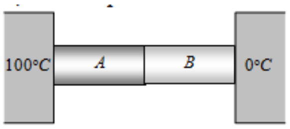
$$Heat\ flux\ is\ given\ as\ H = KA\frac{\Delta\theta}{l}$$

$So\ the\ temperature$ difference will be $\Delta\theta$

$$= \frac{Hl}{KA}$$

$$= \frac{\frac{4000J}{s} \times 0.1m}{\frac{400W}{mC} \times 0.01m^2} = 100℃$$

3. Two metal cubes A and B of same size are arranged as shown in the figure. The extreme ends of the combination are maintained at the indicated temperatures. The arrangement is thermally insulated. The coefficients of thermal conductivity of A and B are 300 W/m °C and 200 W/m °C , respectively. After steady state is reached, the temperature of the interface will be



(a) 45°C **(b) 90°C**

(c) 30°C **(d) 60°C**

Solution:

The correct answer is D.

The heat temperature through A per second

$$Q_1 = \frac{K_1 A(100 - t)}{1}$$

The heat transferred through B per second

$$Q_2 = \frac{K_2 A(t - 0)}{1}$$

At steady state, $\frac{K_1 A(100-t)}{1} = \frac{K_2 A(t-0)}{1}$

$\Rightarrow 300(100 - t) = 200(t - 0)$

$\Rightarrow 300 - 3t = 2t \Rightarrow t = 60℃$

4. A wall is made of two equally thick layers A and B of different materials. The thermal conductivity of A is twice that of B. In the steady state, the temperature difference across the wall is 36°C. The temperature difference across the layer A will be

(a)10℃ **(b) 14℃**

(c)12℃ **(d) 18℃**

Solution:

The correct answer is C.

Here, $K_A = 2K_B, T_A - T_B = 36°\text{C}$

Let T is the temperature of the junction

As

$$\left(\frac{\Delta T}{\Delta t}\right)_A = \left(\frac{\Delta T}{\Delta t}\right)_B$$

$$\therefore \frac{K_A A(T_A - T)}{x} = \frac{K_B A(T - T_B)}{x}$$

$$2K_B(T_A - T) = K_B(T - T_B)$$

$$2(T_A - T) = (T - T_B)$$

Add $(T_A - T)$ on both sides, we get

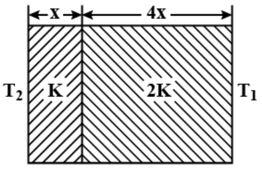
$$3\big((T_A - T)\big) = T_A - T + T - T_B$$

$$3\big((T_A - T)\big) = T_A - T_B$$

$$T_A - T = \frac{T_A - T_B}{3} = \frac{36}{3} = 12°\text{C}$$

Temperature difference across the layer$A = T_A - T = 12°\text{C}$

5. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and 2K and thickness x and 4x, respectively, are T_2 and T_1 ($T_2 > T_1$). The rate of heat transfer through the slab, in a steady state is , with f equal to (see Fig.)



(a) 1 **(b) ½**

(c) 2/3 **(d) 1/3**

Solution:

The correct answer is D.

The thermal resistance of both the blocks will be

$$\frac{x}{KA} \ \ and\ \frac{4x}{2KA} = \frac{2x}{KA}$$

$respectively. Since\ the\ two$

$resistors\ are\ in\ series,$

$(as\ same\ heat\ current\ will$

Flow through both), equivalent thermal resistance will be

$$R_{eq} = \frac{x}{KA} + \frac{2x}{KA} = \frac{3x}{KA}$$

Thus, heat current will be

$$\frac{\text{dQ}}{\text{dt}} = \frac{T_2 - T_1}{R_{eq}} = \frac{1}{3}\frac{KA(T_2 - T_1)}{x}$$

Hence,

$$f = \frac{1}{3}$$

6. A double pane window used for insulating a room thermally from outside consists of two glass sheets each of area 1 m^2 and thickness 0.01 m separated by a 0.05 m thick stagnant air space. In the steady state the room glass interface and glass outdoor interface are at 27°C and 0°C respectively. Calculate the rate of flow of heat through the windowpane. Also find the flow of heat through the windowpane. Also find the temperature of other interface if, conductivities of glass and air are 0.8 and 0.08 Wm^{-1} K^{-1} respectively.

(a) 0.72°C **(b) 0.52°C**

(c) 0.192°C **(d) 0.32°C**

Solution:

The correct answer is B.

$$As, \frac{\text{dQ}}{\text{dt}} = K\frac{A\Delta T}{L} = \frac{\Delta T}{R}$$

$$R_{eqivalent} = \sum \frac{L}{KA} = \frac{1}{A}\left[\frac{0.01}{0.8} \times 2 + \frac{0.05}{0.08}\right]$$

$$A = 1m^2, R_{eqivalent} = \frac{1}{40} + \frac{5}{8} = \frac{26}{40}$$

$$\frac{\text{dQ}}{\text{dt}} = \frac{\Delta T}{R} = \frac{(27-0)\times 40}{26} = 41.5W$$

$$Now, 41.5 = 0.8 \times 1^3\,\frac{(27 - T_2)}{0.01}\ or\ T_2$$
$$= 26.48°\text{C}$$

$$41.5 = \frac{0.8 \times 1^3(T_1 - 0)}{0.01}, T_1 = 0.52°\text{C}$$

$$\frac{\text{dQ}}{\text{dt}} = 41.5W$$

$$T_2 = 26.48°\text{C}$$

$$T_1 = 0.52°\text{C}$$

7. A copper rod 2 m long has a circular cross section of radius 1 cm. One end is kept at 100°C and the other at 0°C, and the surface is insulated so that negligible heat is lost through the surface. Thermal conductivity of copper is 401 W/m–K. Find the temperature 25 cm from the hot end.

(a) 20.5°C **(b) 40.5°C**

(c) 77.5°C **(d) 87.5°C**

Solution:

The correct answer is D.

(a) Thermal resistance

$$R = \frac{1}{KA} = \frac{1}{K(\pi r^2)}$$

Or/

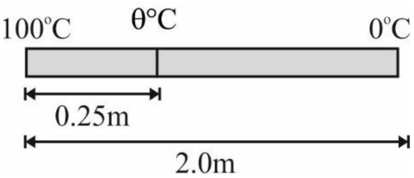
$$R = \left(\frac{2}{(401)(\pi)(10^{-2})^2}\right) = \frac{15.9K}{W}$$

(b)Temperature gradient

$$= \frac{0-100}{2} = -\frac{50K}{m}$$

$$= -50°\mathrm{C}\frac{}{m}$$

(c) Let θ be the temperature at 25 cm from the hot end, then

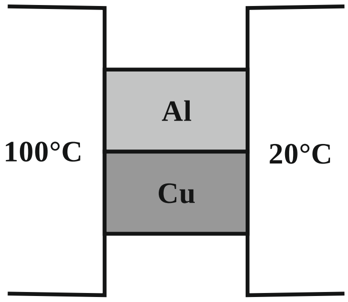


$$(\theta - 100) = (temperature\ gradient) \times (\text{distance})$$

$$or\ \theta - 100 = (-50)(0.25)$$

$$or\ \ \theta = 87.5°\mathrm{C}$$

8. Two metal cubes with 3 cm–edges of copper and aluminium are arranged as shown in figure. Thermal conductivity of copper is 401 W/m–K and that of aluminium is 237 W/m–K. Find the total thermal current from one reservoir to the other.



(a) 1.6 K/W **(b) 2.3 K/W**
(c) 2. K/W **(d) 0.94 K/W**

Solution:

The correct answer is A

(a) Thermal resistance of aluminium cube,

$$R_1 = \frac{l}{KA}$$

$$Or\ \ R_1 = \frac{3.0 \times (10^{-2})}{(273)(3.0 \times 10^{-2})^2}$$

$$= \frac{0.14K}{W}$$

And thermal resistance of copper cube,

$$R_2 = \frac{l}{KA}$$

$$Or\ \ R_2 = \frac{3.0 \times 10^{-2}}{(401)(3.0 \times 10^{-2})^2} = \frac{0.08K}{W}$$

As these two resistance are in parallel, their equivalent resistance will be

$$R = \left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$= \frac{(0.14)(0.08)}{(0.14) + (0.08)}$$

$$= \frac{0.05K}{W}$$

$$\therefore Thermal\ current, H = \frac{Thermal\ \text{difference}}{Thermal\ resistance}$$

$$= \left(\frac{100-20}{0.05}\right) = 1.6 \times \frac{(10^3)}{W}$$

$$= \frac{1.6K}{W}$$

9. Water is being boiled in a flat-bottomed kettle placed on a stove. The area of the bottom is 300 cm^2 and the thickness is 2 mm. If the amount of steam produced is 1 g/min, calculate the difference of temperature between the inner and other outer surfaces of the bottom. The thermal conductivity of the material of kettle = 0.5 cal cm^{-1} s^{-1} (°C)$^{-1}$ and the latent heat of steam = 540 cal g^{-1}.

(a) 0.012 °C **(b) 0.0.04 °C**
(c) 0.02 °C **(d) 0.0.08 °C**

Solution:

Correct option is A

Let the temperature difference be t,

So the rate of heat conduction is given as $\frac{Q}{t} = \frac{KA}{l}$ where $K = 0.5cal$ is the thermal conductivity, A = 300cm^2 is the area of surface, $l = 0.2cm$ is the thickness.

Now rate of vaporization is m$_1$ = 1gm min^{-1}, so heat absorption rate is $1 \times \frac{540}{60} = 9cals^{-1}$

Equating the heat rate, we get $0.5 \times 300 \times \frac{t}{0.2} = 9 \Rightarrow t = 0.012°\mathrm{C}$

10. A cylinder of radius R made of a material of thermal conductivity k$_1$ is surrounded by a cylindrical sheet of inner radius R and outer radius 2R made of material of thermal conductivity k$_2$.The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. Calculate the effective thermal conductivity of the system.

(a) 4K = 2K$_1$ + 3K$_2$ (b) 4K = 5K$_1$ + 3K$_2$
(c) 4K = 6K$_1$ + 4K$_2$ (d) 4K = K$_1$ + 3K$_2$

Solution:

Correct option is D.

Two cylinders are in parallel, therefore equivalent thermal resistance R is given by

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

But

$$R=\frac{l}{kA}$$

$$\therefore \frac{kA}{l}=\frac{k_1A_1}{l_1}+\frac{k_2A_2}{l_2}$$

Hence,

$$l_1=l_2=l\ and\ A_1=\pi R^2$$

$$A_2=\pi(2R)^2-\pi R^2=3\pi R^2\ and\ \mathrm{A}=\pi(2R)^2 = 4\pi R^2$$

$$\therefore \frac{k4\pi R^2}{l}=\frac{k_1\pi R^2}{l}+\frac{k_2 3\pi R^2}{l}$$

$$i.e\ \ k=\frac{k_1+3k_2}{4}$$

$$4k=k_1+3k_2$$

11. A lake is covered with ice 2 cm thick. The temperature of ambient air is –15°C. Find the rate of thickening of ice. For ice k = 4 × 10^{-4} k-cal-m^{-1} s^{-1}(°C)$^{-1}$. Density =0.9 × 10^3 kg/m^3 and latent heat L = 80 Kilo Cal/Kg.

(a) 2.5 cm/ hour **(b) 1.5 cm/ hour**

(c) 3.5 cm/ hour **(d) 4.5 cm/ hour**

Solution:

Correct option is B.

Heat energy flowing per sec given by

$$H=\frac{\mathrm{dQ}}{\mathrm{dt}}=KA\frac{\Delta\theta}{x}\ ----(i)$$

If dm is the mass of ice formed in time dt, then

$$\frac{\mathrm{dm}}{\mathrm{dt}}=\frac{A\mathrm{dx}\rho}{\mathrm{dt}}=A.\rho.\frac{\mathrm{dX}}{\mathrm{dt}}$$

Since,

$$H=\left(\frac{\mathrm{dm}}{\mathrm{dt}}\right)L$$

$$\therefore H=A\rho\frac{\mathrm{dx}}{\mathrm{dt}}L\ ----(ii)$$

From equ (i) and (ii)

$$A\rho L\frac{\mathrm{dx}}{\mathrm{dt}}=KA\frac{\Delta\theta}{x}$$

$$=\frac{4\times10^{-4}}{0.9\times10^3\times80}\times\left(\frac{0-(-15)}{2\times10^{-2}}\right)$$

$$=4.166\times\frac{10^{-6}m}{s}$$

$$=1.45\frac{cm}{hour}$$

12. One end of a uniform brass rod 15 cm long and 20 cm^2 cross-sectional area is kept at 100°C. the other end is at perfect contact with an iron rod of identical cross-section, but length 8 cm. the lateral surface of the composite rod is surrounded by a heat insulator and the free end of the iron rod is kept in ice at 0°C. if 684 gm of ice melts in 1 h, Determine the thermal conductivity of iron. Thermal conductivity of brass = .25 cal/sec-cm°C and latent heat of ice = 80cal/gm.

Solution:

Let the thermal conductivity of iron be

$$K=\frac{cal}{s}-cm°\mathrm{C}.$$

The thermal resistance of brass rod

$$R_1=\frac{l_1}{K_1A}=\frac{15cm}{0.25\ \times20}=3s-°\mathrm{C}\frac{}{cal}$$

And that of iron rod

$$R_2=\frac{l_2}{K_2A}=\frac{8cm}{K\times20}=\frac{2}{5K}s-°\mathrm{C}\frac{}{cal}$$

Since, the two rods are arranged in series, their effective thermal resistance is given by

$$R=R_1+R_2=\left(3+\frac{2}{5K}\right)s-°\mathrm{C}\frac{}{cal}$$

$$Now, rate\ of\ heat\ flow\ through\ the\ rods =\frac{\Delta Q}{\Delta t}$$

But

$$=\frac{(684g)}{3600s}\left(80\frac{cal}{g}\right)=\frac{15.2cal}{s}$$

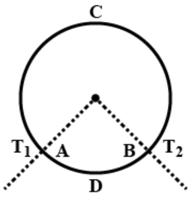
Since

$$\frac{\Delta Q}{\Delta t}=\frac{(T_1-T_2)}{R}\ \therefore R=\frac{\frac{(T_1-T_2)}{\Delta Q}}{(\Delta t)}$$

$$\left(3+\frac{2}{5K}\right)s°\mathrm{C}\frac{}{cal}=\frac{(100-0)}{15.2cal-s}$$

$$K=111.8\times\frac{10^{-3}cal}{s}-cm°\mathrm{C}$$

13. A ring consisting of two parts ADB and ACB of same conductivity k carries an amount of heat H. The ADB part is now replaced with another metal keeping the temperatures T_1 and T_2 constant. The heat carried increases to 2H. What ACB should be the conductivity of the new ADB part? Given $\frac{ACB}{ADB}$ = 3:1



(a) $\frac{7}{3}$ k (b) 2 k

(c) $\frac{5}{2}$ k (d) 3 k

Solution:

Correct option is A.

Here, according to question

$$H_1 + H_2 = \frac{KA(T_1 - T_2)}{3l} + \frac{KA(T_1 - T_2)}{l}$$

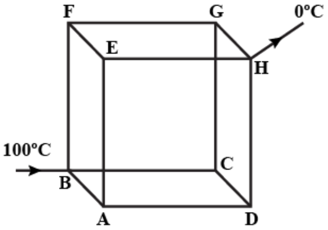
$$= \frac{4}{3l}KA(T_1 - T_2)$$

Now,

$$H_2 = 2H - H_1 = \frac{7KA(T_1 - T_2)}{3l}$$
$$= \frac{KA(T_1 - T_2)}{l}$$

$$\therefore K = \frac{7}{3}K$$

14. Twelve conducting rods form the riders of a uniform cube of side 'l'. If in steady state, B and H ends of the rod are at 100°C and 0°C. Find the temperature of the junction 'A'.



(a) 80°C **(b) 60°C**

(c) 40°C **(d) 70°C**

Solution:

Correct option is B.

$$For\ the\ cube, net\ resistance = \frac{5R}{6}$$

(Where R = thermal resistance of each side)

$$H = \frac{100 - 0}{\frac{5R}{6}}$$

For side A

$$\frac{H}{3} = \frac{100 - \theta_A}{R} \Rightarrow \theta_A = 60°\mathrm{C}$$

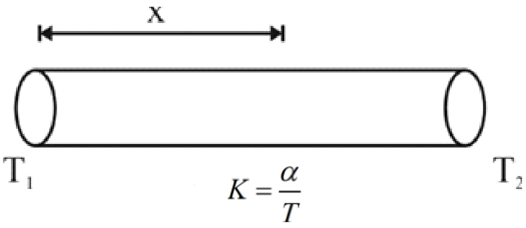
15. A rod of length L with sides fully insulated is of a material whose thermal conductivity varies with temperature as K = $\frac{\alpha}{T}$, where α is a constant. The ends of the rod are kept at temperature T_1 and T_2. The temperature T at x, where x is the distance from the end whose temperature is T_1 is

(a) $T_1\left(\frac{T_2}{T_1}\right)^{\frac{x}{L}}$ **(b)** $\frac{x}{L}\ln\frac{T_2}{T_1}$

(c) $T_1e^{\frac{T_2x}{T_1L}}$ **(d)** $T_1 + \frac{T_2 - T_1}{L}x$

Solution:

Option A is correct.



$$Q = KA\frac{\mathrm{dT}}{\mathrm{dx}}$$

$$Q = A\alpha\frac{1}{T}\frac{\mathrm{dT}}{\mathrm{dx}}$$

$$Q\int_0^x \mathrm{dx} = \mathrm{A\alpha}\int_{T_1}^{T}\frac{\mathrm{dT}}{T}$$

$$Q(x) = A\alpha\ln\left(\frac{T}{T_1}\right) ------(i)$$

At $x = L$, $T = T_2$

$$Q(L) = A\alpha\ln\left(\frac{T_2}{T_1}\right)$$

Substituting in equ (i)

$$\left[\frac{A\alpha}{L}\ln\left(\frac{T_2}{T_1}\right)\right]x = \Delta Q\ln\left(\frac{T}{T_1}\right)$$

$$\frac{x}{L}\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{T}{T_1}\right)$$

$$\left(\frac{T_2}{T_1}\right)^{\frac{x}{L}} \times T_1 = T$$

16. A thin walled metal tank of surface area 5m^2 is filled with water tank and contains an immersion heater dissipating 1 kW. The tank is covered with 4 cm thick layer of insulation whose thermal conductivity is 0.2 W/m/K. The outer face of the insulation is 25 ^{0}C. Find the temperature of the tank in the steady state.

Solution:

$$\frac{dQ}{dt} = \boldsymbol{KA}\frac{dT}{dx} \quad ------(\boldsymbol{i})$$

Given,

$$\frac{dQ}{dt} = 1kW = 10^3 W$$

$dx = 4 \text{ cm} = 4 \times 10^{-2}\, m$

$K = 0.2 W m^{-1} K^{-1}$

$dT = T_2 - T_1 = T_2 - 25$

Substituting the values in (i)

$$10^3 = \frac{0.2 \times 5 \times (T_2 - 25)}{4 \times 10^{-2}}$$

$\therefore 40 = T_2 - 25$

$\therefore\ T_2 = 65°C$

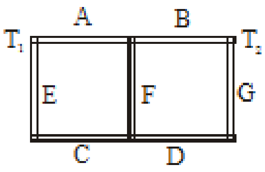
17. A cylindrical rod with one end in a stream chamber and other end in ice cause melting of 0.1 gm of ice/sec. If the rod is replaced with another rod of half the length and double the radius of first and thermal conductivity of second rod is 1/4 that of first, find the rate of ice melting in gm/sec.

Solution:

$$R = \frac{l}{KA}$$

l is halved, A is four times and K is ¼ times. R will become half. Hence, heat current will become two times. Therefore, rate of melting ice will also become two times or $0.2 g - s$.

18. Seven rods A, B, C, D, E, F and G are joined as shown in figure. All the rods have equal cross-sectional area A and length l. The thermal conductivities of the rods are $K_A = K_C = K_0$, $K_B = K_D = 2K_0$, $K_E = 3K_0$, $K_F = 4K_0$ and $K_G = 5K_0$ The rod E is kept at a constant temperature T_1 and the rod G is kept at a constant temperature T_2 ($T_2 > T_1$).



(a) Show that the rod F has a uniform temperature $T = \dfrac{T_1 + 2T_2}{3}$.

(b) Find the rate of heat flow from the source which maintains the temperature T_2.

Solution:

The temperature at both ends of bar F is the same

Rate of heat flow to right = rate of heat flow through left

$$\Rightarrow \left(\frac{Q}{t}\right)_A + \left(\frac{Q}{t}\right)_c = \left(\frac{Q}{t}\right)_b + \left(\frac{Q}{t}\right)_D$$

$$\Rightarrow \frac{K_A(T_1 - T)A}{l} + \frac{K_c(T_1 - T)A}{l} = \frac{K_B(T - T_2)A}{l} + \frac{K_D(T - T_2)A}{l}$$

$\Rightarrow 2K_0(T_1 - T) = 2 \times 2K_0(T - T_2)$

$T_1 - T = 2T - 2T_2$

$$T = \frac{T_1 + 2T_2}{3}$$

(b) As F have same temperature at both the ends, it acts like wheatstone bridge (no heat flows through F)

R_A and R_B are in series, R_C and R_D are in series

Equivalent thermal resistance is

$$R = \frac{(R_A + R_B)(R_C + R_D)}{(R_A + R_B + R_C + R_D)}$$

$$\left(Thermal\ resistance = \frac{1}{KA}\right)$$

By putting the values of R_A, R_B, R_C, R_D

$$R = \frac{3L}{4K_0A}$$

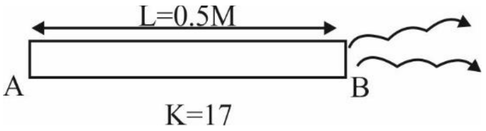
$$q = \frac{\Delta T}{R}$$

$$q = \frac{(T_1 - T_2)4K_0 4}{3L}$$

19. End A of a rod AB of length L = 0.5 m and of uniform cross-sectional area is maintained at some constant temperature. The heat conductivity of the rod is k = 17 J/s-m°K. The other end B of this rod is radiating energy into vacuum and the wavelength with maximum energy density emitted from this end is 75000 Å. If the emissivity of the end B is e = 1, determine the temperature of the end A. Assuming that except the ends, the rod is thermally insulated.

Solution:

Heat flowing through rod = Heat radiation through B



$$\left(\frac{dQ}{dt}\right)_{A \to B} = \sigma T^4$$

$$\frac{K\Delta T}{l} = \sigma {T_B}^4$$

$$T_B = \left(\frac{6}{\lambda_m}\right)$$

$$\Delta T = \left(\frac{\sigma {T_B}^4 l}{K}\right)$$

$$T_A - T_B = \left(\frac{\sigma b^4 . l}{\lambda^4 K}\right)$$

$$T_A = T_B + \left(\frac{\sigma b^4 . l}{\lambda^4 K}\right)$$

$$= \frac{b}{\lambda} + \frac{\sigma b^4 . l}{\lambda^4 K}$$

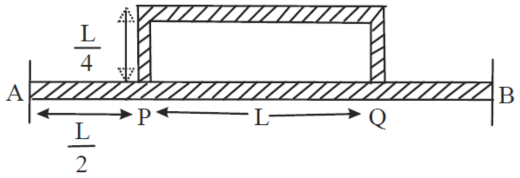
$$= \left(\frac{2.89 \times 10^{-3}}{7.5 \times 10^{-6}}\right) + \left(\frac{5.67 \times 10^{-8} \times 0.5 \times 2.89^4 \times 10^{-12}}{(7.5 \times 10^{-6})^4 \times 17}\right)$$

$$= \left(\frac{2.89 \times 10^{-3}}{7.5}\right) + \left(\frac{5.67 \times 0.5 \times 2.89^4 \times 10^{-20} 10^4}{7.5^4 \times 17 \times 10^{-24}}\right)$$

$$= \left(\frac{2.89 \times 10^{-3}}{7.5}\right) + \left(\frac{5.67 \times .5 \times 2.89^4 \times 10^8}{7.5^4 \times 17}\right)$$

$$= 422.09K$$

20. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and will be close to:

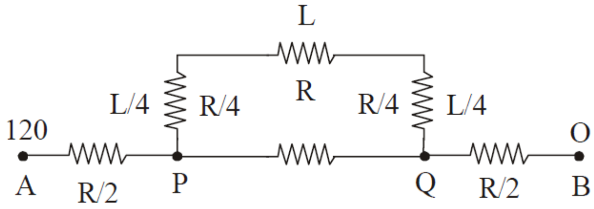


(a) 45°C **(b) 75°C**

(c) 60°C **(d) 35°C**

Solution:

(a) $\frac{\Delta T_{AB}}{R_{AB}} = \frac{120}{\frac{8}{5}R} = \frac{120 \times 5}{8R}$



In steady state temperature difference between P and Q, $\Delta T_{PQ} = \frac{120 \times 5}{8R} \times \frac{3}{5} R = \frac{360}{8}$ = 45°C

21. Three rods of Copper, Brass and Steel are welded together to form a Y shaped structure. Area of cross – section of each rod = 4 cm^2. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings excepts at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is:

(a) 1.2 cal/s **(b) 2.4 cal/s**

(c) 4.8 cal/s **(d) 6.0 cal/s**

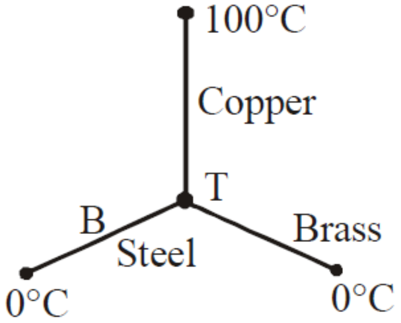
Solution:

(c) Rate of heat flow is given by,

$$Q = \frac{KA(\theta_1 - \theta_2)}{l}$$

Where, K = coefficient of thermal conductivity

l = length of rod and A = area of cross-section of rod



If the junction temperature is T, then

$Q_{Copper} = Q_{Brass} + Q_{Steel}$

$$\frac{0.95 \times 4(100 - T)}{46} = \frac{0.26 \times 4 \times (T - 0)}{13} + \frac{0.12 \times 4 \times (T - 0)}{12}$$

200 – 2T = 2T + T

T = 40 °C

$$\therefore Q_{\text{Copper}} = \frac{0.92 \times 4 \times 60}{46} = 4.8 \text{ cal/s}$$

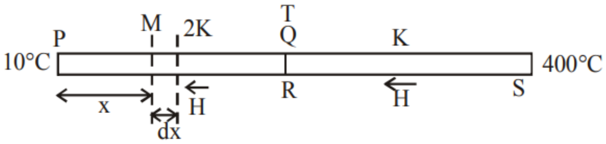
22. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400 °C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5} K^{-1}$, the change in length of the wire PQ is

(a) 0.78mm **(b) 0.90 mm**

(c) 1.56 mm **(d) 2.34 mm**

Solution:

At steady state, heat flow rate is same from *P* to *Q* and from *Q* to *S*.



$$\therefore \frac{KA(400 - T)}{\ell} = \frac{2KA(T - 10)}{\ell} \Rightarrow T = 140°C$$

The temperature gradient

$$\frac{dT}{dx} = \frac{140 - 10}{1} \qquad \therefore dt = 130 \text{ dx}$$

Therefore, change temperature at a cross-section M distant ‘x’ from P is

$\Delta T = 130x$

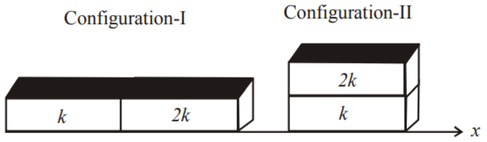
Extension in a small elemental length ‘dx’ is

$dl = dx\alpha \, \Delta T = dx \, \alpha \, (130x)$

$$\therefore \int dl = 130\alpha \int_0^1 x dx$$

$$\therefore \Delta l = 130 \times 1.2 \times 10^{-5} \times \frac{1}{2} = 78 \ \times 10^{-5}$$

23. Two rectangular blocks, having identical dimensions, can be arranged either in configuration-I or in configuration-II as shown in the figure. One of the blocks has thermal conductivity k and the other $2k$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration-I. The time to transport the same amount of heat in the configuration-II is



(a) 2.0s **(b) 4.5s**

(c) 3.0s **(d) 6.0s**

Solution:

(a) Equivalent thermal resistance in configuration-I

$$R_s = R_1 + R_2 = \frac{L}{KA} + \frac{L}{2KA} = \frac{2}{3}\frac{L}{KA}$$

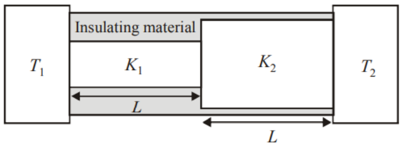
Equivalent thermal resistance in configuration-II

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l} \text{ or}, R_P = \frac{l}{3KA} = \frac{R_s}{4.5}$$

i.e., Thermal resistance in configuration-II, R_p is 4.5 times less than thermal resistance in configuration-I R_s

$$\therefore 4.5t_p = t_s \Rightarrow t_p = \frac{t_s}{4.5} = \frac{9}{4.5}s = 2s$$

24. Two conducing cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300$K and $T_2 = 100$K, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then $K_1/K_2 =$ ________________________.



Solution:

(4.00)

Rate of the heat flow will be same,

Rate of the heat flow $\frac{dQ}{dt} = \frac{\text{temp. difference}}{\text{thermal resistance}} = \frac{1}{R}(T_2 - T_1)$

Where $R = \frac{L}{KA}$

$$\frac{300 - 200}{R_1} = \frac{200 - 100}{R_2} \text{ or } R_1 = R_2$$

$$\frac{L_1}{K_1 A_1} = \frac{L_2}{K_2 A_2} \therefore \frac{K_1}{K_2} = \frac{A_2}{A_1} = \frac{\pi(2r)^2}{\pi r^2} = 4$$

$[\therefore L_1 = L_2 = L]$

Beginner **Heat Transfer - Radiation**

1. For a perfectly black body, its absorptive

power is

(a) 1 **(b) 0.5**

(c) 0 **(d) Infinity**

Solution:

The answer is A.

The perfectly black body neither reflects nor transmit, (i.e, r = 0 , t = 0) any part of the incident radiative energy but absorb whole of it or we can say its absorptive power is 1.

2. There is a rough black spot on a polished metallic plate. It is heated upto 1400 K approximately and then at once taken in a dark room. Which of the following statements is true?

(a) In comparison with the plate, the spot will shine more

(b) In comparison with the plate, the spot will appear more black

(c) The spot and the plate will be equally bright

(d) The plate and the black spot can not be seen in the dark room

Solution:

(a)

3. An ideal black body at room temperature is thrown into a furnace. It is observed that

(a) Initially it is the darkest body and at later times the brightest

(b) It is the darkest body at all times

(c) It cannot be distinguished at all times

(d) Initially it is the darkest body and at later times it cannot be distinguished

Solution:

(a)

4. Relation between emissivity e and absorptive power a is (for black body)

(a) e = a **(b)** $e = \frac{1}{a}$

(c) $\mathbf{e = a^2}$ **(d)** $\mathbf{a = e^2}$

Solution:

The answer is A.

For a black body , emissivity = absorptive power. Hence, e = a.

5. Which of the following statements is wrong

(a) Rough surfaces are better radiators than smooth surface

(b) Highly polished mirror like surfaces are very good radiators

(c) Black surfaces are better absorbers than white ones

(d) Black surfaces are better radiators than white

Solution:

The answer is B .

Depending on the nature of surfaces the incident radiation falling on the body are either absorbed or reflected. The substances which absorbs more energy that they reflect are known as good absorbers.

Rough surfaces are good absorbers as they absorb more incident radiations while highly mirror polished surfaces reflects more incident radiations, thus they are bad absorbers.

Also, every hot body emits heat radiations from its surface. The radiations depend on nature of the surface, its size and surroundings of the body. The body which emits such radiations is known as Radiator.

A good absorber is a good radiator and a bad absorber is a bad radiator. e.g a black body is one which absorbs all the radiation incident on it and after absorbing the incident radiations.

Black surfaces appear in black colour, independent of the colour of incident radiations. Thus, they are better absorbers and radiators than white surfaces.

Therefore, highly polished mirror like surfaces are bad radiators.

6. A hot body will radiate heat most rapidly if its surface is

(a) White & polished (b) White & rough

(c) Black & polished (d) Black & rough

Solution:

The correct answer is D.

Radiation heat transfer increases with increase in surface area and emissivity. Black bodies have highest emissivity (equal to 1) and rough surfaces have higher surface area than polished surfaces. So, black and rough surfaces emit radiations more rapidly.

7. Which of the following law states that “good absorbers of heat are good emitters”

(a) Stefan’s law **(b) Kirchoff’s law**

(c) Planck’s law **(d) Wein’s law**

Solution:

The correct answer is B.

Kirchhoff's law states that, the ratio of emissive power to the absorptive power corresponding to a particular wavelength and at a given temperature is constant for all bodies:

$$\frac{e_\lambda}{a_\lambda} = constant = E_\lambda$$

Where E_λ = emissive power of a perfect black body corresponding to same wavelength and temperature, it is clear from above relation that if a_λ (absorptive power) is larger for a body, e_λ (emissive power) will also be large to keep the ratio constant i.e good absorbers are good emitters.

8. The amount of radiation emitted by a perfectly black body is proportional to

(a) Temperature on ideal gas scale

(b) Fourth root of temperature on ideal gas scale

(c) Fourth power of temperature on ideal gas scale

(d) Source of temperature on ideal gas scale

Solution:

The correct answer is C.

Stefan-Boltzmann law states that total power radiated by a perfectly black body is

$$P = A\sigma T^4$$

So, the radiation emitted by a perfectly black body is proportional to fourth power of temperature on ideal gas scale.

9. A metal ball of surface area 200 cm^2 and temperature 527°C is surrounded by a vessel at 27°C . If the emissivity of the metal is 0.4, then the rate of loss of heat from the ball is (σ = 5.67 × 10^{-8} J/m^2 –s – k^4)

(a) 108 joules approx.

(b) 168 joules approx.

(c) 182 joules approx.

(d) 192 joules approx.

Solution:

The correct answer is C.

From Stefan's law we know that, rate of loss of heat is given as,

$$\frac{Q}{t} = \sigma A\left({T_1}^4 - {T_2}^4\right) \times e$$

So,

$T_1 = 273 + 527K = 800K$

$T_2 = 273 + 27K = 300K$

And

$A = 200 \times 10^{-4} m^2$

So,

$$\frac{Q}{t} = 5.67 \times 10^{-8} \times 2 \times 10^{-2}[(800^4) - (300^4)] \times (0.4)$$

$= (182.12)\, Joule$

10. The rate of radiation of a black body at 0°C is EJ/sec. The rate of radiation of this black body at 273°C will be

(a) 16E **(b) 8E**

(c) 4E **(d) E**

Solution:

The correct answer is A.

$E \,\alpha\, T^4$

Given:

$T_1 = 0℃ = 273K$

Given:

$T_2 = 273℃ = 546K$

$$\frac{E_1}{E_2} = \frac{{T_1}^4}{{T_2}^4} = \left(\frac{273}{546}\right)^4 = \frac{1}{16}$$

$\Rightarrow E_2 = 16E_1$

$\Rightarrow E_2 = 16E$

11. A black body at a temperature of 227°C radiates heat energy at the rate of 5 cal/cm^2-sec. At a temperature of 727°C, the rate of heat radiated per unit area in cal/cm^2-sec will be

(a) 80 **(b) 160**

(c) 250 **(d) 500**

Solution:

The correct answer is A.

$Heat\ energy\ per\ unit\ time \quad R_1 = \sigma AeT_1^4$

Thus

$Heat\ energy\ per\ unit\ time \quad R_2 = \sigma AeT_2^4$

$$\Rightarrow \frac{R_2}{R_1} = \left(\frac{T_2}{T_1}\right)^4$$

$Initial\ temperature\ of\ the\ bo$dy

$T_1 = 227℃ = (227 + 273)K = 500K$

$Final\ temperature\ of\ the\ bo$dy

$T_2 = 727℃ = (727 + 273)K = 1000K$

$$\therefore \frac{R_2}{5} = \left(\frac{1000}{500}\right)^4$$

$$\frac{R_2}{5} = 16$$

$R_2 = 80 \frac{cal}{cm^2}$-sec

12. Energy is being emitted from the surface of a black body at 127°C temperature at the rate of 1.0 × 10^6 J/sec m^2. Temperature of the

black body at which the rate of energy emission is 16.0 × 10^6 J/sec–m^2 will be

(a) 254°C **(b) 508°C**

(c) 527°C **(d) 727°C**

Solution:

The correct answer is C.

Here,

$T_1 = 127°\text{C} = 400K$

$E_2 = 16 \times 10^6 \frac{J}{sm^2}$

$E_1 = 1 \times 10^6 \frac{J}{sm^2}$

Using the relation

$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4$

$\frac{T_2}{T_1} = (\frac{E_2}{E_1})^{1/4}$

$\frac{T_2}{T_1} = \left(\frac{16.0\times10^6}{1\times10^6}\right)^{\frac{1}{4}} = 2$

$T_2 = 2 \times T_1 = 2 \times 400 = 800K$

$T_2 = 527°\text{C}$

13. If temperature of a black body increases from 7°C to 287°C , then the rate of energy radiation increases by

(a) $\left(\frac{287}{7}\right)^4$ **(b) 16**

(c) 4 **(d) 2**

Solution:

The correct answer is B.

For black body radiation

$E \ \alpha \ T^4$

E is energy radiated per unit time per unit area, T is temperature of the body

$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 \Rightarrow \frac{E_2}{E_1} = \left(\frac{273+287}{273+7}\right)^4$

$= \left(\frac{560}{280}\right)^4 = \frac{16}{1} \Rightarrow E_2 = 16E_1$

14. The temperatures of two bodies A and B are 727°C and 127°C . The ratio of rate of emission of radiations will be

(a) 727/127 **(b) 625/16**

(c) 1000/400 **(d) 100/16**

Solution:

The correct answer is B.

$Rate\ of\ energy\ \frac{Q}{t} = P = A \in \sigma T^4 \Rightarrow E \propto T^4$

$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{727+273}{127+273}\right)^4 = \frac{(1000)^4}{(400)^4}$

$= \frac{10^4}{4^4} = \frac{625}{16}$

15. A black body at a high temperature T K radiates energy at the rate of E W/m^2. When the temperature falls to T/2 K, the radiated energy in W/m^2 will be

(a) E/4 **(b) E/2**

(c) 2E **(d) E/16**

Solution:

(d)

16. A spherical body with radius 12 cm radiates 500 W power at 500 K. If the radius were halved and the temperature doubled, what would be the power radiated?

(a) 450 **(b) 900**

(c) 1800 **(d) 2000**

Solution:

(d)

$u = \sigma AT^4$

$u = \sigma\left(4\pi r^2\right)T^4$ (i)

$u' = \sigma\left(4\pi\left(\frac{r}{2}\right)^2\right)\left(2T\right)^4$

$= \sigma\left(4\pi r^2\right)T^4 \times \frac{2^4}{2^2}$

= u × 2^2 = 500 × 4

= 2000

17. The rate of radiation of a black body at 0°C is E joule per sec. Then the rate of radiation of this black body at 273°C will be –

(a) 2E **(b) 4E**

(c) 8E **(d) 16E**

Solution:

(d)

18. If the temperature of the sun were to increase from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on earth to what it was previously will be –

(a) 4 **(b) 16**

(c) 32 **(d) 64**

Solution:

(d)

19. Half part of ice block is covered with black

cloth and rest half is covered with white cloth and then it is kept in sunlight. After some time clothes are removed to see the melted ice. Which of the following statements is correct –

(a) Ice covered with white cloth will melt more

(b) Ice covered with black cloth will melt more

(c) Equal ice will melt under both clothes

(d) It will depend on the temperature of surroundings of ice

Solution:

(b)

20. Two thermometers A and B are exposed in sunlight. The valve of A is painted black, but that of B is not painted. The correct statement regarding this case is –

(a) Temperature of A will rise faster than B but the final temperature will be the same in both

(b) Both A and B show equal rise in beginning

(c) Temperature of A will remain more than B

(d) Temperature of B will rise faster.

Solution:

(a)

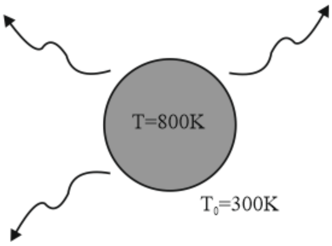
21. A metal ball of surface area 200 cm^2 and temperature 527°C is surrounded by a vessel at 27°C. If the emissivity of the metal is 0.4, then the rate of loss of heat from the ball is ($\sigma = 5.67 \times 10^{-8}$ J/m^2 – s – K^4)

(a) 108 joule approx **(b) 168 joule approx**

(c) 182 joule approx **(d) 192 joule approx**

Solution:

(c)



$U_{net} = e\sigma A\left(T^4 - T_0^4\right)$

$= 0.4 \times 5.67 \times 10^{-8} (2 \times 10^{-2}\ m^2)$ $[8^4 - 3^4] \times 10^8$

$= 5.67 \times 0.8 \times 10^{-2} [4096 - 81]$

$= 5.67 \times 0.8 \times 4015 \times 10^{-2}$

22. The area of a hole of heat furnace is 10^{-4} m^2. It radiates 1.58×10^5 calories of heat per hour. If the emissivity of the furnace is 0.80, then its temperature is

(a) 1500 K **(b) 2000 K**

(c) 2500 K **(d) 3000 K**

Solution:

The correct answer is C.

According to Stefan's law $E = \sigma \in AT^4$

$E = \frac{1.58\times10^5\times4.2}{60\times60}$

$= 5.6 \times 10^{-8} \times 10^{-4} \times 0.8 \times T^4$

$T \approx 2500\ K$

23. Two spheres P and Q of same colour having radii 8 cm and 2 cm are maintained at temperatures 127°C and 527°C respectively. The ratio of energy radiated by P and Q is

(a) 0.054 **(b) 0.0034**

(c) 1 **(d) 2**

Solution:

The correct answer is C.

$Total\ energy\ ra$diated from a body Q = A $\in \sigma T^4 t$

$Q \propto AT^4 \propto r^2 T^4 \quad (\therefore A = 4\pi r^2)$

$$\Rightarrow \frac{Q_P}{Q_Q} = \left(\frac{r_P}{r_Q}\right)^2 \left(\frac{T_P}{T_Q}\right)^4 = \left(\frac{8}{2}\right)^2 \left\{\frac{273+127}{273+527}\right\}^4 = 1$$

24. The energy emitted per second by a black body at 27°C is 10J. If the temperature of the black body is increased to 327°C , the energy emitted per second will be

(a) 20J **(b) 40 J**

(c) 80 J **(d) 160 J**

Solution:

The correct answer is D.

$Energy\ emitte$d = E = σ ∈ T^4

$T = 27°C = 27 + 273$

$T = 300\ K$

$E = 10\ J$

$10 = \sigma E(300)^4$

$$\sigma E = \frac{10}{(300)^4}$$

$T = 327°C = 327 + 273 = 600K$

$$E = \sigma E(600)^4 = \frac{10}{(300)^4}(600)^4$$

$E = 10 \times 2^4$

$E = 160\, J$

25. A spherical black body with a radius of 12 cm radiates 450 W power at 500 K . If the radius were halved and the temperature doubled, the power radiated in watt would be

(a) 225 **(b) 450**

(c) 900 **(d) 1800**

Solution:

The correct answer is D.

We know that radiation from the body is given as $E = \in \sigma A T^4$

$$\frac{E_1}{E_2} = \frac{{r_1}^2 \times {T_1}^4}{{r_2}^2 \times {T_2}^4} = \frac{0.12^2 \times 500^4}{0.06^2 \times 1000^4}$$

$$E_2 = \frac{450 \times 0.06^2 \times 1000^4}{0.12^2 \times 500^4} = 1800\, watt$$

26. If the temperature of the sun (black body) is doubled, the rate of energy received on earth will be increased by a factor of

(a) 2 **(b) 4**

(c) 8 **(d) 16**

Solution:

The correct answer is D.

$For\ hot\ objects, the\ power\ ra$diated per unit area = $\frac{P}{A} = e\sigma T^4$

This is called Stefan-Boltzmann Law, Hence

$$P \propto T^4$$

This means that the rate of energy received on Earth becomes $2^4 = 16\ times$

When the temperature of Sun gets doubled.

27. The radiation emitted by a star A is 10,000 times that of the sun. If the surface temperatures of the sun and the star A are 6000 K and 2000 K respectively, the ratio of the radii of the star A and the sun is

(a) 300 : 1 **(b) 600 : 1**

(c) 900 : 1 **(d) 1200 : 1**

Solution:

The correct answer is C.

We know that $E = \sigma \epsilon A T^4$ and it is given that

$E_{star} = 10000 \times E_{sun}$

$T_{star} = 2000K, r_{star} = r_1$

$T_{sun} = 6000K, r_{sun} = r_2$

$$\frac{E_{star}}{E_{sun}} = \frac{{r_1}^2 \times {T_1}^4}{{r_2}^2 \times {T_2}^4}$$

$$\frac{10000 \times E_{sun}}{E_{sun}} = \frac{{r_1}^2 \times (2000)^4}{{r_2}^2 \times (6000)^4}$$

$$\Rightarrow \frac{r_1}{r_2} = 900$$

So, ratio will be, 900: 1

28. Star A has radius r surface temperature T while star B has radius 4r and surface temperature T/2. The ratio of the power of two starts, P_A: P_B is

(a) 16 : 1 **(b) 1 : 16**

(c) 1 : 1 **(d) 1 : 4**

Solution:

The correct answer is C.

$Power\ P \propto AT^4 \propto r^2 T^4$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{r_2}{r_1}\right)^2 \times \left(\frac{T_2}{T_1}\right)^4 = \left(\frac{4r}{r}\right)^2 \times \left(\frac{\frac{T}{2}}{T}\right)^4 = 1$$

29. If the temperature of the sun were to be increased from T to 2T and its radius from R to 2R, then the ratio of the radiant energy received on the earth to what it was previously will be

(a) 4 **(b) 16**

(c) 32 **(d) 64**

Solution:

The correct answer is D.

$Rate \propto AT^4 \Rightarrow R^2 T^4 \Rightarrow 64\ Times$

30. At 127°C radiates energy is 2.7 × 10^{-3} J/s. At what temperature radiated energy is 4.32 × 10^6 J/s

(a) 400 K **(b) 4000 K**

(c) 80000 K **(d) 40000 K**

Solution:

The correct answer is C.

Temperature $T_1 = 127°C = 127 + 273 = 400K$

Energy radiated from a body

$Q = Ae\sigma T^4 t$

$$\Rightarrow \frac{Q_2}{Q_1} = \left(\frac{T_2}{T_1}\right)^4$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{Q_2}{Q_1}\right)^{\frac{1}{4}} = \left(\frac{4.32 \times 10^6}{2.7 \times 10^{-3}}\right)^{\frac{1}{4}}$$

$$\frac{T_2}{400} = \left(\frac{16 \times 27}{27} \times 10^8\right)^{\frac{1}{4}} = 2 \times 10^2$$

$T_2 = 200 \times 400 = 80000K$

31. If the sun's surface radiates heat at 6.3 × 10^7 Wm^{-2} . Calculate the temperature of the sun assuming it to be a black body

($\sigma = 5.7 \times 10^{-8}\ Wm^{-2}\ K^{-4}$)

(a) 5.8×10^3 K **(b) 8.5×10^3 K**
(c) 3.5×10^8 K **(d) 5.3×10^8 K**

Solution:

The correct answer is A.

From Stefan's law $E = \sigma T^4$

$$T^4 = \frac{E}{\sigma} = \frac{6.3 \times 10^7}{5.7 \times 10^{-8}} = 1.105 \times 10^{15}$$

$$T = 0.58 \times 10^4 = 5.8 \times 10^3 \text{K}$$

32. Two spheres of same radius R have their densities in the ratio 8: 1 and the ratio of their specific heats are 1:4. If by radiation their rates of fall of temperature are same, then find the ratio of their rates of losing heat.

Solution:

$$\left(\frac{\text{dQ}}{\text{dt}}\right)_1 = m_1 s_1 \left(\frac{\text{dT}}{\text{dt}}\right)_1$$

$$\left(\frac{\text{dQ}}{\text{dt}}\right)_2 = m_2 s_2 \left(\frac{\text{dT}}{\text{dt}}\right)_2$$

Given,

$$\left(\frac{\text{dT}}{\text{dt}}\right)_1 = \left(\frac{\text{dT}}{\text{dt}}\right)_2$$

$$\therefore \frac{\left(\frac{\text{dQ}}{\text{dt}}\right)_1}{\left(\frac{\text{dQ}}{\text{dt}}\right)_2} = \frac{m_1 s_1}{m_2 s_2} = \frac{m_1}{m_2} \times \frac{s_1}{s_2} = \frac{\text{d}_1}{\text{d}_2} \times \frac{s_1}{s_2}$$

$$= \frac{8}{1} \times \frac{1}{4} = \frac{2}{1}$$

33. The operating temperature of a tungsten filament in an incandescent lamp is 2000 K and its emissivity is 0.30. Find the surface area of the filament of a 25 watt lamp. Stefan constant = 5.67×10^{-8} in SI units.

Solution:

Rate of emission of energy is equal to the power of the lamp

$\Rightarrow W = A\sigma e T^4$

Where e → emissivity

$$A = \frac{W}{e\sigma T^4}$$

$$A = \frac{25}{0.3 \times 5.67 \times 10^{-8} \times (2000)^4}$$

$$\therefore A = 0.918 \times 10^{-4} m^2$$

$$A = 9.185 \times 10^{-5} m^2$$

34. A thin brass rectangular sheet of sides 15.0 cm and 12.0 cm is heated in a furnace to 600ºC and taken out. How much electric power is needed to maintain the sheet at this temperature, given that its emissivity is 0.250? Neglect heat loss due to convection. (Stefan – Boltzmann constant = 5.67×10^{-8} $\text{Wm}^{-2}\,\text{K}^{-4}$)

Solution:

Area of both sides of the plate

$$A = 2 \times (15.0) \times (12.0) \times 10^{-4} m^2$$
$$= 3.60 \times 10^{-2} m^2$$

The energy radiated by the plate

$$= e\sigma A T^4 = 0.250 \times 5.67 \times 10^{-8} \times 3.60 \times 10^{-2} \times (600 + 273)^4$$

$$= 5.10 \times 10^{-10} \times 873^4 = 296.4W$$

35. Two identical object A and B are at temperature T_A and T_B respectively. Both objects are placed in a room with perfectly absorbing walls maintained at a temperature T ($\text{T}_\text{A} > \text{T} > \text{T}_\text{B}$). The objects A and B attains the temperature T eventually. Select the correct statements from the following

(a) A only emits radiation, while B only absorbs it until both attain the same temperature

(b) a loses more heat by radiation that it absorbs, while B absorbs more radiation that it emits, until they attain the temperature T.

(c) both A and B only absorb radiation, but do not emit it, until they attain the temperature T.

(d) each object continuous to emit and absorb radiation even after attaining the temperature T.

Solution:

The correct options are B and D.

1. Since the temperature of A is higher than the temperature of the surrounding hence A radiates heat much larger than it absorbs heat. Since the temperature of B is lower than the temperature of the surrounding hence B absorbs heat much larger than it radiates.This process goes on until both A and B reach the temperature T.

2.Even after reaching thermal equilibrium, both bodies keep radiating and absorbing.

36. A solid sphere and a hollow sphere of the same material and of equal radii are heated to the same temperature.

(a) both will emit equal amount of radiation per unit time in the beginning.

(b) Both will absorb equal amount of radiation from the surrounding in the beginning

(c) the initial rate of cooling (dT/dt) will be the same for the two sphere.

(d) the two sphere will have equal temperatures at any instant.

Solution:

Correct options are A and B.

According to many radiation law like Stefan Boltzmann we know that radiation emission and absorption are a purely surface phenomenon. Since the two bodies are of same material, same radii, and same temperature they will at that instant radiate and absorb at the same rates.

But however since the hollow sphere has lesser mass, the rate at which it’s temperature will rise will be different from that of the solid sphere. Hence their rates of cooling would be varied and they would have different temperature at different times.

37. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 104 times the power emitted from B. The ratio $\frac{\lambda_A}{\lambda_B}$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

Solution:

(2) From (i) Stefan- Boltzmann law, $P = \sigma A T^4$ and (ii) Wein′s displacement law $= \lambda_m \times T =$ constant

$$\frac{P_A}{P_B} = \frac{A_A}{A_B}\frac{T_A^4}{T_B^4} = \frac{A_A}{A_B} \times \frac{\lambda_B^4}{\lambda_A^4}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \left[\frac{A_A}{A_B} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{R_A^2}{R_B^2} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}}$$

$$= \left[\frac{400 \times 400}{10^4}\right]^{\frac{1}{4}}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = 2$$

38. A spherical black body with a radius of 12 cm radiates 450 watt power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(a) 225 **(b) 450**

(c) 1000 **(d) 1800**

Solution:

(d) Radiated power of a black body,

$P = \sigma A T^4$

where, A = surface area of the body

T = temperature of the body

and σ = Stefan's constant

When radius of the sphere is halved, new area,

$$A' = \frac{A}{4}$$

∴ Power radiated,

$P' = \sigma \frac{A}{4} (2T)^4 = 16/4\ (\sigma A T^4)$

$= 4P = 4 \times 450 = 1800$ watts

Expert **Heat Transfer - Radiation**

1. A thin square steel plate with each side equal to 10 cm is heated by a blacksmith. The rate of radiated energy by the heated plate is 1134 W. The temperature of the hot steel plate is (Stefan's constant σ = 5.67 × 10^{-8} watt m^{-2} K^{-4}, emissivity of the plate = 1)

(a) 1000 K **(b) 1189 K**

(c) 2000 K **(d) 2378 K**

Solution:

The answer is B.

$$The\ rate\ of\ ra\text{diated energy}\frac{Q}{t} = P = A \in \sigma T^4$$

$$\Rightarrow 1134 = 5.67 \times 10^{-8} \times (0.1)^2 T^4 = 1189K$$

2. There are two parallel black sheets of infinite dimensions at temperature T K and 2T K. The temperature of 3rd identical sheet place mid-way and parallel to the sheets at equilibrium will be $\left[\frac{n}{2}\right]^{1/4} T.$ Value of n, is

Solution:

$$Q_{mt_1} = Q_{mt_2}$$

$$\sigma A\left(x^4 - T^4\right) = \sigma A\left((2T)^4 - x^4\right)$$

$$2x^4 = 9(16+1)T^4$$

$$x^4 = \frac{17}{2}T^4$$

$$x = \sqrt[4]{\frac{17}{2}}.T$$

3. Two metallic sphere A and B are made of same material and have got identical surface finish. The mass of sphere A is four times that of B. Both the spheres are heated to the same temperature and placed in a room having lower temperature but thermally insulated from each other.

(a) The ratio of heat loss of A to that of B is $2^{4/3}$.

(b) The ratio of heat loss of A to that of B is $2^{2/3}$.

(c) The ratio of the initial rate of cooling of A to that of B is $2^{-2/3}$.

(d) The ratio of the initial rate of cooling of A to that of B is $2^{-4/3}$.

Solution:

Correct options are A and C.

Heat loss, as per Stephan Boltzmann law is given by the relation:

$$q' = e\sigma AT^4$$

$$q' = e\sigma T^4(4\pi r^2)$$

Also

$$\frac{4}{3}\pi r^3 = \frac{m}{\rho}$$

$$r = km^{\frac{1}{3}}$$

Hence

$$\frac{q_1'}{q_2'} = \frac{r_1^2}{r_2^2} = \left(\frac{m_1^2}{m_2^2}\right)^{\frac{2}{3}} = 4^{\frac{2}{3}} = 2^{\frac{4}{3}}$$

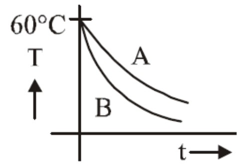
$$R = \frac{q'}{mc} = = \frac{Aq'}{m} \text{ where A is some constant.}$$

Hence,

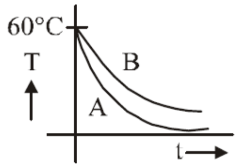
$$\frac{R_1}{R_2} = \left(\frac{m_2}{m_1}\right)^{\frac{1}{3}} = 2^{-\frac{2}{3}}$$

4. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

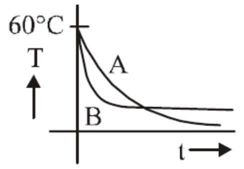
(a)



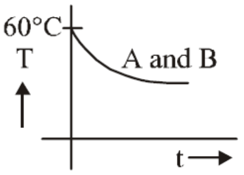
(b)



(c)



(d)



Solution:

(b) Rate of Heat loss = $mS\left(\frac{dT}{dt}\right) = e\sigma AT^4$

$$-\frac{dT}{dt} = \frac{e\sigma \times A \times T^4}{\rho \times Vol. \times S} \Rightarrow -\frac{dT}{dt} \propto \frac{1}{\rho S}$$

$$\frac{\left(-\frac{dT}{dt}\right)_A}{\left(-\frac{dT}{dt}\right)_B} = \frac{\rho_B}{\rho_A} \times \frac{S_B}{S_A} = \frac{10^9}{8\times10^2} \times \frac{4000}{2000}$$

$$\left(-\frac{dT}{dt}\right)_A > \left(-\frac{dT}{dt}\right)_B$$

So, A cools down at faster rate.

5. The energy spectrum of a black body exhibits a maximum around a wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum around a wavelength $\frac{3\lambda_0}{4}$. The power radiated by the black body will now increase by a factor of

(a) 256/81 **(b) 64/27**

(c) 16/9 **(d) 4/3**

Solution:

The answer is A.

From Wien's displacement law :

$$\lambda T = b$$

Stefan's law $E = \sigma T^4$

$$ratio = \frac{E_2}{E_1} = \frac{\sigma {T_2}^4}{\sigma {T_1}^4} = \frac{{T_2}^4}{{T_1}^4} = \left(\frac{\frac{b}{\lambda_2}}{\frac{b}{\lambda_1}}\right)^4 = \left(\frac{\lambda_1}{\lambda_2}\right)^4$$

$$= \left(\frac{\lambda_0}{\frac{3\lambda_0}{4}}\right)^4 = \left(\frac{4}{3}\right)^4 = \frac{256}{81}$$

6. Two identical objects A and B are at temperatures T_A and T_B respectively. Both objects are placed in a room with perfectly absorbing walls maintained at temperatures T ($T_A > T > T_B$) . The objects A and B attain temperature T eventually which one of the following is correct statement

(a) 'A' only emits radiations while B only absorbs them until both attain temperature

(b) A loses more radiations than it absorbs while B absorbs more radiations that it emits until temperature T is attained

(c) Both A and B only absorb radiations until they attain temperature T

(d) Both A and B only emit radiations until they attain temperature T

Solution:

The correct answer is B.

1. Since the temperature of A is higher than the temperature of the surrounding hence A radiates heat much larger than it absorbs heat. Since the temperature of B is lower than the temperature of the surrounding hence B absorbs heat much larger than it radiates. This process goes on until both A and B reach the temperature T.

2. Even after reaching thermal equilibrium, both bodies keep radiating and absorbing.

7. When the body has the same temperature as that of surroundings

(a) It does not radiate heat

(b) It radiates the same quantity of heat as it absorbs

(c) It radiates less quantity of heat as it receives from surroundings

(d) It radiates more quantity of heat as it receives heat from surroundings

Solution:

The correct answer is B.

Since the bodies has the same temperature as that of surrounding, it is in thermal equilibrium with the

surroundings.

So, there is no net exchange of heat,

So, amount of heat radiated = amount of heat received from the surroundings.

Pro Heat Transfer - Radiation

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1. The rectangular surface of area 8 cm × 4cm of a black body at a temperature of 127°C emits energy at the rate of E per second. If the length and breadth of the surface are each reduced to half of the initial value and the temperature is raised to 327°C , the rate of emission of energy will become

(a) $\frac{3}{8}E$ **(b)** $\frac{81}{16}E$

(c) $\frac{9}{16}E$ **(d)** $\frac{81}{64}E$

Solution:

The answer is D.

$E \propto AT^4$

Let,

$E = kAT^4$

$E = k(8\times4)(400)^2 = 8192\times10^8$

$E_1 = k\left(\frac{8}{2}\times\frac{4}{2}\right)(600)^4 = 10368\times10^8$

$$\frac{E_1}{E} = \frac{10368}{8192} = \frac{81}{64}$$

Hence,$E_1 = \frac{81}{64}E$

2. A solid copper sphere (density ρ and specific heat C) of radius r at an initial temperature 200 K is suspended inside a chamber whose walls are at almost 0 K. What is the required for temperature of the sphere to drop to 100 K?

(a) $\frac{7r\rho C}{72\times10^6\sigma}$ **(b)** $\frac{7r\rho C}{72\times10^5\sigma}$

(c) $\frac{7r\rho C}{72\times10^4\sigma}$ **(d)** $\frac{7r\rho C}{72\times10^7\sigma}$

Solution:

The correct answer is A.

In this question, we will equate the heat loss from the sphere to heat loss due to radiation.

$$mC\frac{\mathrm{dT}}{\mathrm{dt}} = \sigma AT^4$$

$$\left(\frac{4}{3}\pi r^3\right)(\rho)C\frac{\mathrm{dT}}{\mathrm{dt}} = \sigma(4\pi r^2)T^4$$

$$\frac{r\rho C}{3\sigma}\int_{100}^{200}\frac{\mathrm{dT}}{T^4} = \int_0^t \mathrm{dt}$$

$$\frac{rC\rho}{9\sigma\times10^6}\left(1-\frac{1}{8}\right) = t$$

$$t = \frac{7r\rho C}{72\times10^6\sigma}$$

3. A space object has the space of a sphere of radius R. Heat sources ensure that the heat evolution at a constant rate is distributed uniformly over its volume. The amount of heat liberated by a thermodynamic temperature. In what proportion would the temperature of the object change if its radius is decreased to half?

(a) 2.19 **(b) 5.19**

(c) 1.19 **(d) 4.19**

Solution:

The correct answer is C.

The total amount of heat q liberated by the space object per unit time is proportional to its volume: $q = \alpha R^3$, where α is a coefficient. Since the amount of heat given away per unit surface the area is proportional to T^4, and in equilibrium, the entire amount of the liberated heat s dissipated into space, we can write $q = \beta R^2 T^4$ (the area of the surface is proportional to R^2, and β is a coefficient). Equating these two expressions for q, we obtain

$$T^4 = \frac{\alpha}{\beta}R$$

Consequently, the fourth power of the temperature of the object is proportional to its radius, and hence in radius by half leads to a decrease in temperature only by a factor of $(2)^{\frac{1}{4}} = 1.19$.

4. Two bodies A and B have thermal emissivities of 0.01 and 0.81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength λ_B corresponding to maximum spectral radiancy in the radiation from B is shifted from the wavelength corresponding to maximum spectral radiancy in the radiation from A by 1.00 μm. If the temperature of A is 5802 K, calculate (a) the temperature of B and (b) wavelength λ_B.

(a) 2000K, 1.6 μ m (b) 3000K, 1.8 μm

(c) 1934K, 1.5 μ m (d) 4000K, 1.9 μ m

Solution:

The correct answer is C.

Power radiated and surface area is same for both A and B,

Therefore,

$$e_A\sigma T_A^4 A = e_B\sigma T_B^4 A$$

$$\therefore \frac{T_A}{T_B} = \left(\frac{e_B}{e_A}\right)^{\frac{1}{4}} = \left(\frac{0.81}{0.01}\right)^{\frac{1}{4}} = 3$$

$$\therefore T_B = \frac{T_A}{3} = \frac{5802}{2} = 1934K$$

$$T_B = 1934K$$

According to Wien's displacement law,

$$\lambda_m T = \text{constant}$$

$$\lambda_B T_B = \text{b}$$

$$\lambda_B = \frac{2.89\times10^{-3}}{1934} \sim 1.5\ \mu m$$

NOTE $\lambda_m T = b =$ when's constant value of this constant for perfectly black body is $2.89\times10^{-3} m.K$. For other bodies of this constant will have some different value.

5. Two bodies A and B have thermal emissivity of .01 and .81 respectively. The outer surface areas of the two bodies are same. The two bodies emit total radiant power at the same rate. The wavelength of B corresponding to maximum spectral radiation is shifted from the wavelength corresponding to maximum spectral radiation from A by 10^{-6} m. The temperature of A is 5802K. Find temperature of B and wavelength corresponding to maximum spectral radiation.

Solution:

$$Power\ radiated: \text{P} = \text{eA}\sigma T^4$$

$$\frac{P_A}{P_B} = \frac{e_A}{e_B}\frac{T_A^4}{T_B^4} = \frac{0.01}{0.81}\frac{T_A^4}{T_B^4}$$

Given,

$$P_A = P_B$$

$$\Rightarrow \left(\frac{T_A}{T_B}\right)^4 = 81 = 3^4$$

$$\frac{T_A}{T_B} = 3$$

Given, $T_A = 5802K$

$$\Rightarrow T_B = \frac{T_A}{3} = \frac{5802}{3}K$$

$$(\lambda_m)_A T_A = (\lambda_m)_B T_B = ((\lambda_m)_A + 1)\frac{T_A}{3}$$

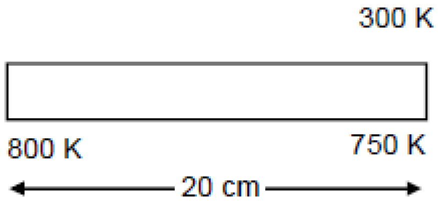
$$\frac{((\lambda_m)_A + 1)}{(\lambda_m)_A} = 3$$

$$\frac{1}{(\lambda_m)_A} = 2$$

$$\Rightarrow (\lambda_m)_B = (\lambda_m)_A + 1 = 1.5\mu m$$

6. One end of a rod of length 20 cm is inserted in a furnace at 800 K. the sides of the rod are covered with an insulating material and the other end emits radiation like a blackbody. The temperature of this end is 750 K in the steady state. The temperature of the surrounding air is 300 K. Assuming radiation to be the only important mode of energy transfer between the surrounding and the open end of the rod, find the thermal conductivity of the rod. Stefan constant $\sigma = 6.0\times10^{-8} W/m^2 - K^4$

Solution:



$$\sigma = 6\times10^{-8}\frac{w}{m^2} - K^4$$

$$L = 20cm = 0.2m$$

$$K = ?$$

$$\Rightarrow E = \frac{KA(\theta_1 - \theta_2)}{\text{d}} = A\sigma\left({T_1}^4 - {T_2}^4\right)$$

$$\Rightarrow K = \frac{s(T_1 - T_2)}{\theta_1 - \theta_2}$$

$$= \frac{6 \times 10^{-8} \times (750^4 - 300^4) \times 2 \times 10^{-1}}{50}$$

$$\Rightarrow K = 73.993 \approx 74$$

7. A solid receives heat by radiation over its surface at the rate of 4 kW. The heat convection rate from the surface of solid to the surrounding is 5.2 kW, and heat is generated at a rate of 1.7 kW over the volume of the solid. The rate of change of the average temperature of the solid is 0.5°Cs^{-1}. Find the heat capacity of the solid.

Solution:

$$H_{\text{Radiation}} + H_{Generated} - H_{Convection} = H_{Retained}$$

$$4 + 1.7 - 5.2 = 0.5kW = 0.5 \times 10^3 W = H_{Retained}$$

$$Heat\ Capacity\ C = \frac{H_{Retained}}{\Delta T}$$

Substituting $\Delta T = 0.5$ as given

$$C = \frac{0.5 \times 10^3}{0.5} = \frac{1000J}{°\text{C}}$$

8. A copper sphere is suspended in an evacuated chamber maintained at 300 K. The sphere is maintained at a constant temperature of 500 K by heating it electrically. A total of 210 W of electric power is needed to do it. When the surface of the copper sphere is completely blackened, 700 W is needed to maintain the same temperature of the sphere. Calculate the emissivity of copper.

Solution:

$$Q = e\sigma A\left({T_2}^4 - {T_1}^4\right)$$

For any body, $210 = eA\sigma[(500)^4 - (300)^4]$

For black body, $700 = 1 \times A\sigma[(500)^4 - (300)^4]$

Dividing $\frac{210}{700} = \frac{e}{1} \Rightarrow e = 0.3$

9. Three bodies A, B and C have equal surface area and thermal emissivity's in the ratio e_A: e_B: e_C = $1 : \frac{1}{2} : \frac{1}{4}$. All the three bodies are radiating at same rate. Their wavelengths corresponding to maximum intensity are λ_A, λ_B and λ_C respectively and their temperatures are T_A, T_B and T_C on kelvin scale, then select the incorrect statement.

(a) $\sqrt{T_A T_C} = T_B$

(b) $\sqrt{\lambda_A \lambda_C} = \lambda_B$

(c) $\sqrt{e_A T_A}\sqrt{e_C T_C} = e_B T_B$

(d) $\sqrt{e_A \lambda_A T_A \cdot e_B \lambda_B T_B} = e_C \lambda_C T_C$

Solution:

Correct option is D.

For same power of radiation

$$P_A = P_B = P_C \Rightarrow e_A\sigma {T_A}^4 = e_B\sigma {T_B}^4 = e_C\sigma {T_C}^4$$

$$\lambda_A T_A = \lambda_B T_B = \lambda_C T_C \left(e_A : e_B : e_C = 1 : \frac{1}{2} : \frac{1}{4}\right)$$

$$\Rightarrow \sqrt{T_A T_C} = T_B \ or \ \sqrt{e_A T_A}\sqrt{e_A T_A} = e_B T_B \ \& \ \sqrt{\lambda_A \lambda_C} = \lambda_B$$

10. A black coloured solid sphere of radius R and mass M is inside a cavity with vacuum inside. The walls of the cavity are maintained at temperature T0. The initial temperature of the sphere is 3T0. If the specific heat of the material of the sphere varies as α T3 per unit mass with the temperature T of the sphere, where α is a constant, then the time taken for the sphere to cool down to temperature 2T0 will be (σ is Stefan Boltzmann constant)

(a) $\frac{M\alpha}{4\pi R^2\sigma}\ln\left(\frac{3}{2}\right)$ **(b)** $\frac{M\alpha}{4\pi R^2\sigma}\ln\left(\frac{16}{3}\right)$

(c) $\frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{16}{3}\right)$ **(d)** $\frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{3}{2}\right)$

Solution:

(c) In the given problem, fall in temperature of sphere,

dT = $(3T_0 - 2T_0) = T_0$

Temperature of surrounding, $T_{\text{surr}} = T_0$

Initial temperature of sphere, $T_{\text{initial}} = 3T_0$

Specific heat of the material of the sphere varies as,

c = αT^3 per unit mass (α = a constant)

Applying formula,

$$\frac{dT}{dt} = \frac{\sigma A}{McJ}\left(T^4 - T_{surr}^4\right)$$

$$\frac{T_0}{dt} = \frac{\sigma 4\pi R^2}{M\alpha\left(3T_0\right)^3 J}\left[\left(3T_0\right)^4 - \left(T_0\right)^4\right]$$

$$dt = \frac{M\alpha 27T_0^4 J}{\sigma 4\pi R^2 \times 80T_0^4}$$

Solving we get,

Time taken for the sphere to cool down temperature $2T_0$,

$$t = \frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{16}{3}\right)$$

11. Parallel rays of light of intensity *I*
$912\ \mathrm{Wm^{-2}}$ are incident on a spherical black body kept in surroundings of temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\mathrm{Wm^{-2}K^{-4}}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to

(a) 330K **(b) 660 K**

(c) 990 K **(d) 1550K**

Solution:

(a) Let T be the final steady state temperature of the black body.

In steady state.

Energy lost = Energy gained

$\sigma(T^4 - T_0^4) \times 4\pi R^2 = I(\pi R^2)$

$\therefore 5.7 \times 10^{-8}[T^4 - (300)^4] \times 4 = 912$

$\therefore T = 330\ \mathrm{K}$

12. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2$, (P/P_0), where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

Solution:

(9) According to stefan's law, $P \propto T^4$ or $P = P_0T^4$

$\therefore \log_2 P = \log_2 P_0 + \log_2 T^4 \quad \therefore \log_2 \frac{P}{P_0} = 4\log_2 T$

At $T = 487℃ = 760\mathrm{K}, \log_2 \frac{P}{P_0} = 4\log_2 760 = 1$ ... (i)

At $T = 2767℃ = 3040\mathrm{K}$,

$\log_e \frac{\mathrm{p}}{\mathrm{p_0}} = 4\log_2 3040 = 4\log_2(760 \times 4)$

$= 4[\log_2 760 + \log_2 2^2] = 4\log_2 760 + 8 = 1 + 8 = 9$

13. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of $\sigma T_0^4 = 460\mathrm{Wm^{-2}}$ (where *σ* is the Stefan-Boltzmann constant). Which of the following options is/are correct?

(a)The amount of energy radiated by the body in 1 second is close to 60 joules.

(b)If the surrounding temperature reduces by a small amount $\Delta T_0 \ll T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$, more energy per unit time

(c)Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(d)If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

Solution:

(c) Energy radiated by the body $= \sigma A(\mathrm{T}^4 - T_0^4)\mathrm{t}$

[For a black body e = 1]

$= \sigma A[(\mathrm{T_0} - 10)^4 - T_0^4]\mathrm{t}$

$= \sigma A T_0^4 \left[\frac{40}{T_0}\right] \times t$

$= 460 \times 1 \times \frac{40}{300} \times 1 = 61.33J$

$P = \frac{\text{Energy radiated}}{\text{time}}$

$= \sigma AT^4 - \sigma AT_0^4$

$\therefore \left|\frac{dp}{d\mathrm{T_0}}\right| = \sigma A(4T_0^3)$

$\therefore |dp| = \sigma(4T_0^3)d\mathrm{T_0}$

$\therefore |\Delta P| = 4\sigma AT_0^3$

Here as human body is not a black body. So, option (a) and (b) are incorrect.

Energy radiated ∝ A where A is the surface area of the body. Hence option (c) is correct.

Beginner **Newton's Law of Cooling**

1. Newton's law of cooling is a special case of

(a) Stefan's law **(b) Kirchhoff's law**
(c) Wien's law **(d) Planck's law**

Solution:

The answer is A.

Newton's law of cooling is just a simplified version of Stefan's Boltzmann's law. We apply binomial

approximation assuming that the temperature difference between the object and the surroundings is small enough so that we can apply binomial approximation.

2. A body cools in 10 min from 60°C to 40°C. What will be its temperature after next 10 min. $T_s = 10°C$.

Solution:

The answer is 28°C.

According to Newton's law of cooling,

$$\left(\frac{\theta_1-\theta_2}{t}\right)=\alpha\left[\frac{\theta_1+\theta_2}{t}-\theta_0\right]$$

For the given condition,

$$\frac{60-40}{10}=\alpha\left[\frac{60+40}{2}-10\right]\ldots.(i)$$

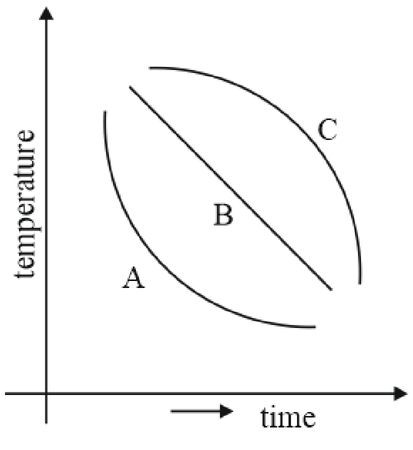
Let θ be the temperature after next 10 minutes. Then,

$$\frac{40-\theta}{10}=\alpha\left[\frac{40+\theta}{2}-10\right]\ldots.(ii)$$

Solving Eqn. (i) and (ii), we get

$\theta = 28°C$

3. A block of steal heated to 100°C is left in a room to cool. Which of the curves shown in fig., represents the correct behavior –



(a) A **(b) B**
(c) C **(d) None of these**

Solution: (a)

$$\frac{dT}{dt}=-k\left(T-T_0\right)$$

$$T\downarrow\rightarrow\left(T-T_0\right)\downarrow\downarrow$$

|slope|↓↓

|slope| ∝ $(T - T_0)$

4. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surrounding is

(a) 5°C **(b) 10°C**
(c) 15°C **(d) 20°C**

Solution:

The answer is B.

We know from Newton's law of cooling that

$T = T_f + (T_i - T_f)e^{-kt}$

$50 = T_f + (60 - T_f)e^{-kt}$

$50 = y + 60x \quad ------ \ (i)$

$where \ x = e^{-10k}$

and $y = T_f(1 - e^{-10k})$

Also,

$42 = T_f + (50 - T_f)e^{-10k}$

$42 = y + 50x \quad ------- \ (ii)$

Solving (i) and (ii), we get

$x = e^{-10k} = \frac{4}{5}$

And,

$y = T_f(1 - e^{-10k}) = 2$

$T_f = 10°C$

5. A body takes 4 minutes to cool from 100°C to 70°C . To cool from 70°C to 40°C it will take (room temperature is 15°C)

(a) 7 minutes **(b) 6 minutes**
(c) 5 minutes **(d) 4 minutes**

Solution:

The correct answer is B.

$$\frac{\theta_1-\theta_2}{t}=K\left(\frac{\theta_1-\theta_2}{2}-\theta_0\right)$$

$$\therefore\ \frac{100-70}{4}=K\left(\frac{100+70}{2}-15\right)=60K\Rightarrow K=\frac{1}{8}$$

$$Again\ \frac{70-40}{t}=\frac{1}{8}\left(\frac{70+40}{2}-15\right)=5\Rightarrow t=6min$$

6. A cup of tea cools from 80°C to 60°C in one

minute. The ambient temperature is 30°C . In cooling from 60°C to 50°C it will take

(a) 30 seconds **(b) 60 seconds**

(c) 90 seconds **(d) 48 seconds**

Solution:

The correct answer is D.

$$\frac{\Delta t}{time} = k\left(\frac{t_1 + t_2}{2} - t_2\right)$$

$$\frac{80 - 60}{60} = k\left(\frac{80 + 60}{2} - 30\right)$$

$$\frac{1}{3} = k(40)$$

$$k = \frac{1}{120}$$

$$\frac{60 - 50}{time} = \frac{1}{120}\left(\frac{60 + 50}{2} - 30\right)$$

$$time = 48\ secs$$

7. If a metallic sphere gets cooled from 62°C to 50°C in 10 minutes and in the next 10 minutes gets cooled to 42°C , then the temperature of the surroundings is

(a) 30°C **(b) 36°C**

(c) 26°C **(d) 20°C**

Solution:

The correct answer is C.

26℃

$$\frac{\theta_1 - \theta_2}{t} = K\left(\frac{\theta_1 - \theta_2}{2} - \theta_0\right)$$

In the first 10 $minute$

$$\frac{62 - 50}{10} = K\left[\frac{60 + 50}{2} - \theta_0\right] \Rightarrow 1.2$$
$$= K[56 - \theta_0] - - - (i)$$

$$\frac{50 - 42}{10} = K\left[\frac{50 + 42}{2} - \theta_0\right] \Rightarrow 0.8$$
$$= K[46 - \theta_0] - - - -(ii)$$

From equation (i) and (ii)

$$\frac{1.2}{0.8} = \frac{(55 - \theta_0)}{(46 - \theta_0)} \Rightarrow \theta_0 = 26℃$$

8. A body cools from 60°C to 50°C in 10 minutes. If the room temperature is 25°C and assuming Newton's law of cooling to hold good, the temperature of the body at the end of the next 10 minutes will be

(a) 38.5°C **(b) 40°C**

(c) 42.85°C **(d) 45°C**

Solution:

The correct answer is C.

From Newton's law of cooling

$$\frac{\theta_1 - \theta_2}{t} = K\left(\frac{\theta_1 - \theta_2}{2} - \theta_0\right)$$

In the first 10 minutes, from the data given in the question, we can say that

$$\frac{50 - 60}{10} = K\left[\frac{60 + 50}{2} - 25\right]$$

$$\Rightarrow K = \frac{1}{30} - - - - - - (1)$$

Let θ be the temperature at the end of next 10 minutes.

$$\Rightarrow \frac{50 - \theta}{10} = K\left[\frac{50 + \theta}{2} - 25\right]$$

Using (1) in the above equation

$$\frac{50-\theta}{10} = \frac{\theta}{60} \Rightarrow 7\theta = 300 \Rightarrow \theta \approx 42.9℃$$

9. In a room where the temperature is 30°C , a body cools from 61°C to 59°C in 4 minutes. The time (in min.) taken by the body to cool from 51°C to 49°C will be

(a) 4 min **(b) 6 min**

(c) 5 min **(d) 8 min**

Solution:

The correct answer is B.

The equation given by Newton's law of cooling:

$$\frac{dT}{dt} = -bA(T - T_0)$$

$$\frac{dT}{T-T_0} =$$
$$-bAdt\ \ integrating\ between\ appropriate\ limits.$$

$$\frac{1}{t}\ln\left(\frac{59 - 30}{61 - 30}\right) = \frac{1}{t}\ln\left(\frac{49 - 30}{51 - 30}\right)$$

Solving for t, we get $t\ =\ 6\ minutes.$

10. A body cools from 60°C to 50°C in 10 minutes when kept in air at 30°C . In the next 10 minutes its temperature will be

(a) Below 40°C **(b) 40°C**

(c) Above 40°C **(d) Cannot be predicted**

Solution:

The correct answer is C.

Here from newton's law of cooling

$$\frac{\mathrm{dT}}{\mathrm{dt}} = -K\left(\frac{T_f + T_i}{2} - T_s\right)$$

$$\therefore\ \frac{T_f - T_i}{\Delta\mathrm{t}} = -K\left(\frac{T_f + T_i}{2} - T_s\right)$$

$$\therefore K = \frac{1}{20}$$

$$\Rightarrow \frac{60 - T}{10} = \frac{1}{20}\left(\frac{60 + T}{2} - 30\right)$$

$$\Rightarrow 120 - 2T = \left(\frac{60 + T}{2} - 30\right)$$

$$\frac{5T}{2} = 120$$

$$T = 48°\text{C}$$

11. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to :

(a) 31°C (b) 33°C

(c) 28°C (d) 35°C

Solution:

(b) From Newton's Law of cooling,

$$\frac{T_1 - T_2}{t} = K\left[\frac{T_1 + T_2}{2} - T_0\right]$$

Here, $T_1 = 50°C$, $T_2 = 40°C$

and $T_0 = 20°C$, t = 600S = 5 minutes

$$\frac{50-40}{5\,Min} = K\left(\frac{50+40}{2} - 20\right)s \quad \text{...}$$

(i)

Let T be the temperature of sphere after next 5 minutes. Then

$$\frac{40-T}{5} = K\left(\frac{40+T}{2} - 20\right) \quad \text{... (ii)}$$

Dividing eqn. (ii) by (i), we get

$$\frac{40-T}{10} = \frac{40+T-40}{50+40-40} = \frac{T}{50}$$

40-T=T/5 , 200 = 6T , T= 33°C $\therefore T = \frac{200}{6} =$ 33.3 °C

12. A body takes 10 minutes to cool from 60°C to 50°C. The temperature of surroundings is constant at 25°C.Then, the temperature of the body after next 10 minutes will be approximately

(a) 43°C (b) 47°C

(c) 41°C (d) 45°C

Solution:

(a) According to Newton's law of cooling,

$$\left(\frac{\theta_1 - \theta_2}{t}\right) = K\left(\frac{\theta_1 + \theta_2}{2} - \theta_0\right)$$

$$\left(\frac{60-50}{10}\right) = K\left(\frac{60+50}{2} - 25\right)\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

... (i)

$$\text{and, } \left(\frac{50-\theta}{10}\right) = K\left(\frac{50+\theta}{2} - 25\right) \quad \text{... (ii)}$$

Dividing eq. (i) by (ii),

$$\frac{10}{(50-\theta)} = \frac{60}{\theta}$$

θ = 42.85°C ≅ 43°C

13. A liquid cools down from 70°C to 60°C in 5 minutes. The time taken to cool it from 60°C to 50°C will be

(a) 5 minutes

(b) Lesser than 5 minutes

(c) Greater than 5 minutes

(d) Lesser or greater than 5 minutes depending upon the density of the liquid

Solution:

The correct answer is C.

Greater than 5 minutes.

According to Newton's law of cooling

$Rate\ of\ cooling$

$\propto means\ temperature\ difference.$

Initially, means temperature difference

$$= \left(\frac{70+60}{2} - \theta_0\right) = (65 - \theta_0)$$

Finally, means temperature difference

$$= \left(\frac{60+50}{2} - \theta_0\right) = (55 - \theta_0)$$

In second case mean temperature difference decreases,So rate of fall of temperature decreases,,

So it takes more time to cool through t and it will be Greater than 5 minutes

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Expert **Newton's Law of Cooling**

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1. A pan filled with hot food cools from 50.1°C to 49.9°C in 5 sec. How long will it take to cool from 40.1 ° to 39.9°C if room temperature is 30°C?

Solution:

The mathematical interpretation of Newton's law of cooling states that

$$\frac{\text{dT}}{\text{dt}} = -bA(T - T_0)$$

Solving the differential equation, with basic integration and applying the limits:

$T; varying\ from\ T_1 and$ d T_2

T ; varying from 0 to t. the time it takes to change from T_1 to T_2, T_0 is the room temperature.

Now applying this here:

$$\frac{1}{5}\ln\left(\frac{19.9}{20.1}\right)=\frac{1}{t}\ln\left(\frac{9.9}{10.1}\right)$$

Solving this we get $t = 10s$

2. Two metallic spheres S_1 and S_2 are made of the same material and have got identical surface finish. The mass of S_1 is thrice that of S_2. Both the spheres are heated to the same high temperature and placed in the same room having lower temperature, but are thermally insulated from each other. The ratio of initial rate of cooling of S_1 to that of S_2 is

(a) 1/3 **(b) 1/√3**

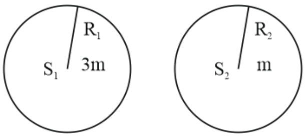
(c) √3/1 **(d) $(1/3)^{1/3}$**

Solution:(d)

Cooling of S_1 to that of S_2 is

$$\frac{dT}{dt}=\frac{-4e\sigma\left(4\pi R^3\right)T_0^3}{ms}\left(\Delta T\right)$$

$$\frac{\left(\frac{dT}{dt}\right)_1}{\left(\frac{dT}{dt}\right)_2}=\frac{R_1^2}{R_2^2}\times\frac{m}{3m}=\frac{3^{2/3}}{3}=\frac{1}{3^{1/3}}$$



$$t=\frac{3m}{\frac{4}{3}\pi R_1^3}=\frac{m}{\frac{4}{3}\pi R_2^2}$$

$$\frac{R_1}{R_2}=\sqrt[3]{3}$$

3. A mass of 50g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from 30°C to 25°C. A mass of 100g of another liquid in an analytic vessel with identical surroundings takes the same time to cool from 30° C to 25° C. The specific heat of the liquid is : (The water equivalent of the vessel is 30g.)

(a) 2.0 kcal/kg **(b) 7 kcal/kg**

(c) 3 kcal/kg **(d) 0.5 kcal/kg**

Solution:

(d) As the surrounding is identical, vessel is identical time taken to cool both water and liquid (from 30°C to 25°C) is same 2 minutes, therefore

$$\left(\frac{dQ}{dt}\right)_{water}=\left(\frac{dQ}{dt}\right)_{liquid}$$

or, $\frac{\left(m_wC_w+W\right)\Delta T}{t}=\frac{\left(m_lC_l+W\right)\Delta T}{t}$

(W = water equivalent of the vessel)

or, $m_wC_w = m_lC_l$

∴ Specific heat of liquid, $C_l=\frac{m_WC_W}{m_l}$

$=\frac{50\times 1}{100}$ = 0.5 kcal/kg

4. A cup of coffee cools from 90°C to 80°C in t minutes, when the room temperature is 20°C. The time taken by a similar cup of coffee to cool from 80°C to 60°C at a room temperature same at 20°C, is

(a) $\frac{13}{10}t$ **(b)** $\frac{13}{5}t$

(c) $\frac{10}{13}t$ **(d)** $\frac{5}{13}t$

Solution:

(b) In first conditions;

Given, the initial temperature of the cup of coffee, T_i = 90°C

The final temperature of the cup of coffee, T_f = 80°C

The time taken to drop the temperature 90°C to 80°C is t.

The temperature of the surrounding, T_0 = 20°C

Using the Newton's law of cooling,

Rate of cooling = $\frac{dT}{dt}=K\left[\frac{T_i+T_f}{2}-T_0\right]$

Substituting the values in the above equation, we get

$$\frac{90-80}{t}=K\left[\frac{90+80}{2}-20\right]$$

$$\frac{10}{t}=K\left[65\right]\Rightarrow K=\frac{2}{13t}$$

In second conditions;

The initial temperature of the cup of coffee, T_i' = 80°C

The final temperature of the cup of coffee, T_f' = 60°C.

Using the Newton's law of cooling,

Rate of cooling = $\frac{dT}{dt}=K\left[\frac{T_i'+T_f'}{2}-T_0\right]$

Substituting the values in the above equation, we get

$$\frac{80-60}{t_1}=\frac{2}{13t}\left[\frac{60+80}{2}-20\right]$$

$$\frac{20}{t_1}=\frac{2}{13t}[50]\Rightarrow t_1=\frac{13}{5}t$$

5. An object kept in a large room having air temperature of 25°C takes 12 minutes to cool from 80°C to 70°C. The time taken to cool for the same object from 70°C to 60°C would be nearly

(a) 10 min **(b) 12 min**

(c) 20 min **(d) 15 min**

Solution:

(d) From Newton's law of cooling, the time taken (t) by a body to cool from T_1 to T_2 when placed in a medium of temperature T_0 can be calculated from relation

$$\frac{T_1-T_2}{t}=\frac{1}{K}\left(\frac{T_1+T_2}{2}-T_0\right)$$

When the object cool from 80°C to 70°C in 12 minutes, then from Newton's law of cooling,

$$\frac{80-70}{12}=\frac{1}{K}\left(\frac{80+70}{2}-25\right)$$

[$\because T_0 = 25\ °C$]

$$\frac{5}{6}=\frac{1}{K}50 \qquad \dots \text{(i)}$$

Similarly, when object cool from 70°C to 60°C we get

$$\frac{70-60}{t}=\frac{1}{K}\left(\frac{70+60}{2}-25\right)$$

$$\frac{10}{t}=\frac{1}{K}40 \qquad \dots \text{(ii)}$$

Divide Eq. (i) and (ii), we get

$$\frac{5}{6}\times\frac{t}{10}=\frac{50}{40}$$

$$t=\frac{5}{4}\times 12 = 15 \text{ minutes}$$

6. A body cools from a temperature 3T to 2T in 10 minutes. The room temperature is T. Assume that Newton's law of cooling is applicable. The temperature of the body at the end of next 10 minutes will be

(a) $\frac{7}{4}T$ **(b)** $\frac{3}{2}T$

(c) $\frac{4}{3}T$ **(d) T**

Solution:

(b) According to Newton's law of cooling,

$\Delta T = \Delta T_0\, e^{-\lambda t}$

$3T - 2T = (3T - T)\, e^{-\lambda \times 10}$ … (i)

Again for next 10 minutes

$T' - T = (2T)\times e^{-\lambda(20)}$ … (ii)

From Eqs. (i) and (ii), we get

$$T'-T=(2T)\left(e^{-\lambda\times10}\right)^2=(2T)\left(\frac{1}{2}\right)^2$$

$$=\frac{T}{2}$$

$$\therefore \quad T'=T+\frac{T}{2}=\frac{3T}{2}$$

Pro Newton's Law of Cooling

1. Liquid is filled in a vessel which is kept in a room with temperature 20°C .When the temperature of the liquid is 80°C , then it loses heat at the rate of 60 cal/sec . What will be the rate of loss of heat when the temperature of the liquid is 40°C

(a) 180 cal/sec **(b) 40 cal/sec**

(c) 30 cal/sec **(d) 20 cal/sec**

Solution:

The correct answer is D.

We know rate of heat transfer

$q = C\Delta T$

Where C is constant

$60 = C(80 - 20)$

$60 = C \times 60$

$C = 1$

$for\ T = 40°$

$q = C(40 - 20)$

$= 1 \times 20$

$q = 20\,\frac{cal}{sec}$

2. In Newton's experiment of cooling, the water equivalent of two similar calorimeters is 10 gm each. They are filled with 350 gm of water and 300 gm of a liquid (equal volumes) separately. The time taken by water and liquid to cool from 70°C to 60°C is 3 min and 95 sec respectively. The specific heat of the liquid will be

(a) 0.3 Cal/gm °C **(b) 0.5 Cal/gm °C**
(c) 0.6 Cal/gm °C **(d) 0.8 Cal/gm °C**

Solution:

The answer is C

$$0.6\frac{Cal}{gm}\ ^\circ\text{C}$$

$$S_l = \frac{1}{m_l}\left[\frac{t_l}{t_w}(M_w C_w + W) - W\right]$$

$$= \frac{1}{300}\left[\frac{95}{3\times 60}(350\times 1 + 10) - 10\right]$$

$$= 0.6\frac{Cal}{gm}\times {}^\circ\text{C}$$

3. The room temperature is +20°C when outside temperature is -20°C and room temperature is + 10°C when outside temperature is -40°C. Find the temperature of the radiator heating the room.

(a) 30°C **(b) 60°C**
(c) 90°C **(d) 45°C**

Solution:

The answer is B.

According to Newton's law of cooling

$$\frac{d\theta}{dt} \propto (\theta - \theta_o)$$

$$\theta \rightarrow temperature$$

$$\theta_o \rightarrow room\ temperature$$

$$T - 20 \propto [20 - (-20)]$$

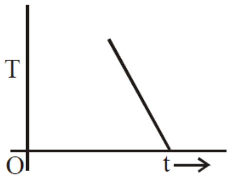
$$T - 10 \propto [10 - (-40)]$$

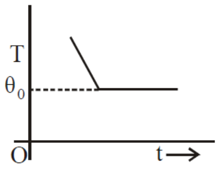
$$\frac{T-20}{T-10} = \frac{40}{50}$$

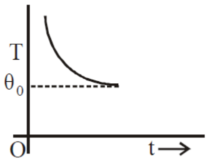
$$5T - 100 = 4T - 40$$

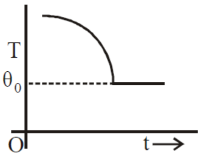
$$T = 60^\circ\text{C}$$

4. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to

(a)


(b)


(c)


(d)


Solution:

(c) According to Newton's law of cooling the temperature goes on decreasing with time non-linearly.

5. A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm^2 and it is absorbed by the water over an effective area of 0.05 m^2. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be_____________. (Ignore effect of the container, and take constant for Newton's law of cooling=0.001 s^{-1}, Heat capacity of water = 4200 J kg^{-1} K^{-1})

Solution:

(8.33)

Rate of loss of heat,

$$\frac{dQ}{\Delta dt} = e\sigma A(T^4 - T_0^4) \qquad \text{... (i}$$

$$\Rightarrow \frac{dQ}{\Delta dt} = e\sigma A(\,T_0 - \Delta T)\ ^4$$

$$= \sigma T_0^4\left[\left(1+\frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$\Rightarrow e\sigma T_0^4\left[\left(1+4\frac{\Delta T}{T_0}\right) - 1\right]$$

$$\frac{dQ}{Adt} = \sigma e T_0^3 \cdot 4\Delta T \qquad \text{... (ii)}$$

Now from eq. (i)

$$ms = \frac{dT}{dt} = \sigma e A(T^4 - T_0^4) \quad [\because Q = ms\Delta T]$$

$$\Rightarrow \frac{dT}{dt} = \frac{\sigma eA}{ms}[(T_0 + \Delta T^4)] - T_0^4$$

$$= \frac{\sigma eA}{ms}T_0^4 \times \left[\left(1+\frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$\frac{dT}{dt} = \frac{\sigma eA}{ms}T_0^4 \cdot 4\Delta T$$

$$\frac{dT}{dt} = K\Delta T;$$

$$\left(K\frac{4\sigma eAT_0^3}{ms}\text{Constant for Newton's law of cooling}\right)$$

$$\Rightarrow 4\sigma eAT_0^3 = \frac{K}{A}(ms)$$

From eq. (i)

$$\frac{dQ}{Adt} = e\sigma T_0^3 \cdot 4\Delta T$$

Since, rate of loss of heat = heat received per second

$$700 = \left(\frac{K}{A}\right)(ms)\Delta T$$

$$[K \times ms = 4200 \times 10^{-30}]$$

$$\Rightarrow \Delta T = \frac{700 \times A}{K \times ms} = \frac{700 \times 5 \times 10^{-2}}{10^{-3} \times 4200}$$

$$= \frac{50}{6} = \frac{25}{3}$$

$$\therefore \Delta T = 8.33$$

Beginner **Weins Displacement Law**

1. Colour of shining bright star is an indication of its

(a) Distance from the earth

(b) Size

(c) Temperature

(d) Mass

Solution:

The correct answer is C.

The colour of a star mostly indicates a star's, which are blue in colour, are the hottest, and class M stars, which are red in colour, are the coldest.

2. If light from the sun is found to have a maximum intensity near the wavelength of 480 nm. Calculate the temperature of the surface of the sun.

(a) 2000 K **(b) 4000 K**

(c) 6000 K **(d) 8000 K**

Solution:c)

$\lambda_m T = b$

$$T = \frac{b}{\lambda_m} = \frac{2.88 \times 10^{-3}}{480 \times 10^{-9}} = \frac{28800}{48} = 6000 \text{ K}$$

3. The spectral energy distribution of the Sun (temp = 6050 K) has a maximum at 4753Å. The temperature of a Star for which this maximum is at 9506 Å is –

(a) 6050 K **(b) 3025 K**

(c) 12100 K **(d) 24200 K**

Solution: (b)

$\lambda_m T = b = \lambda_m' T'$

$$T' = \frac{\lambda_m}{\lambda_m'} \times 6050 = \frac{4753}{9506} \times 6050 = 3025 \text{ K}$$

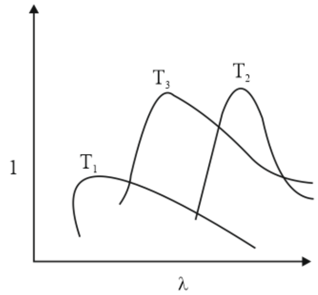
4. Two stars emit maximum radiation at wavelength 3600 Å and 4800 Å respectively. The ratio of their temperature is

(a) 1 : 2 **(b) 2 : 1**

(c) 3 : 4 **(d) 4 : 3**

Solution:(d)

5. The plots of intensity versus wavelength for three black bodies at temperature T_1, T_2 and T_3 respectively are as shown. Their temperature are such that –

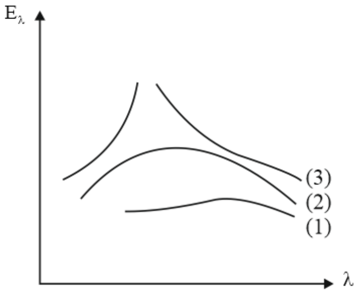


(a) $T_1 > T_2 > T_3$ **(b) $T_1 > T_3 > T_2$**

(c) $T_2 > T_3 > T_1$ **(d) $T_3 > T_2 > T_1$**

Solution:(b)

6. Graph of spectral radiancy (E_λ) Vs Wavelength (λ) is shown in figure. Which of the following combination is correct



(a) 1-Sun, 2-Tungsten wire, 3- welding arc

(b) 1-Tungsten wire, 2-welding arc, 3-sun

(c) 1-Sun, 2-welding arc, 3-Tungsten wire

(d) 1-welding arc, 2-Tungsten wire, 3-Sun

Solution: (b)

7. Three discs A, B and C having radii 2 m, 4 m and 6 m respectively are coated with carbon black on their outer surfaces. The wavelength corresponding to maximum intensity are 300 nm, 400 nm and 500 nm respectively.

The power radiated by them are Q_A, Q_B and Q_C respectively

(a) Q_A is maximum **(b) Q_B is maximum**

(c) Q_C is maximum (d) $Q_A = Q_B = Q_C$

Solution: (b)

8. If wavelengths of maximum intensity of radiations emitted by the sun and the moon are 0.5×10^{-6} and 10^{-4} m respectively, the ratio of their temperatures is

(a) 1/100 (b) 1/200

(c) 100 (d) 200

Solution:

The correct answer is D.

We know that $\lambda T = constant$

$$\Rightarrow \frac{T_{sun}}{T_{moon}} = \frac{\lambda_{moon}}{\lambda_{sun}} = \frac{10^{-4}}{0.5 \times 10^{-6}}$$

$$\Rightarrow \frac{T_{sun}}{T_{moon}} = 200$$

9. The wavelength of radiation emitted by a body depends upon

(a) The nature of its surface

(b) The area of its surface

(c) The temperature of its surface

(d) All the above factors

Solution:

The correct answer is C.

According to Wien's displacement law as temperature of body increases, the maximum intensity of emission shifts towards shorter wavelength.

$\lambda_m T = b\ (constant)$

Where λ_m is the wavelength at which maximum emission takes place at absolute temperature T.

Expert Weins Displacement Law

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1. The wavelength of maximum emitted energy of a body at 700 K is 4.08 µm. If the temperature of the body is raised to 1400 K, the wavelength of maximum emitted energy will be

(a) 1.02 µm (b) 16.32 µm

(c) 8.16 µm (d) 2.04 µm

Solution:

The correct answer is D.

$\lambda_{m1} T_1 = \lambda_{m2} T_2$

$$\Rightarrow \lambda_{m2} = \frac{\lambda_{m1} T_1}{T_2} = 4.08 \times \frac{700}{1400} = 2.04 \mu m$$

2. Two stars emit maximum radiation at wavelength 3600 Å and 4800 Å respectively. The ratio of their temperatures is

(a) 1 : 2 (b) 3 : 4

(c) 4 : 3 (d) 2 : 1

Solution:

The correct answer is C.

By Wein's displacement law

At maximum radiation λT = constant

$\Rightarrow \lambda_1 T_1 = \lambda_2 T_2$

$$\Rightarrow \frac{T_1}{T_2} = \frac{\lambda_2}{\lambda_1}$$

$$Ratio\ of\ temperatures = \frac{T_1}{T_2} = \frac{4}{3}$$

3. Three stars A, B, C have surface temperatures T_A, T_B, T_C, respectively. Star A appears bluish, start B appears reddish and star C yellowish. Hence

(a) $T_A > T_B > T_C$ (b) $T_B > T_C > T_A$

(c) $T_C > T_B > T_A$ (d) $T_A > T_C > T_B$

Solution:

(d) According to Wein's displacement law,

$$\lambda = \frac{b}{T} \text{ i.e., } \lambda \propto \frac{1}{T} \qquad \text{... (i)}$$

We know that,

$\lambda_{bluish} < \lambda_{yellowish} < \lambda_{reddish}$

Hence, using Eq.(i), we have $T_A > T_C > T_B$

4. A black body is at a temperature of 5760 K. The energy of radiation emitted by the body at wavelength 250 nm is U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3. Wien's constant, b = 2.88×10^6 nmK. Which of the following is correct ?

(a) $U_3 = 0$ (b) $U_1 > U_2$

(c) $U_2 > U_1$ (d) $U_1 = 0$

Solution:

(c) Given, temperature, T_1 = 5760 K

Since, it is given that energy of radiation emitted by the body at wavelength 250 nm in U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3.

∵ According to Wien's law, we get

$\lambda_m T = b$

where, b =Wien's constant = 2.88×10^6 nmk

$$\lambda_m = \frac{b}{T}$$

$$\lambda_m = \frac{2.88 \times 10^6 nmK}{5760K}$$

λ_m = 500 nm

∵ λ_m = wavelength corresponding to maximum energy, so, $U_2 > U_1$.

5. A black body emits radiations of maximum

intensity at a wavelength of 5000 Å, when the temperature of the body is 1227°C . If the temperature of the body is increased by 1000℃ , the maximum intensity of emitted radiation would be observed at

(a) 2754.8 Å **(b) 3000 Å**

(c) 3500 Å **(d) 4000 Å**

Solution:

The correct answer is B.

By Wien's displacement law $\lambda_m T = b$ we have

$(5000)(1500) = (\lambda_m)(1500 + 1000)$

$\Rightarrow \lambda_m = \dfrac{(500)(1500)}{2500} = 3000A^0$

6. If the temperature of the sun becomes twice its present temperature, then

(a) Radiated energy would be predominantly in infrared

(b) Radiated energy would be predominantly in ultraviolet

(c) Radiated energy would be predominantly in X-ray region

(d) Radiated energy would become twice the present radiated energy

Solution:

The correct answer is B.

Radiate energy would be predominantly in ultraviolet.The wavelength corresponding to maximum emission of radiation from the sin is $\lambda_{maz} = 4753A^0$ (close to the wavelength of violet colour of visible region). Hence if temperature is doubled λ_m is decreased $\left(\lambda_m \propto \frac{1}{T}\right)$ i.e, mostly ultraviolet radiations emits.

7. A black body has maximum wavelength λ_m at temperature 2000 K. Its corresponding wavelength at temperature 3000 K will be

(a) $\frac{3}{2}\lambda_m$ **(b)** $\frac{2}{3}\lambda_m$

(c) $\frac{4}{9}\lambda_m$ **(d)** $\frac{9}{4}\lambda_m$

Solution:

The correct answer is B.

Applying Wien's displacement law.

$\lambda T = Constant$

$\lambda_{m1} T_1 = \lambda_{m2} T_2$

$\lambda_m \times 2000 = \lambda \times 3000$

$\lambda = \frac{2}{3}\lambda_m$

8. The intensity of radiation emitted by the sun has its maximum value at a wavelength of 510 nm and that emitted by the north star has the maximum value at 350 nm . If these stars behave like black bodies, then the ratio of the surface temperature of the sun and north star is

(a) 1.46 **(b) 0.69**

(c) 1.21 **(d) 0.83**

Solution:

The correct answer is B.

Using Wien's displacement law.

$\lambda T = Constant$

Given:

$\lambda_N = 350\ mm \quad \lambda_S = 510\ mm$

$\therefore \dfrac{T_S}{T_N} = \dfrac{\lambda_N}{\lambda_S}$

OR

$\dfrac{T_S}{T_N} = \dfrac{350}{510} = 0.69$

9. The power radiated by a black body is P and it radiates maximum energy at wavelength, λ_0. If the temperature of the black body is now changed, so that it radiates maximum energy at wavelength $\frac{3}{4}$ λ_0, the power radiated by it becomes nP. The value of n is

(a) $\frac{256}{81}$ **(b)** $\frac{4}{3}$

(c) $\frac{3}{4}$ **(d)** $\frac{81}{256}$

Solution:

(a) According to Wien's law,

$\lambda_{\max} \propto \dfrac{1}{T}$

i.e. $\lambda_{\max} T$ = constant

where, $\lambda_{\max}$ is the maximum wavelength of the radiation emitted at temperature T.

$\therefore \quad \lambda_{\max_1} T_1 = \lambda_{\max_2} T_2$

or $\quad \dfrac{T_1}{T_2} = \dfrac{\lambda_{\max_2}}{\lambda_{\max_1}}$ … (i)

Here, $\lambda_{\max_1} = \lambda_0$ and $\lambda_{\max_2} = \dfrac{3}{4}\lambda_0$

Substituting the above values in Eq. (i), we get

$$\frac{T_1}{T_2} = \frac{\frac{3}{4}\lambda_0}{\lambda_0} = \frac{3}{4}$$

Or $\frac{T_1}{T_2} = \frac{3}{4}$ … (ii)

As we know that, from Stefan's law, the power radiated by a body at temperature T is given as

$P = \sigma A e T^4$

i.e. $P \propto T^4$

($\because$ the quantity σAe is constant for a body)

$$\frac{P_1}{P_2} = \frac{T_1^4}{T_2^4} = \left(\frac{T_1}{T_2}\right)^4$$

From Eq. (i), we get

$$\frac{P_1}{P_2} = \left(\frac{3}{4}\right)^4 = \frac{81}{256}$$

Given, $P_1 = P$ and $P_2 = nP$

$$\frac{P_1}{P_2} = \frac{P}{nP} = \frac{81}{256}$$

or $n = \frac{256}{81}$

Test Yourself

Beginner Test - I

1. Is it correct to call heat as the energy in transit?
2. Can temperature on Celsius scale and kelvin scale related?
3. Why should a thermometer bulb have a small heat capacity?
4. Each side of a cube increases by 0.01% on heating. How much is the area of its faces and volume increased?
5. Find out the temperature which has same numerical value on celsius and fahrenheit scale.
6. Gas thermometers are more sensitive than mercury thermometers. Why?
7. A steel tape 1 m long is correctly calibrated for a temperature of 27.0 °C. The length of a steel rod measured by this tape is found to be 63.0 cm on a hot day when the temperature is 45.0 °C. What is the actual length of the steel rod on that day? What is the length of the same steel rod on a day when the temperature is 27.0°C? Coefficient of linear expansion of steel = $1.20 \times 10^{-5}\ °C^{-1}$?
8. There are two spheres of same radius and material at same temperature but one being solid while the other hollow. Which sphere will expand more if (i) they are heated to the same temperature (ii) same amount of heat is given to each of them?
9. Two identical rectangular strips-one of copper and the other of steel are riveted to form a bimetallic strip. What will happen on heating?
10. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K, respectively. Express these temperatures on the celsius and fahrenheit scales.
11. A metallic ball has a radius of 9.0 cm at 0°C. Calculate the change in its volume when it is heated to 90°C. Given that coefficient of linear expansion of metal of ball is $1.2 \times 10^{-5}\ K^{-1}$.
12. The brass scale of a barometer gives correct reading at 0°C. Coefficient of linear expansion of brass is 2.0×10^{-5} /°C. The barometer reads 75.00 cm at 27°C. What is the true atmospheric pressure at 27°C?
13. A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250°C, if the original length are at 40.0°C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand. Coefficient of linear expansion of brass = $2.0 \times 10^{-5}\ °C^{-1}$ and that of steel = $1.2 \times 10^{-5}\ °C^{-1}$
14. Show that the coefficient of volume expansion for a solid substance is three times its coefficient of linear expansion.
15. Black body radiation is white. Comment.
16. If all the objects radiate electromagnetic energy, why do not the objects around us in everyday life become colder and colder?
17. Can we boil water inside in the earth satellite?
18. Why an ice box is constructed with a double wall?
19. Two bodies at different temperatures T_1 and T_2, if brought in thermal contact do not necessarily settle at the mean temperature $\frac{(T_1+T_2)}{2}$. Why?
20. Place a safety pin on a sheet of paper. Hold the sheet over a burning candle, until the paper becomes yellow and charr. On removing the pin, its white trace is observed on the paper. Why?
21. Is it possible to convert water into vapour form without increasing its temperature, if temperature and pressure of water are 30 ℃ and 1 atm respectively?
22. Which object will cool faster when kept in open air, the one at 300 °C or the one of 100 °C? Why?
23. Two thermos flasks are of the same height and same capacity. One has a circular cross-section while the other has a square cross-section. Which of the two is better?

24. Given below are observations on molar specific heats at room temperature of some common gases.

Gas	**Molar specific heat (C_V) (cal mol^{-1} K^{-1})**
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monoatomic gases. (Typically, molar specific heat of a monoatomic gas is 2.92 cal/mol K). Explain this difference. What can you infer from the somewhat larger than the rest) value for chlorine?

25. Two vessels of different materials are identical in size and wall thickness. They are filled with equal quantities of ice at 0°C. If the ice melts completely in 10 and 25 min respectively, compare the coefficients of thermal conductivity of the materials of the vessels.
26. In a coal fire, the pockets formed by coals appear brighter than the coals themselves. Is the temperature of such a pocket higher than the surface temperature of a glowing coal?
27. Why rooms are provided with the ventilators near the roof?
28. If a drop of water falls on a very hot iron, it does not evaporate for a long time. Give reason.
29. How does tea in a thermo flask remain hot for a long time?
30. Two vessels A and B of different materials but having identical shape, size and wall thickness are filled with ice and kept at the same place. Ice melts at the rate of 100 g min^{-1} and 150g min^{-1} in A and B, respectively. Assuming that heat enters the vessels through the walls only, calculate the ratio of thermal conductivities of their materials.
31. Explain the following
    (i) Hot tea cools rapidly when poured into the saucer from the cup.
    (ii) Temperature of a hot liquid falls rapidly in the beginning but slowly afterwards.
    (iii) A hot liquid cools faster if outer surface of the container is blackened.
32. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500°C and then placed on a large ice block. What is the maximum amount of ice that can melt? (specific heat of copper = 0.39 Jg^{-1} °C^{-1}, and heat of fusion of water = 335 Jg^{-1}).
33. Two rods of the same area of cross-section, but of lengths l_1 and l_2 and conductivities K_1 and K_2 are joined in series. Show that the combination is equivalent of a material of conductivity

$$K = \frac{l_1 + l_2}{\left(\frac{l_1}{K_1}\right) + \left(\frac{l_2}{K_2}\right)}$$

34. A child running a temperature of 101°F is given an antipyretic (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98°F in 20 min. What is the average rate of extra evaporation caused by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water and latent heat of evaporation of water at that temperature is about 580 cal g^{-1}.
35. Explain why
    (i) a body with large reflectivity is a poor emitter.
    (ii) a brass tumbler feels much colder than a wooden tray on a chilly day.

(iii) an optical pyrometer (for measuring high temperature) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.
(iv) the earth without its atmosphere would be inhospitably cold.
(v) heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water.

36. Distinguish between conduction, convection and radiation.

Answer Key

Refer. Solutions

Beginner Test - II

1. The triple points of neon and carbon dioxide are 24.57 K and 216.55 K respectively. Express these temperatures on the Celsius and Fahrenheit scales.
2. Two absolute scales A and B have triple points of water defined to be 200 A and 350 B. What is the relation between T_A and T_B ?
3. The electrical resistance in ohms of a certain thermometer varies with temperature according to the approximate law:
   $R = R_0 [1 + \alpha (T - T_0)]$
   The resistance is 101.6 Ω at the triple-point of water 273.16 K, and 165.5 Ω at the normal melting point of lead (600.5 K). What is the temperature when the resistance is 123.4 Ω?
4. Answer the following:
   The triple-point of water is a standard fixed point in modern thermometry. Why? What is wrong in taking the melting point of ice and the boiling point of water as standard fixed points (as was originally done in the Celsius scale)?
   There were two fixed points in the original Celsius scale as mentioned above which were assigned the number 0 °C and 100 °C respectively. On the absolute scale, one of the fixed points is the triple-point of water, which on the Kelvin absolute scale is assigned the number 273.16 K. What is the other fixed point on this (Kelvin) scale?
   The absolute temperature (Kelvin scale) T is related to the temperature t_c on the Celsius scale by $t_c = T - 273.15$
   Why do we have 273.15 in this relation, and not 273.16?
   What is the temperature of the triple-point of water on an absolute scale whose unit interval size is equal to that of the Fahrenheit scale?
5. Two ideal gas thermometers A and B use oxygen and hydrogen respectively. The following observations are made

Temperature	Pressure thermometer A	Pressure thermometer B
Triple – point of water	1.250×10^5 Pa	0.200×10^5 Pa
Normal melting point of sulphur	1.797×10^5 Pa	0.287×10^5 Pa

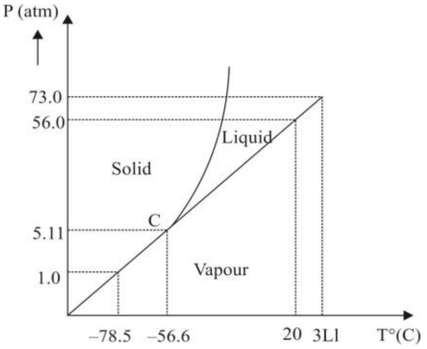
   What is the absolute temperature of normal melting point of sulphur as read by thermometers A and B?
   What do you think is the reason behind the slight difference in answers of thermometers A and B? (The thermometers are not faulty). What further procedure is needed in the experiment to reduce the discrepancy between the two readings?
6. A large steel wheel is to be fitted on to a shaft of the same material. At 27 °C, the outer diameter of the shaft is 8.70 cm and the diameter of the central hole in the wheel is 8.69 cm. The shaft is cooled using ‘dry ice’. At what temperature of the shaft does the wheel slip on the shaft? Assume coefficient of linear expansion of the steel to be constant over the required temperature range: αsteel = 1.20×10^{5} K^{-1} .

7. A hole is drilled in a copper sheet. The diameter of the hole is 4.24 cm at 27.0 °C. What is the change in the diameter of the hole when the sheet is heated to 227 °C? Coefficient of linear expansion of copper = 1.70×10^{-5} K^{-1} .
8. A brass wire 1.8 m long at 27 °C is held taut with little tension between two rigid supports. If the wire is cooled to a temperature of –39°C, what is the tension developed in the wire, if its diameter is 2.0 mm? Co-efficient of linear expansion of brass = 2.0×10^{-5} –1 K ; Young's modulus of brass = 0.91×10^{11} Pa.
9. A brass rod of length 50 cm and diameter 3.0 mm is joined to a steel rod of the same length and diameter. What is the change in length of the combined rod at 250 °C, if the original lengths are at 40.0 °C? Is there a 'thermal stress' developed at the junction? The ends of the rod are free to expand (Co-efficient of linear expansion of brass = 2.0×10^{-5} K^{-1} , steel = 1.2×10^{-5} K^{-1}).
10. The coefficient of volume expansion of glycerine is 49×10^{-5} K^{-1} . What is the fractional change in its density for a 30 °C rise in temperature?
11. A 10 kW drilling machine is used to drill a bore in a small aluminium block of mass 8.0 kg. How much is the rise in temperature of the block in 2.5 minutes, assuming 50% of power is used up in heating the machine itself or lost to the surroundings. Specific heat of aluminium = 0.91 J g-1 K-1.
12. A copper block of mass 2.5 kg is heated in a furnace to a temperature of 500 °C and then placed on a large ice block. What is the maximum amount of ice that can melt? (Specific heat of copper = 0.39 J g^{-1} K^{-1} ; heat of fusion of water = 335 J g^{-1}).
13. In an experiment on the specific heat of a metal, a 0.20 kg block of the metal at 150 °C is dropped in a copper calorimeter (of water equivalent 0.025 kg) containing 150 cm^3 of water at 27°C. The final temperature is 40°C. Compute the specific heat of the metal. If heat losses to the surroundings are not negligible, is your answer greater or smaller than the actual value for specific heat of the metal?
14. Given below are observations on molar specific heats at room temperature of some common gases.

Gas	Molar specific heat (Cv) (cal mol^{-1} K^{-1})
Hydrogen	4.87
Nitrogen	4.97
Oxygen	5.02
Nitric oxide	4.99
Carbon monoxide	5.01
Chlorine	6.17

The measured molar specific heats of these gases are markedly different from those for monatomic gases. Typically, molar specific heat of a monatomic gas is 2.92 cal/mol K. Explain this difference. What can you infer from the somewhat larger (than the rest) value for chlorine?

15. Answer the following questions based on the P-T phase diagram of carbon dioxide:



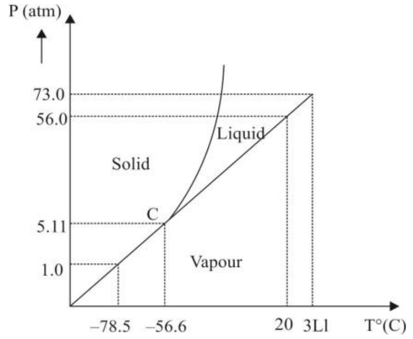
(a)At what temperature and pressure can the solid, liquid and vapour phases of CO2 coexist in equilibrium?

(b)What is the effect of decrease of pressure on the fusion and boiling point of CO2?

(c) What are the critical temperature and pressure for CO_2 What is their significance?

(d)Is CO_2 solid, liquid or gas at (a) –70 °C under 1 atm, (b) –60 °C under 10 atm, (c) 15 °C under 56 atm?

16. Answer the following questions based on the P–T phase diagram of CO_2:



(a)CO_2 at 1 atm pressure and temperature – 60 °C is compressed isothermally. Does it go through a liquid phase?

(b)What happens when CO_2 at 4 atm pressure is cooled from room temperature at constant pressure?

(c) Describe qualitatively the changes in a given mass of solid CO_2 at 10 atm pressure and temperature –65 °C as it is heated up to room temperature at constant pressure.

(d)CO_2 is heated to a temperature 70 °C and compressed isothermally. What changes in its properties do you expect to observe?

17. A child running a temperature of 101°F is given an antipyrin (i.e. a medicine that lowers fever) which causes an increase in the rate of evaporation of sweat from his body. If the fever is brought down to 98 °F in 20 min, what is the average rate of extra evaporation caused, by the drug? Assume the evaporation mechanism to be the only way by which heat is lost. The mass of the child is 30 kg. The specific heat of human body is approximately the same as that of water and latent heat of evaporation of water at that temperature is about 580 cal g^{-1} .

18. A 'thermacole' icebox is a cheap and efficient method for storing small quantities of cooked food in summer in particular. A cubical icebox of side 30 cm has a thickness of 5.0 cm. If 4.0 kg of ice is put in the box, estimate the amount of ice remaining after 6 h. The outside temperature is 45 °C, and co-efficient of thermal conductivity of thermacole is 0.01 J s^{-1} m^{-1} k^{-1}. [Heat of fusion of water = 335×10^3 J kg^{-1}]

19. A brass boiler has a base area of 0.15 m^2 and thickness 1.0 cm. It boils water at the rate of 6.0 kg/min when placed on a gas stove. Estimate the temperature of the part of the flame in contact with the boiler. Thermal conductivity of brass = 10^9 J s^{-1} m^{-1} k^{-1} ; Heat of vaporisation of water = 2256×10^3 J kg^{-1} .

20. Explain why:

(a)A body with large reflectivity is a poor emitter

(b)a brass tumbler feels much colder than a wooden tray on a chilly day

(c)an optical pyrometer (for measuring high temperatures) calibrated for an ideal black body radiation gives too low a value for the temperature of a red hot iron piece in the open, but gives a correct value for the temperature when the same piece is in the furnace.

(d) the earth without its atmosphere would be inhospitably cold

(e)heating systems based on circulation of steam are more efficient in warming a building than those based on circulation of hot water

21. A body cools from 80°C to 50°C in 5 minutes. Calculate the time it takes to cool from 60°C to 30°C. The temperature of the surroundings is 20 °C.

Answer Key

1. $T_C = -248.58°C$, $T_F = 415.44°F$, T_C= –56.60°C, $T_F = -69.88°F$
2. 4 : 7
3. T = 384.61 K
4. 491.69
5. 391.98 K
6. –68.95℃(−69℃)
7. 1.44×10^{-2} cm
8. –3.8 × 102 N
9. 0.346 cm
10. 1.47 × 10–2

11. 103°C
12. 1.45 kg
13. 0.43 J g^{-1} K^{-1}
14. 4.95 cal mol^{-1} K^{-}
15. NA
16. NA
17. 4.3 g/min
18. 3.687 kg
19. 237.98°C
20. NA
21. 10 minutes.

Expert Test - I

1. A copper rod of 88 cm and an aluminum rod of unknown length have their increase in length independent of increase in temperature. The length of aluminum rod is ($\alpha_{Cu} = 1.7 \times 10^{-5}\ K^{-1}$, $\alpha_{Al} = 2.2 \times 10^{-5}\ K^{-1}$)

(a) 113.9 cm
(b) 88 cm
(c) 68 cm
(d) 6.8 cm

2. Coefficient of linear expansion of brass and steel rods are α_1 and α_2 Lengths of brass and steel rods are l_1 and l_2 respectively. If $(l_2 - 1_1)$ is maintained same at all temperatures, which one of the following relations holds good?

(a) $\alpha_1 l_2^2 = \alpha_2 l_1^2$
(b) $\alpha_1^2 l_2 = \alpha_2^2 l_1$
(c) $\alpha_1 l_1 = \alpha_2 l_2$
(d) $\alpha_1 l_2 = \alpha_2 l_1$

3. The quantities of heat required to raise the temperature of two solid copper spheres of radii r_1 and r_2 ($r_1 = 1.5r_2$) through 1K are in the ratio

(a) $\frac{9}{4}$
(b) $\frac{3}{2}$
(c) $\frac{5}{3}$
(d) $\frac{27}{8}$

4. A piece of ice falls from a height h so that it melts completely. Only one-quarter of the heat produced is absorbed by the ice and all energy of ice gets converted into heat during its fall. The value of his [Latent heat of ice is 3.4×10^5 J/kg and g = 10N/kg]

(a) 544 km
(b) 136 km
(c) 68 km
(d) 34 km

5. Two identical bodies are made of a material for which the heat capacity increases with temperature. One of these is at 100°C, while the other one is at 0°C. If the two bodies are brought into contact, then assuming no heat loss, the final common temperature is

(a) 50°C
(b) more than 50°C
(c) less than 50°C but greater than 0°C
(d) 0°C

6. A cup of coffee cools from 90°C to 80°C in t minutes, when the room temperature is 20°C. The time taken by a similar cup of coffee to cool from 80°C to 60°C at a room temperature same at 20°C, is

(a) $\frac{13}{10}t$
(b) $\frac{13}{5}t$
(c) $\frac{10}{13}t$
(d) $\frac{5}{13}t$

7. Three stars A, B, C have surface temperatures T_A, T_B, T_C, respectively. Star A appears bluish, start B appears reddish and star C yellowish. Hence

(a) $T_A > T_B > T_C$
(b) $T_B > T_C > T_A$
(c) $T_C > T_B > T_A$
(d) $T_A > T_C > T_B$

8. An object kept in a large room having air temperature of 25°C takes 12 minutes to cool from 80°C to 70°C. The time taken to cool for the same object from 70°C to 60°C would be nearly

(a) 10 min
(b) 12 min
(c) 20 min
(d) 15 min

9. A deep rectangular pond of surface area A, containing water (density = ρ, specific heat capacity = s), is

located in a region where the outside air temperature is a steady value at the –26°C. The thickness of the frozen ice layer in this pond, at a certain instant is x. Taking the thermal conductivity of ice as K, and its specific latent heat of fusion as L, the rate of increase of the thickness of ice layer, at this instant would be given by

(a) 26K/ρ r(L–4s) (b) $26K/(\rho x^2 - L)$

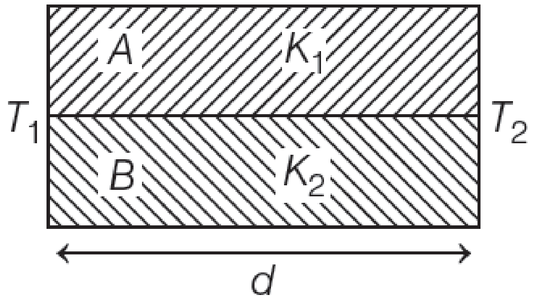
(c) 26K/(ρxL) (d) 26K/ρr(L + 4s)

10. The power radiated by a black body is P and it radiates maximum energy at wavelength, λ_0. If the temperature of the black body is now changed, so that it radiates maximum energy at wavelength $\frac{3}{4}\lambda_0$, the power radiated by it becomes nP. The value of n is

(a) $\frac{256}{81}$ (b) $\frac{4}{3}$

(c) $\frac{3}{4}$ (d) $\frac{81}{256}$

11. Two rods A and B of different materials are welded together as shown in figure. Their thermal conductivities are K_1 and K_2. The thermal conductivity of the composite rod will be



(a) $\frac{K_1+K_2}{2}$ (b) $\frac{3(K_1+K_2)}{2}$

(c) $K_1 + K_2$ (d) $2(K_1 + K_2)$

12. A spherical black body with a radius of 12 cm radiates 450 watt power at 500 K. If the radius were halved and the temperature doubled, the power radiated in watt would be

(a) 225 (b) 450

(c) 1000 (d) 1800

13. A black body is at a temperature of 5760 K. The energy of radiation emitted by the body at wavelength 250 nm is U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3. Wien's constant, $b = 2.88 \times 10^6$ nmK. Which of the following is correct ?

(a) $U_3 = 0$ (b) $U_1 > U_2$

(c) $U_2 > U_1$ (d) $U_1 = 0$

14. A body cools from a temperature 3T to 2T in 10 minutes. The room temperature is T. Assume that Newton's law of cooling is applicable. The temperature of the body at the end of next 10 minutes will be

(a) $\frac{7}{4}T$ (b) $\frac{3}{2}T$

(c) $\frac{4}{3}T$ (d) T

15. A piece of iron is heated in a flame. If first becomes dull red then becomes reddish yellow and finally turns to white hot. The correct explanation for the above observation is possible by using

(a) Stefan's law (b) Wien's displacement law

(c) Kirchhoff's law (d) Newton's law of cooling

Answer Key

1. (c) 2. (c) 3. (d) 4. (b)
5. (b) 6. (b) 7. (d) 8. (d)
9. (c) 10. (a) 11. (a) 12. (d)
13. (c) 14. (b) 15. (b)

Expert Test - II

1. Two different wires having lengths L_1 and L_2 and respective temperature coefficient of linear expansion α_1 and α_2 are joined end-to-end. Then the effective temperature coefficient of linear expansion is:

(a) $\frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$
(b) $2\sqrt{\alpha_1\alpha_2}$
(c) $\frac{\alpha_1 + \alpha_2}{2}$
(d) $4\frac{\alpha_1\alpha_2}{\alpha_1 + \alpha_2}\frac{L_2 L_1}{\left(L_2 + L_1\right)^2}$

2. When the temperature of a metal wire is increased from 0°C to 10°C, its length increased by 0.02%. The percentage change in its mass density will be closest to :

(a) 0.06
(b) 2.3
(c) 0.008
(d) 0.8

3. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass, M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C it regains its original length of 0.2 m. The value of M is close to: (Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2, respectively; $g = 10\ \text{ms}^{-2}$)

(a) 9 kg
(b) 0.5 kg
(c) 1.5 kg
(d) 0.9 kg

4. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then the new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4:3, then the value of T is :

(a) 230°C
(b) 270°C
(c) 200°C
(d) 250°C

5. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y, for this metal is:

(a) $\frac{F}{A\alpha\Delta T}$
(b) $\frac{F}{A\alpha\left(\Delta T - 273\right)}$
(c) $\frac{F}{2A\alpha\Delta T}$
(d) $\frac{2F}{A\alpha\Delta T}$

6. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by:

(a) $\frac{3\alpha}{PK}$
(b) $3PK\alpha$
(c) $\frac{P}{3\alpha K}$
(d) $\frac{P}{\alpha K}$

7. A compressive force, F is applied at the two ends of a long thin steel rod. It is heated, simultaneously, such that

its temperature increases by ΔT. The net change in its length is zero. Let l be the length of the rod, A its area of cross-section, Y its Young's modulus, and α its coefficient of linear expansion. Then, F is equal to :

(a) $l^2Y\alpha\,\Delta T$ (b) $lA\,Y\alpha\,\Delta T$

(c) $AY\alpha\,\Delta T$ (b) $\frac{AY}{\alpha\Delta T}$

8. On a linear temperature scale Y, water freezes at – 160° Y and boils at – 50°Y. On this Y scale, a temperature of 340K would be read as : (water freezes at 273 K and boils at 373 K)

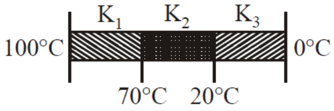
(a) – 73.7° Y (b) – 233.7° Y

(c) – 86.3°Y (d) – 106.3° Y

9. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}\ °C^{-1}$ and $\gamma_{(mercury)} = 1.5 \times 10^{-4}\ °C^{-1}$, where γ is the coefficient of volume expansion, then V_m (in cc) is close to ______________.

10. A non-isotropic solid metal cube has coefficients of linear expansion as: 5×10^{-5}/°C along the x-axis and 5×10^{-6}/ °C along the y and the z-axis. If the coefficient of volume expansion of the solid is $C \times 10^{-6}$/°C then the value of C is ___________.

11. Three rods of identical cross-section and lengths are made of three different materials of thermal conductivity K_1, K_2 and K_3, respectively. They are joined together at their ends to make a long rod (see figure). One end of the long rod is maintained at 100°C and the other at 0°C (see figure). If the joints of the rod are at 70°C and 20°C in steady state and there is no loss of energy from the surface of the rod, the correct relationship between K_1, K_2 and K_3 is:



(a) $K_1 : K_3 = 2 : 3$, $K_1 < K_3 = 2 : 5$ (b) $K_1 < K_2 < K_3$

(C) $K_1 : K_2 = 5 : 2$, $K_1 : K_3 = 3 : 5$ (d) $K_1 > K_2 > K_3$

12. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetics energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal (g – °C)(1 cal = 4.2×10^7 ergs) close to:

(a) 87.5°C (b) 83.3°C

(c) 119.2°C (d) 38.4°C

13. The specific heat of water = 4200 $Jkg^{-1}\ K^{-1}$ and the latent heat of ice = $3.4 \times 10^5\ J\ kg^{-1}$. 100 grams of ice at 0°C is placed in 200 g of water at 25°C. The amount of ice that will melt as the temperature of water reaches 0°C is close to (in grams):

(a) 61.7 (b) 63.8

(c) 69.3 (d) 64.6

14. A calorimeter of water equivalent 20 g contains 180 g of water at 25°C. 'm' grams of steam at 100°C is mixed in it till the temperature of the mixture is 31°C. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal $g^{-1}\ °C^{-1}$)

(a) 2 (b) 4

(c) 3.2 (d) 2.6

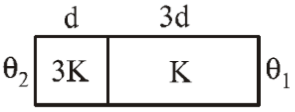
15. M grams of steam at 100°C is mixed with 200 g of ice at its melting point in a thermally insulated container. If it produces liquid water at 40°C [heat of vaporization of water is 540 cal/g and heat of fusion of ice is 80 cal/g], the value of M is ______________.

16. When M_1 gram of ice at –10°C (Specific heat = 0.5 cal $g^{-1}°C^{-1}$) is added to M_2 gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is:

(a) $\frac{50M_2}{M_1}-5$ (b) $\frac{5M_1}{M_2}-50$

(c) $\frac{50M_2}{M_1}$ (d) $\frac{5M_2}{M_1}-5$

17. Two materials having coefficients of thermal conductivity '3K' and 'K' and thickness 'd' and 3d', respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is:



(a) $\frac{\theta_1}{10}+\frac{9\theta_2}{10}$ (b) $\frac{\theta_2+\theta_1}{2}$

(c) $\frac{\theta_1}{6}+\frac{5\theta_2}{6}$ (d) $\frac{\theta_1}{3}+\frac{2\theta_2}{3}$

18. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius 2R. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is:

(a) $\frac{K_1+K_2}{2}$ (b) $K_1 + K_2$

(c) $\frac{2K_1+3K_2}{5}$ (d) $\frac{K_1+3K_2}{4}$

19. Ice at –20°C is added to 50 g of water at 40°C, When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to

(Specific heat of water = 4.2J/g/°C
Specific heat of Ice = 2.1 J/g/°C
Heat of fusion of water at 0°C = 334J/g)

(a) 50g (b) 100 g

(c) 60 g (d) 40 g

20. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C, will be:

(a) 85°C (b) 60°C

(c) 80°C (d) 70°C

21. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is 0.1 $WK^{-1} m^{-1}$, the energy flux through it in the steady state is:

(a) 90 Wm^{-2} (b) 120 Wm^{-2}

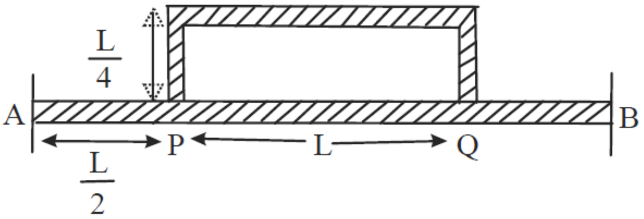
(c) 65 Wm^{-2} (d) 200 Wm^{-2}

22. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal if water temperature stabilizes at 21.5°C. (Specific heat of brass is 394 J kg^{-1} K^{-1})

(a) 458 J kg^{-1} K^{-1} (b) 1232 J kg^{-1} K^{-1}

(c) 916 J kg^{-1} K^{-1} (d) 654 J kg^{-1} K^{-1}

23. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length 2L. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$, is connected across AB (See figure). In steady state, temperature difference between P and will be close to:



(a) 45°C (b) 75°C
(c) 60°C (d) 35°C

24. In an experiment a sphere of aluminium of mass 0.20 kg is heated upto 150°C. Immediately, it is put into water of volume 150 cc at 27°C kept in a calorimeter of water equivalent to 0.025 kg. Final temperature of the system is 40°C. The specific heat of aluminium is : (take 4.2 Joule = 1 calorie)

(a) 378 J/kg – °C (b) 315J/kg – °C
(c) 476J/kg – °C (d) 434 J/kg – °C

25. A copper ball of mass 100 gm is at a temperature T. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is given by (Given : room temperature = 30° C, specific heat of copper=0.1 cal/gm °C

(a) 1250°C (b) 825°C
(c) 800°C (d) 885°C

26. An experiment takes 10 minutes to raise the temperature of water in a container from 0°C to 100°C and another 55 minutes to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of water to be 1 cal/g °C, the heat of vapourization according to this experiment will come out to be :

(a) 560 cal/g (b) 550 cal/ g
(c) 540 cal/g (d) 530 cal/ g

27. Three rods of Copper, Brass and Steel are welded together to form a Y shaped structure. Area of cross - section of each rod = 4 cm^2. End of copper rod is maintained at 100°C where as ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cms respectively. The rods are thermally insulated from surroundings excepts at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units respectively. Rate of heat flow through copper rod is:

(a) 1.2 cal/s (b) 2.4 cal/s
(c) 4.8 cal/s (d) 6.0 cal/s

28. A black coloured solid sphere of radius R and mass M is inside a cavity with vacuum inside. The walls of the cavity are maintained at temperature T0. The initial temperature of the sphere is 3T0. If the specific heat of the material of the sphere varies as αT3 per unit mass with the temperature T of the sphere, where α is a constant, then the time taken for the sphere to cool down to temperature 2T0 will be (σ is Stefan Boltzmann constant)

(a) $\frac{M\alpha}{4\pi R^2\sigma}\ln\left(\frac{3}{2}\right)$ (b) $\frac{M\alpha}{4\pi R^2\sigma}\ln\left(\frac{16}{3}\right)$

(c) $\frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{16}{3}\right)$ (d) $\frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{3}{2}\right)$

29. Assume that a drop of liquid evaporates by decrease in its surface energy , so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

(a) ρL/T (b) $\sqrt{T / \rho L}$

(c) T/ρL (d) 2T/ρL

30. A mass of 50g of water in a closed vessel, with surroundings at a constant temperature takes 2 minutes to cool from 30°C to 25°C. A mass of 100g of another liquid in an analytic vessel with identical surroundings takes the same time to cool from 30° C to 25° C. The specific heat of the liquid is : (The water equivalent of the vessel is 30g.)

(a) 2.0 kcal/kg (b) 7 kcal/kg

(c) 3 kcal/kg (d) 0.5 kcal/kg

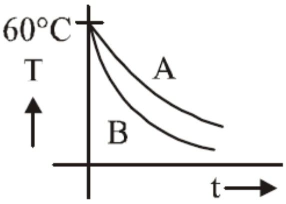
31. A metallic sphere cools from 50°C to 40°C in 300 s. If atmospheric temperature around is 20°C, then the sphere's temperature after the next 5 minutes will be close to :

(a) 31°C (b) 33°C

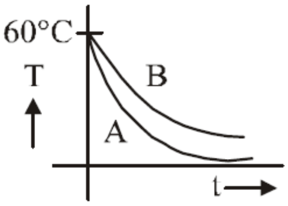
(c) 28°C (d) 35°C

32. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of 8×10^2 kg/m^3 and specific heat of 2000 J kg^{-1} K^{-1} while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1} K^{-1}. Which of the following best describes their temperature versus time graph schematically? (assume the emissivity of both the beakers to be the same)

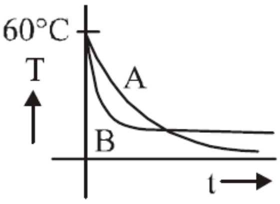
(a)



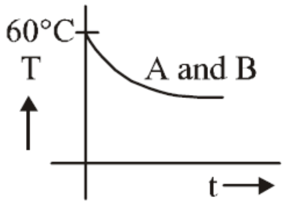
(b)



(c)



(d)



33. A body takes 10 minutes to cool from 60°C to 50°C. The temperature of surroundings is constant at 25°C. Then, the temperature of the body after next 10 minutes will be approximately

(a) 43°C (b) 47°C

(c) 41°C (d) 45°C

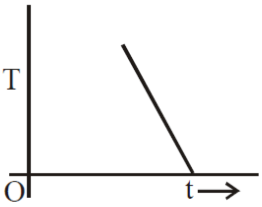
34. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is:

(a) 25°C (b) 10°C

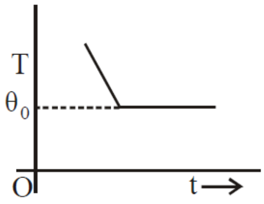
(c) 15°C (d) 20°C

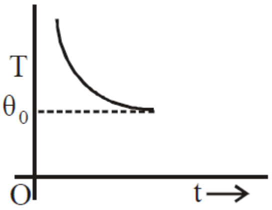
35. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closest to

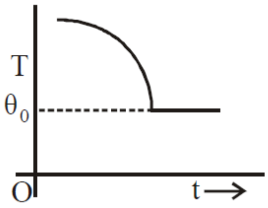
(a)



(b)



(c)


(d)


Answer Key

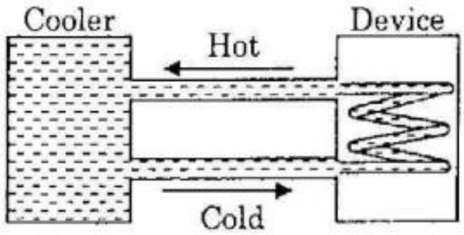
1. (a)	2. (a)	3. (Bonus)	4. (a)
5. (a)	6. (c)	7. (c)	8. (c)
9. (20.00)	10. (60.00)	11. (a)	12. (a)
13. (a)	14. (a)	15. (40)	16. (a)
17. (a)	18. (d)	19. (d)	20. (c)
21. (a)	22. (c)	23. (a)	24. (d)
25. (d)	26. (b)	27. (c)	28. (c)
29. (d)	30. (d)	31. (b)	32. (b)
33. (a)	34. (b)	35. (c)	

Pro Test - I

1. The ends Q and R of two thin wires, PQ and RS, are soldered (joined) together. Initially each of the wires has a length of 1 m at 10°C. Now the end P is maintained at 10°C, while the end S is heated and maintained at 400 °C. The system is thermally insulated from its surroundings. If the thermal conductivity of wire PQ is twice that of the wire RS and the coefficient of linear thermal expansion of PQ is $1.2 \times 10^{-5}\mathrm{K}^{-1}$, the change in length of the wire PQ is

(a) 0.78mm
(b) 0.90 mm
(c) 1.56 mm
(d) 2.34 mm

2. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is (Specific heat of water is 4.2 kj kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3}

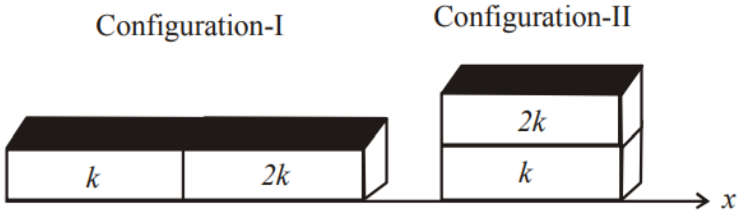


(a) 1600
(b) 2067
(c) 2533
(d) 3933

3. Parallel rays of light of intensity $I = 912\ \mathrm{Wm}^{-2}$ are incident on a spherical black body kept in surroundingsof temperature 300 K. Take Stefan-Boltzmann constant $\sigma = 5.7 \times 10^{-8}\mathrm{Wm}^{-2}\mathrm{K}^{-4}$ and assume that the energy exchange with the surroundings is only through radiation. The final steady state temperature of the black body is close to

(a) 330K
(b) 660 K
(c) 990 K
(d) 1550K

4. Two rectangular blocks, having identical dimensions, can be arranged either in configuration-I or in configuration-II as shown in the figure. One of the blocks has thermal conductivity k and the other $2k$. The temperature difference between the ends along the x-axis is the same in both the configurations. It takes 9 s to transport a certain amount of heat from the hot end to the cold end in the configuration-I. The time to transport the same amount of heat in the configuration-II is **[Adv. 2013]**



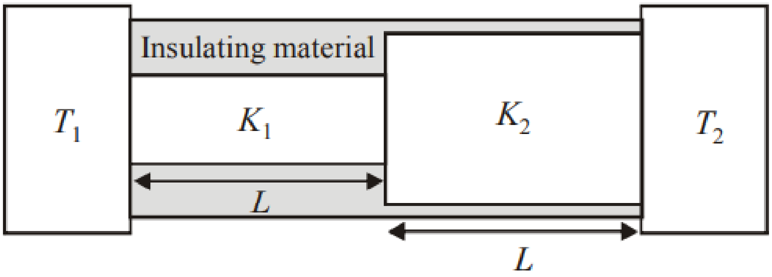
(a) 2.0s (b) 4.5s
(c) 3.0s (d) 6.0s

5. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2$, (P/P_0), where P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

6. Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 104 times the power emitted from B. The ratio $\frac{\lambda_A}{\lambda_B}$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

7. A liquid at 30°C is poured very slowly into Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5gm of the liquid completely evaporates. After pouring another 80gm of the liquid to its specific heat will be ________________°C. [Neglect the heat exchange with surrounding]

8. Two conducing cylinders of equal length but different radii are connected in series between two heat baths kept at temperatures $T_1 = 300\text{K}$ and $T_2 = 100\text{K}$, as shown in the figure. The radius of the bigger cylinder is twice that of the smaller one and the thermal conductivities of the materials of the smaller and the larger cylinders are K_1 and K_2 respectively. If the temperature at the junction of the two cylinders in the steady state is 200 K, then $K_1/K_2 =$ _____________________.



9. A current carrying wire heats a metal rod. The wire provides a constant power P to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) is the metal rod changes with time (t) as $T(t) = T_0(1 + \beta t^{1/4})$ Where β is a constant with appropriate dimensions while T_0 is a constant with dimensions of temperature. The heat capacity of metal is:

(a) $\frac{4P(T(t)-T_0)}{\beta^4 T_0^2}$ (b) $\frac{4P(T(t)-T_0)^2}{\beta^4 T_0^3}$
(c) $\frac{4P(T(t)-T_0)^4}{\beta^4 T_0^5}$ (d) $\frac{4P(T(t)-T_0)^3}{\beta^4 T_0^4}$

10. A container with 1 kg of water in it is kept in sunlight, which causes the water to get warmer than the

surroundings. The average energy per unit time per unit area received due to the sunlight is 700 Wm^2 and it is absorbed by the water over an effective area of 0.05 m^2. Assuming that the heat loss from the water to the surroundings is governed by Newton's law of cooling, the difference (in °C) in the temperature of water and the surroundings after a long time will be_____________. (Ignore effect of the container, and take constant for Newton's law of cooling=0.001 s^{-1}, Heat capacity of water = 4200 J kg^{-1} K^{-1})

11. A human body has a surface area of approximately 1 m^2. The normal body temperature is 10 K above the surrounding room temperature T0. Take the room temperature to be T_0 = 300 K. For T_0 = 300 K, the value of $\sigma T_0^4 = 460 Wm^{-2}$ (where σ is the Stefan-Boltzmann constant). Which of the following options is/are correct?

(a)The amount of energy radiated by the body in 1 second is close to 60 joules.

(b)If the surrounding temperature reduces by a small amount $\Delta T_0 \ll T_0$, then to maintain the same body temperature the same (living) human being needs to radiate $\Delta W = 4\sigma T_0^3 \Delta T_0$, more energy per unit time

(c)Reducing the exposed surface area of the body (e.g. by curling up) allows humans to maintain the same body temperature while reducing the energy lost by radiation

(d)If the body temperature rises significantly then the peak in the spectrum of electromagnetic radiation emitted by the body would shift to longer wavelengths

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Answer Key

1. (a)	2. (b)	3. (a)	4. (a)
5. (9)	6. (2)	7. (270°C)	8. (4.00)
9. (d)	10. (8.33)	11. (c)	

Solution:
Test Papers

Solutions: Beginner Test - I

1.

Yes, it is perfect correct to call heat as the energy in transit because it is continuously flowing on account of temperature difference between bodies or parts of a system.

2.

$t(°C) = T(k) - 273.15$

or $T(K) = t(°C) + 273.15$

3.

The thermometer bulb having small heat capacity will absorb less heat from the body whose temperature is to be measured. Hence, the temperature of that body will practically remain unchanged.

4.

The area of the faces will increased by 0.02% and the volume by 0.03%.

5.

Let θ be the same numerical value of temperature on the both scales.

$\therefore \quad \frac{T_C}{5} = \frac{T_F - 32}{9}$

$\frac{\theta}{5} = \frac{\theta - 32}{9}$ $[\because \theta°\text{C} = \theta°\text{F} = \theta,$ given]

$9\theta = 5\theta - 160$

$-4\theta = 160$

$\therefore \theta = -40°$

$\theta = -40°\text{C} = -40°\text{F}$

6.

The coefficient of increase of pressure (or volume) of a gas is $\frac{1}{273.15}$ $°\text{C}^{-1}$. It is very large as compared to coefficient of expansion of mercury. Therefore, for a certain increase in temperature, increase in volume of the gas will be large compared to that of mercury and hence a gas thermometer is more sensitive.

7.

Here, $t_1 = 27°C, l_1 = 1\,m = 100cm,$

$t_2 = 45°C \ , l_2 = 63cm,$

$\alpha = 1.20 \times 10^{-5}\,°\text{C}^{-1}$

Length of the rod on the hot day is

$$l_2 = l_1\,[1 + \alpha(t_2 - t_1)]$$

$= \quad 100[1 + 1.20 \times 10 - 5\,(45 - 27)] = 100.0216\,cm$

As the steel tape has been calibrated for a temperature of 45°C, can be calculated as

$$L2 = \left(\frac{100.0216}{100}\right)63 = 63.0136cm$$

So, length of the steel rod at $27°C = 63\ cm.$

And length of rod at $45°C = 63.0136cm.$

8.

(i) As thermal expansion of isotropic solids is similar to true photographic enlargement, the expansion of a cavity is same as if it were a solid body of the same material i.e. $\Delta V = \gamma V\,\Delta T$. As here V, γ and ΔT are same for both solid and hollow spheres, so the expansions of both will be equal.

(ii) If same amount of heat is given to the two spheres, then due to lesser mass, rise in temperature of hollow sphere will be more (as $\Delta T = \frac{Q}{Mc}$) and hence the expansion will be more as

$\Delta V = \gamma V\,\Delta T.$

9.

The coefficient of linear expansion of copper is more than steel. On heating, the expansion in copper strip is more than the steel strip. The bimetallic strip will bend with steel strip on inner (concave) side.

10.

For neon Triple point $T = 24.57\ K$

$\therefore T_C = T(K) - 273.15$

$= 24.57 - 273.15 = -248.58°C$

$$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5} \times (-248.58) + 32$$

$$= -415.44°F$$

For carbon dioxide Triple point, $T = 216.55\ K$

$\therefore T_C = T(K) - 273.15$

$= 216.55 - 273.15 = -56.6°C$

$$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5} \times (-56.6) + 32$$

$$= -69.88°C$$

11.

As radius of ball, $r_O = 9.0\ cm = 0.090\ m$ at 0°C, hence its volume,

$$V_0 = \frac{4}{3}\pi r_0^3 = \frac{4}{3} \times 3.14 \times (0.090)^3$$

$$= 3.05 \times 10^{-3}\ m^3$$

Again as, $\alpha = 1.2 \times 10^{-5} K^{-1}$,

∴ $\gamma = 3\alpha = 3 \times 1.2 \times 10^{-5} = 3.6 \times 10^{-5} K^{-1}$

Moreover rise in temperature

$$\Delta T = 90°C - 0°C = 90°\,C = 90\,K$$

∴ Increase in volume, $\Delta V = V\gamma\Delta T$

$$= 3.05 \times 10^{-3} \times 3.6 \times 10^{-5} \times 90$$

$$= 9.88 \times 10^{-6}\,m^3 = 9.88\,cm^3$$

12.

As the brass scale of a barometer gives correct reading at $T_1 = 0°C$, hence at temperature $T_2 = 27°C$, the scale will expand and will not give correct reading.

In such a case, true value

= observed scale reading × $(1 + \alpha\Delta T)$

∴ True pressure

$= 75.00\,cm \times (1 + 2.0 \times 10^{-5} \times (27 - 0)]$

= 75 × (1 + 2.0 × 10^{-5} × 27)

$= 75.00\,(1 + 54 \times 10^{-5})cm$

$= 75.04\,cm$

13.

For brass rod.

$l = 50\,cm, t_1 = 40°C, t_2 = 250°C$

$\alpha = 2.0 \times 10^{-5}\,°C^{-1}$

Change in length of brass rod is

$\Delta l = \alpha l(t_2 - t_1)$

$= 2.0 \times 10^{-5} \times 50 \times (250 - 40) = 0.21\,cm$

For steel rod,

$l = 50\,cm, t_1, = 40°C, t_2 = 250°C,$

$\alpha = 1.2 \times 10^{-5}\,°C^{-1}$

Change in length of steel rod is

$\Delta l' = \alpha l(t_2 - t_1)$

$= 1.2 \times 10^{-5} \times 50 \times (250 - 40) = 0.13\,cm$

Change in length of the combined rod at 250°C

$= \Delta l + \Delta l' = 0.21 + 0.13 = 0.34\,cm$

As the rods expand freely, so no thermal stress is developed at the junction.

14.

Consider a solid in the form of a rectangular parallelopiped of sides a, b and c respectively so that its volume $V = abc$.

If the solid is heated so that its temperature rises by ΔT, then increase in its sides will be

$\Delta a = a\alpha\Delta T, \Delta b = b\alpha\Delta T\ and\ \Delta c = c\alpha\Delta T$

or $a' = a + \Delta a = a(1 + \alpha\Delta T)$

$b' = b + \Delta b = b(1 + \alpha\Delta T)$

and $c' = c + \Delta c = c(1 + \alpha\,\Delta T)$

∵ New volume, $V' = V + \Delta V = a'b'c'$

$= abc\,(1 + \alpha\Delta T)^3$

∴ Increase in volume,

$\Delta V = V' - V = [abc\,(1 + \alpha\Delta T)^3 - abc]$

∴ Coefficient of volume expansion,

$$\gamma = \frac{\Delta V}{V \cdot \Delta T} = \frac{abc\left(1+\alpha \cdot \Delta T\right)^3 - abc}{abc \cdot \Delta T}$$

$$\therefore \quad \gamma = \frac{\left(1+\alpha \cdot \Delta T\right)^3 - 1}{\Delta T}$$

$$= \frac{\left(1+3\alpha \cdot \Delta T + 3\alpha^2 \cdot \Delta T^2 + \alpha^3 \cdot \Delta T^3\right) - 1}{\Delta T}$$

$$= 3\alpha + 3\alpha^2\,\Delta T + \alpha^3\,\Delta T^2$$

However, as α has an extremely small value for solids, hence terms containing higher powers of α may be neglected. Therefore, we obtain the relation $\gamma = 3\alpha$

i.e. coefficient of volume expansion of a solid is three times of its coefficient of linear expansion.

15.

The statement is true. A black body absorbs radiations of all wavelengths. When heated to a suitable temperature, it emits radiations of all wavelengths. Hence, a black body radiation is white.

16.

According to the Principal of heat exchange, all the objects (above 0K) not only radiate electromagnetic energy but also absorb at the same rate from their surroundings. Thus, they do not become colder.

17.

No, the process of transfer of heat by convection is based on the fact that a liquid becomes lighter on becoming hot and rise up. In condition of weightlessness, this is not possible. So, transfer of heat by convection is not possible in the earth satellite.

18.

An ice box is made of double wall and the space in between the walls is filled with some non-conducting material to provide heat insulation, so that the loss of heat can be minimised.

19.

The two bodies may have different masses and different materials i.e. they may have different thermal capacities. In case the two bodies have equal thermal capacities, they would settle at the mean temperature $\frac{T_1+T_2}{2}$.

20.

The safety pin is made of steel which is good conductor of heat. So, the safety pin takes heat from the paper under it and transfer it away to the surroundings. The portion of the paper under the safety pin remains comparatively colder than the remaining part.

21.

Yes, water at $30°C$ can be converted into vapour by reducing its pressure until it equals to the vapour pressure of water at 30°C.

22.

The object at 300°C will cool faster than the object at $100°C$. This is in accordance with Newton's law of cooling.

As we know, rate cooling of an object ∝ temperature between the object and its surroundings.

23.

As both flasks have same height and capacity, the area of the cylindrical wall will be less than that of the square wall. Hence, the thermos flask of circular cross-section will transmit less heat as compared to the thermos flask of square cross-section and it will be better.

24.

A monoatomic gas has three degrees of freedom, while a diatomic gas possesses five degrees of freedom.

Therefore, molar specific heat of a diatomic gas (at constant volume).

$$C_V=\frac{f}{2}R=\frac{5}{2}R=\frac{5}{2}\times\frac{8.31}{4.2} = 5\ cal\ mol^{-1}\ K^{-1}$$

In the given table, all the gases are diatomic gases and for all of them (except chlorine), the value of C_V is about 5 $cal\ mol^{-1}\ K^{-1}$.

The slightly higher value of Cy for chlorine is due to the fact that even at room temperature, a chlorine gas molecule possesses the vibrational mode of motion also.

25.

Let K_1 and K_2 be the coefficients of thermal conductivity of the materials and t_1 and t_2 be the times in which ice melts in the two vessels.

As the same quantity of ice melts in the two vessels, the quantity of heat flowed into the vessels must be same.

$$\because\ Q=\frac{K_1A(T_1-T_2)t_1}{x}=\frac{K_2A(T_1-T_2)t_2}{x}$$

$$\Rightarrow\ K_1t_2\ =\ K_2t_2$$

$$\therefore\quad \frac{K_1}{K_2}=\frac{t_2}{t_1}=\frac{25\,\text{min}}{10\,\text{min}}=5:2$$

26.

The temperature of pockets formed by coals are not appreciably different from the surface temperature of glowing coals.

However, the pockets formed by coals act as cavities. The radiations from these cavities are black body radiations and so have maximum intensity. Hence, the pockets appear brighter than the glowing coals.

27.

It is done so to remove the harmful impure air and to replace it by the cool fresh air. The air we breadth out is warm and so it is lighter. It rises upwards and can go out through the ventilator provided near the roof.

The cold fresh air from outside enters the room through the doors and windows. Thus, the convection current is set up in the air.

28.

When a drop of water falls on a very hot iron, it gets insulated from the iron by a layer of poor conducting water vapour. As the heal is conducted very slowly through this layer, it takes quite long for the drop to evaporate. But if the drop of water falls on iron which is not very hot, then it comes in direct contact with iron and evaporates immediately.

29.

The air between the two walls of the Thermos-flask is evacuated. This prevents heat loss due to conduction and convection.

The loss of heat due to radiation is minimised by silvering the inside surface of the double wall. As the loss of heat due to the three processes is minimised and the tea remains hot for a long time.

30.

Let m_1 and m_2 be the masses of ice melted in same time ($t = 1\,min$) in vessels A and B, respectively.

Then, the amounts of heat flowed into the two vessels will be

$$Q_1 = \frac{K_1 A(T_1 - T_2)t}{x} = m_1 L \qquad \text{... (i)}$$

$$Q_2 = \frac{K_2 A(T_1 - T_2)t}{x} = m_2 L \qquad \text{... (ii)}$$

Where, L is latent heat of ice.

Dividing Eq. (i) by Eq. (ii), we get,

$$\frac{K_1}{K_2} = \frac{m_1}{m_2} = \frac{100g}{150g} = \frac{2}{3} = 2:3$$

31.

(i) As surface area increases on pouring hot tea in saucer from the cup and the rate of loss of heat is directly proportional to surface area of the radiating surface, so the tea will cool faster in the saucer.

(ii) Temperature of a hot liquid falls exponentially in accordance with Newton's law of cooling. In other words, rate of cooling is directly proportional to the temperature difference between hot liquid and the surroundings. It is due to this reason that a hot liquid cools rapidly in the beginning but slowly afterwards.

(iii) When outer surface of container is blackened, the surface becomes good emitter of heat and so the hot liquid in it cools faster.

32.

Mass of copper block,

$M = 2.5\,kg = 2.5 \times 10^3\,g$

Specific heat of copper, $c = 0.39\,J\,g^{-1}\,°\text{C}^{-1}$

Fall in temperature, $\Delta T = 500 - 0 = 500°C$

Heat lost by copper block $= mc\,\Delta T$

$= 2.5 \times 10^3 \times 0.39 \times 500\,J$

Let mass of ice melted = $M\ gram$

Heat of fusion of ice, $L = 335\,Jg^{-1}$

Heat gained by ice $= ML = M \times 335\,J$

∵ Heat gained = Heat lost

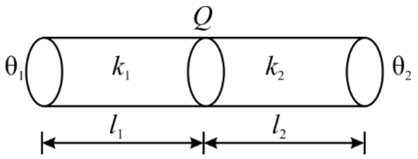
$\therefore M \times 335 = 2.5 \times 103 \times 0.39 \times 500$

or $\quad M = \dfrac{2.5 \times 10^3 \times 0.39 \times 500}{335}$

$= 1455.2\,g = 1.455\,kg$

33.

It is given that conductivities K_1 and K_2 are in series, so rate of flow of heat energy is same. But the sum of the difference in temperature is the difference across their free ends.



$\therefore \quad (\theta_1 - \theta) + (\theta - \theta_2) = (\theta_1 - \theta_2)$

$$\frac{\theta}{t}\frac{l_1}{k_1 A} + \frac{\theta}{t}\frac{l_2}{k_2 A} = \frac{\theta}{t}\frac{(l_1 + l_2)}{k_{eq} A}$$

$$\frac{l_1}{K_1} + \frac{l_2}{K_2} = \frac{l_1 + l_2}{K_{eq}}$$

$$\therefore \quad K_{eq} = \frac{l_1 + l_2}{\left(\dfrac{l_1}{K_1} + \dfrac{l_2}{K_2}\right)}$$

34.

Mass of child, M = 30 kg = 30×10^3 g

Fall in temperature, ΔT = 101 – 98 = 3°F = $3 \times \frac{5}{9} = \frac{5}{3}$ °C

Specific heat of human body,

c = specific heat of water = 1 cal g^{-1} $°C^{-1}$

Heat lost by child in the form of evaporation of sweat,

Q = Mc ΔT = $30 \times 10^3 \times 1 \times \frac{5}{3}$

= 50000 cal

If M' gram of sweat evaporates from the body of the child, then heat gained by sweat

Q = M'L = M' × 580 cal [∵ L = 580 calg^{-1}]

∵ Heat gained = Heat lost

M' × 580 = 50000 $\Rightarrow M' = \frac{50000}{580}$ = 86.2 g

Time taken by sweat to evaporate = 20 min

∴ Rate of evaporation of sweat = $\frac{86.2}{20}$ = 4.31 g min^{-1}

35.

(i) A body with large reflectivity is a poor absorber of heat. According to Kirchhoff's law, a poor absorber of heat is a poor emitter. Hence, a body with large reflectivity is a poor emitter.

(ii) Brass is a good conductor of heat. When a brass tumbler is touched, heat quickly flows from human body to tumble. Consequently, the tumbler appears colder. Wood is a bad conductor. So, heat does not flow from the human body to the tray in this case. Thus, it appears comparatively hotter.

(iii) Let T be the temperature of the hot iron in the furnace. Heat radiated per second per unit area,

$E = \sigma T4$

When the body is place in the open at temperature T$_0$, the heat radiated/second/unit area,

$$E' = \sigma(T^4 - T_o^4)$$

Clearly, $E' < E$. So, the optical pyrometer gives too low a value for the temperature in the open.

(iv) Heat radiated out by the earth is reflected back by the atmosphere. In the absence of atmosphere, at night all heat would escape from the earth's surface and thereby the earth's surface would be inhospitably cold. Also, atmosphere helps in maintaining the temperature through convection current.

(v) Though steam and boiling water are at the same temperature but each unit mass of steam contains a larger amount of additional heat called the latent heat. e.g. each gram of steam has 540 cal of more heat than each gram of boiling water. Hence, steam loses more heat than boiling water.

36.

	Conduction	**Convection**	**Radiation**
1.	It is the transfer of heat by direct physical contact.	It is the transfer of heat by the motion of a fluid	It is the transfer of heat by electromagnetic waves.
2.	It is due to temperature difference. Heat flows from high temperature region to low temperature region.	It is due to difference in density. Heat flows from low density region to high density region.	It occurs from all bodies at temperatures above 0 K.
3.	It occurs in solids through molecular collisions, without actual flow of matter.	It occurs in fluids by actual flow of matter.	It can take place at large distances and does not heat the intervening medium.
4.	It is a slow process.	It is also a slow process.	It propagates at the speed of light.
5.	It does not obey the laws of reflection and refraction.	It does not obey the laws of reflection and refraction.	It obeys the laws of reflection and refraction.

Solutions: Beginner Test - II

1.

Kelvin and Celsius scales are related as:

$T_C = T_K - 273.15$ … (i)

Celsius and Fahrenheit scales are related as:

$T_F = \frac{9}{5}T_C + 32$ ... (i)

For neon:

$T_K = 24.57\ K$

$\therefore T_C = 24.57 - 273.15 = -248.58°C$

$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(-248.58) + 3$

$= 415.44°F$

For carbon dioxide:

$T_K = 216.55\ K$

$\therefore T_C = 216.55 - 273.15 = -56.60°C$

$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(-56.60) + 32$

$= -69.88°F$

2.

Triple point of water on absolute scale A,

$T_1 = 200\ A$

Triple point of water on absolute scale B,

$T_2 = 350\ B$

Triple point of water on Kelvin scale,

$T\ K = 273.15\ K$

The temperature $273.15\ K$ on Kelvin scale is equivalent to $200\ A$ on absolute scale A.

$T_1 = T_K$

$200\ A = 273.15\ K$

$\therefore A = \frac{273.15}{200}$

The temperature 273.15 K on Kelvin scale is equivalent to 350 B on absolute scale B

$T_1 = T_K$

$350\ B = 273.15$

$\therefore B = \frac{273.15}{350}$

T_A is triple point of water on scale A.

T_B is triple point of water on scale B.

$$\therefore \frac{273.15}{200} \times T_A = \frac{273.15}{350} T_B$$

$$T_A = \frac{200}{350} T_B$$

Therefore, the ratio $T_A : T_B$ is given as 4 : 7.

3.

It is given that:

$R = R_0\ [1 + \alpha\ (T - T_0)]$ … (i)

Where,

R_0 and T_0 are the initial resistance and temperature

respectively R and T are the final resistance and temperature respectively α is a constant

At the triple point of water, $T_o = 273.15\ K$

Resistance of lead, $R_0 = 101.6\ \Omega$

At normal melting point of lead, $T = 600.5\ K$

Resistance of lead, $R = 165.5\ \Omega$

Substituting these values in equation (i), we get:

$R = R_0[1 + \alpha(T - T_O)]$

$165.5 = 101.6[1 + \alpha(600.5 - 273.15)]$

$165.5 = 101.6[1 + \alpha(327.35)]$

$$\therefore \alpha = \frac{0.629}{327.35} = 1.92 \times 10^{-3} K^{-1}$$

For resistance, $R_1 = 123.4\ \Omega$

$R_1 = R_0[1 + \alpha(T - T_0)]$

Where, T is the temperature when the resistance of lead is $123.4\ \Omega$

$123.4 = 101.6[1 + 1.92 \times 10^{-3}(T - 273.15)]$

$1.214 = 1 + 1.92 \times 10^{-3}(T - 273.15)$

$\frac{0.214}{1.92\times10^{-3}} = T - 273.15 \therefore T = 384.61\ K$

4.

The triple point of water has a unique value of $273.16\ K$. At particular values of volume and pressure, the triple point of water is always $273.16\ K$. The melting point of ice and boiling point of water do not have particular values because these points depend on pressure and temperature.

The absolute zero or 0 K is the other fixed point on the Kelvin absolute scale.

The temperature 273.16 K is the triple point of water. It is not the melting point of ice. The temperature 0°C on Celsius scale is the melting point of ice. Its corresponding value on Kelvin scale is 273.15 K.

Hence, absolute temperature (Kelvin scale) T, is related to temperature t_c, on Celsius scale as:

$t_c = T - 273.15$

Let T_F be the temperature on Fahrenheit scale and T_K be the temperature on absolute scale. Both the temperatures can be related as:

$$\frac{T_F - 32}{180} = \frac{T_K - 273.15}{100} \text{ ... (i)}$$

Let TT_{F1} be the temperature on Fahrenheit scale and T_{K1} be the temperature on absolute scale. Both the temperatures can be related as:

$$\frac{T_{F1} - 32}{180} = \frac{T_{K1} - 273.15}{100} \text{ ... (ii)}$$

It is given that:

$T_{K1} - T_K = 1\ K$

Subtracting equation (i) from equation (ii), we get:

$$\frac{T_{F1} - T_F}{180} = \frac{T_{K1} - T_K}{100} = \frac{1}{100}$$

$$T_{F1} - T_F = \frac{1 \times 180}{100} = \frac{9}{5}$$

Triple point of water $= 273.16\ K$

∴Triple point of water on absolute scale

$= 273.16 \times \frac{9}{5} = 491.69$

5.

Triple point of water, $T = 273.16\ K$.

At this temperature, pressure in thermometer A,

P_A = 1.250 × 10^5 Pa

At this temperature, pressure in thermometer A,

P_1 = 1.797 × 10^5 Pa

According to Charles' law, we have the relation:

$$\frac{P_A}{T} = \frac{P_1}{T_1}$$

$$\therefore T_1 = \frac{P_1 T}{P_A} = \frac{1.797 \times 10^5 \times 273.16}{1.250 \times 10^5}$$

$= 392.69\ K$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer A is 392.69 K.

At triple point 273.16 K, the pressure in thermometer B, P_B = 0.200 × 10^5Pa

At temperature T_1, the pressure in thermometer B, P_2= 0.287 × 10^5 Pa

According to Charles' law, we can write the relation:

$$\frac{P_B}{T} = \frac{P_1}{T_1}$$

$$\frac{0.200 \times 10^5}{273.16} = \frac{0.287 \times 10^5}{T_1}$$

$$\therefore T_1 = \frac{0.287 \times 10^5}{0.200 \times 10^5} \times 273.16 = 391.98\ K$$

Therefore, the absolute temperature of the normal melting point of sulphur as read by thermometer B is 391.98 K.

The oxygen and hydrogen gas present in thermometers A and B respectively are not perfect ideal gases. Hence, there is a slight difference between the readings of thermometers A and B.

To reduce the discrepancy between the two readings, the experiment should be carried under low pressure conditions. At low pressure, these gases behave as perfect ideal gases.

6.

The given temperature, $T = 27°C$ can be written in Kelvin as:

$27 + 273 = 300\ K$

Outer diameter of the steel shaft at T,

$d_1 = 8.70\ cm$

Diameter of the central hole in the wheel at T,

$d_2 = 8.69\ cm$

Coefficient of linear expansion of steel,

$\alpha_{\text{steel}} = 1.20 \times 10^{-5} K^{-1}$

After the shaft is cooled using 'dry ice', its temperature becomes T_1.

The wheel will slip on the shaft, if the change in diameter,

$\Delta d = 8.69 - 8.70 = -\ 0.01\ cm$

Temperature T_1, can be calculated from the relation:

$\Delta d = d1\alpha steel\ (T_1 - T)$

$= 8.70 \times 1.20 \times 10^{-5}\ (T_1 - 300)$

$(T_1 - 300) = -95.78$

$\therefore T_1 = 204.21\ K$

$= 204.21 - 273.16$

$= -68.95°C$

Therefore, the wheel will slip on the shaft when the temperature of the shaft is –69°C.

7.

Initial temperature, $T_1 = 27.0°C$

Diameter of the hole at T_1, $d_1 = 4.24\ cm$

Final temperature,$T_2 = 227°C$

Diameter of the hole at ,T_2 = d_2

Co-efficient of linear expansion of copper, α_{Cu}= 1.70 × $10^{-5} K^{-1}$.

For co-efficient of superficial expansion β and change in temperature ΔT, we have the relation:

$$\frac{Change\ in\ Area(\Delta A)}{Original\ area(A)} = \beta \Delta T$$

$$\frac{\left(\pi \frac{d_2^2}{4} - \pi \frac{d_1^2}{4}\right)}{\pi \frac{d_1^2}{4}} = \frac{\Delta A}{A}$$

$$\therefore \frac{\Delta A}{A} = \frac{d_2^2 - d_1^2}{d_1^2}$$

$$\frac{d_2^2}{d_1^2} - 1 = 2\alpha(T_2 - T_1)$$

$$\frac{d_2^2}{(4.24)^2} = 2 \times 1.7 \times 10^{-5}(227 - 27) + 1$$

$d_2^2 = 17.98 \times 1.0068 = 18.1$

$\therefore d_2 = 4.2544\ cm$

Change in diameter = $d_2 - d_1$

$= 4.2544 - 4.24 = 0.0144\ cm$

Hence, the diameter increases by 1.44 × 10^{-2} cm

8.

Initial temperature, $T_1 = 27^oC$

Length of the brass wire at T_1 is L=1.8m

Final temperature, $T_2 = 39^oC$

Diameter of the wire, d=2.0mm=2×10^{-3} m

Tension developed in the wire =F

Coefficient of linear expansion of brass =2.0×10^{-5}

Youngs modulus of brass, Y=0.91×10^{11} Pa

Youngs modulus is given by the relation:

Y= Stress / Strain

$$Y=\frac{F/A}{\Delta L/L} \quad \text{------------(i)}$$

$$\Delta L = FxL/(A\,x\,Y)$$

Where,

F= Tension developed in the wire

A= Area of cross-section of the wire.

ΔL= Change in the length, given by the relation:

$$\Delta L = \alpha L(T_2 - T_1) \quad \text{------------(ii)}$$

Equating equations (i) and (ii), we get:

$$\alpha L(T_2 - T_1) = \frac{FL}{\pi(d/2)^2 Y}$$

$$F = \alpha L(T_2 - T_1)Y\pi(d/2)^2$$

$$F = 2x10^{-5}x(-39-27)x3.14x0.91x10^{11}x(2x10^{-3}/2)^2$$

$$F = -3.8x10^2\ N$$

9.

Initial temperature, T_1 = 40°C

Final temperature, T_2 = 250°C

Change in temperature, $\Delta T = T_2 - T_1 = 210°C$

Length of the brass rod at T_2, l_1 = 50 cm

Diameter of the brass rod at T_1, d_1 = 3.0 mm

Length of the steel rod at T_2, l_2 = 50 cm

Diameter of the steel rod at T_2, d_2 = 3.0 mm

Coefficient of linear expansion of brass, α_1 =

Coefficient of linear expansion of steel, α_2= 1.2 $\times 10^{-5}K^{-1}$

For the expansion in the brass rod, we have:

$$\frac{Change\ in\ length(\Delta l_1)}{Original\ length(l_1)} = \alpha_1 \Delta T$$

$$\therefore \Delta l_1 = 50 \times (2.1 \times 10^{-5}) \times 210 = 0.2205\ cm$$

$$\Delta l_2 = 50 \times (1.2 \times 10^{-5}) \times 210 = 0.126\ cm$$

Total change in the lengths of brass and steel,

$\Delta l = \Delta l_1 + \Delta l_2$

$= 0.2205 + 0.126 = 0.346\ cm$

Total change in the length of the combined rod = $0.346\ cm$

Since the rod expands freely from both ends, no thermal stress is developed at the junction.

10.

Coefficient of volume expansion of glycerin, αV = $49 \times 10^{-5}K^{-1}$

Rise in temperature, $\Delta T = 30°C$

Fractional change in its volume = $\frac{\Delta V}{V}$

This change is related with the change in temperature as:

$$\frac{\Delta V}{V} = \alpha_V \Delta T$$

$$V_{T_2} - V_{T_1} = V_{T_1}\alpha_V \Delta T$$

$$\frac{m}{\rho_{T_2}} - \frac{m}{\rho_{T_1}} = \frac{m}{\rho_{T_1}}\alpha_V \Delta T$$

Where, m = Mass of Glycerine

$\rho_{T_1} = Initial\ density\ at\ T_1$

$\rho_{T_2} = Final\ density\ at\ T_2$

Where,

$$\frac{\rho_{T_1} - \rho_{T_2}}{\rho_{T_2}} = Fractional\ change\ in\ density$$

∴

$$Fractional\ change\ in\ density\ of\ glycerin = 49 \times 10^{-5} \times 30 = 1.47 \times 10^{-2}$$

11.

Power of the drilling machine,

$$P = 10\,kW = 10 \times 10^3\,W$$

Mass of the aluminum block,

$$m = 8.0\,kg = 8 \times 10^3\,g$$

Time for which the machine is used,

$$t = 2.5\,\text{min} = 2.5 \times 60 = 150\,s$$

Specific heat of aluminum, c = 0.91 J $g^{-1}K^{-1}$

Rise in the temperature of the block after drilling

$= \delta T$

Total energy of the drilling machine

$= Pt = 10 \times 10^3 \times 150 = 1.5 \times 10^6\,J$

It is given that only 50% of the power is useful.

$$Useful\ energy, \Delta Q = \frac{50}{100} \times 1.5 \times 10^6 = 7.5 \times 10^5 J$$

$$But\ \Delta Q = mc\Delta T$$

$$= \frac{7.5 \times 10^5}{8 \times 10^3 \times 0.91} = 103°C$$

Therefore, in 2.5 minutes of drilling, the rise in the temperature of the block is 103°C.

12.

Mass of the copper block,

$m = 2.5\, kg = 2500\, g$

Rise in the temperature of the copper block,

$\Delta\theta = 500°C$

Specific heat of copper, $C = 0.39\, J\, g^{-1}\, °C^{-1}$

Heat of fusion of water, L = 335 J g^{-1}

The maximum heat the copper block can lose,

$Q = mC\Delta\theta$

$= 2500 \times 0.39 \times 500 = 487500\, J$

Let m_1 g be the amount of ice that melts when the copper block is placed on the ice block.

$The\ heat\ gained\ by\ the\ melted\ ice, Q = m_1 L$

$\therefore m_1 = \frac{Q}{L} = \frac{487500}{335} = 1455.22\, g$

Hence, the maximum amount of ice that can melt is $1.45\, kg$.

13.

Mass of the metal, $m = 0.20\, kg = 200\, g$

Initial temperature of the metal, T_1 = 150°C

Final temperature of the metal, T_2 = 40°C

Calorimeter has water equivalent of mass, $m = 0.025\, kg = 25\, g$

Volume of water, V = 150 cm^3

Mass (M) of water at temperature T = 27°C:

$150 \times 1 = 150\, g$

Fall in the temperature of the metal:

ΔT = $T_1 - T_2$ = 150 – 40 = 110°C

Specific heat of water, $\frac{Cw = 4.186\frac{J}{g}}{°K}$

Specific heat of the metal $= C$

Heat lost by the metal, $\theta = mC\Delta T \ ...\ (i)$

Rise in the temperature of the water and calorimeter system:

ΔT’ = 40 – 27 = 13°C

Heat gained by the water and calorimeter system:

$m_1 Cw\Delta T$

$= (M + m')Cw\, \Delta T' \ ...\ (ii)$

Heat lost by the metal = Heat gained by the water and colorimeter system

$mC\Delta T = (M + m')Cw\, \Delta T'$

$200 \times C \times 110 = (150 + 25) \times 4.186 \times 13$

$\therefore C = \frac{175 \times 4.186 \times 13}{110 \times 200} = 0.43\, Jg^{-1}K^{-1}$

If some heat is lost to the surroundings, then the value of C will be smaller than the actual value.

14.

The gases listed in the given table are diatomic. Besides the translational degree of freedom, they have other degrees of freedom (modes of motion).

Heat must be supplied to increase the temperature of these gases. This increases the average energy of all the modes of motion.

Hence, the molar specific heat of diatomic gases is more than that of monatomic gases.

If only rotational mode of motion is considered, then the molar specific heat of a diatomic

Gas = $\frac{5}{2}$R

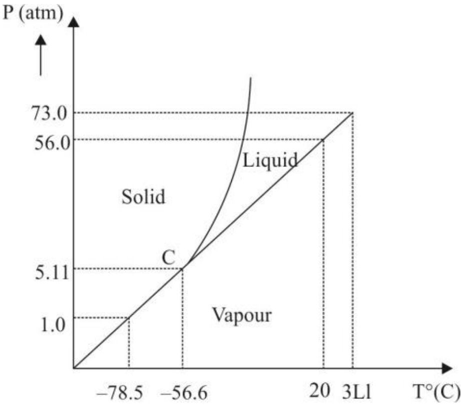
$= \frac{5}{2} \times 1.98 = 4.95\, cal\, mol^{-1}K^{-1}$

With the exception of chlorine, all the observations in the given table agree with($\frac{5}{2}R$)

This is because at room temperature, chlorine also has vibrational modes of motion besides rotational and translational modes of motion.

15.

The P-T phase diagram for CO_2 is shown in the following figure.



(a)C is the triple point of the CO_2 phase diagram. This means that at the temperature and pressure corresponding to this point (i.e., at –56.6°C and 5.11 atm), the solid, liquid, and vaporous phases of CO_2 co-exist in equilibrium.

(b)The fusion and boiling points of CO_2 decrease with a decrease in pressure.

© The critical temperature and critical pressure of CO_2 are 31.1°C and 73 atm respectively. Even if it is compressed to a pressure greater than 73 atm, CO_2 will not liquefy above the critical temperature.

(d)It can be concluded from the P-T phase diagram of CO_2 that: CO_2 is gaseous at –70°C, under 1 atm pressure

CO_2 is solid at –60°C, under 10 atm pressure

CO_2 is liquid at 15°C, under 56 atm pressure

16.

No

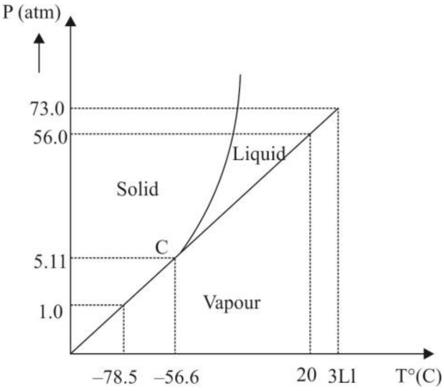
It condenses to solid directly.

The fusion and boiling points are given by the intersection point where this parallel line cuts the fusion and vaporization curves.

It departs from ideal gas behavior as pressure increases.

Explanation:

The P-T phase diagram for CO_2 is shown in the following figure.



(a)At 1 atm pressure and at –60°C, CO_2 lies to the left of –56.6°C (triple point C). Hence, it lies in the region of vaporous and solid phases. Thus, CO_2 condenses into the solid state directly, without going through the liquid state.

(b) At 4 atm pressure, CO_2 lies below 5.11 atm (triple point C). Hence, it lies in the region of vaporous and solid phases. Thus, it condenses into the solid state directly, without passing through the liquid state.

(c)When the temperature of a mass of solid CO_2 (at 10 atm pressure and at –65°C) is increased, it changes to the liquid phase and then to the vapour phase. It forms a line parallel to the temperature axis at 10 atm. The fusion and boiling points are given by the intersection point where this parallel line cuts the fusion and vaporization curves.

(d) If CO_2 is heated to 70°C and compressed isothermally, then it will not exhibit any transition to the liquid state. This is because 70°C is higher than the critical temperature of CO_2. It will remain in the vapour state, but will depart from its ideal behavior as pressure increases.

17.

Initial temperature of the body of the child,

$T_1 = 101°F$

Final temperature of the body of the child,

T_2 = $98°F$

Change in temperature, ΔT = $\left[(101-98)\times\frac{5}{9}\right]°C$

Time taken to reduce the temperature, t = 20 min

Mass of the child, $m = 30\ kg = 30 \times 10^3\ g$

Specific heat of the human body = Specific heat of water $\frac{= c = 1000\frac{cal}{kg}}{}\,°C$

Latent heat of evaporation of water,

$$L = 580\ calg^{-1}$$

The heat lost by the child is given as:

$\Delta\theta = mc\Delta T$

$= 30 \times 1000 \times (101-98) \times \frac{5}{9} = 50000\ cal$

Let m_1 be the mass of the water evaporated from the child's body in 20 min

Loss of heat through water is given by:

$\Delta\theta = m_1 L$

$\therefore m_1 = \frac{\Delta\theta}{L} = \frac{50000}{580} = 86.2\ g$

∴Average rate of extra evaporation caused by the drug= $\frac{m_1}{t}$

$= \frac{86.2}{20} = 4.3\frac{g}{min}$.

18.

Side of the given cubical ice box,

$s = 30\ cm = 0.3\ m$

Thickness of the ice box, $l = 5.0\ cm = 0.05\ m$

Mass of ice kept in the ice box, $m = 4\ kg$

Outside temperature, $T = 45°C$

Coefficient of thermal conductivity of thermacole, K = 0.01 J $s^{-1}m^{-1}k^{-1}$

Heat of fusion of water, $L = 335 \times 10^3$ J kg^{-1}. Let
m be the total amount of ice that melts in 6 h.
The amount of heat lost by the food:

$$\theta = \frac{KA(T-0)t}{l}$$

Where,
A = Surface area of the box = $6s^2 = 6 \times (0.3)^2$
= 0.54 m^3

$$\theta = \frac{0.01 \times 0.54 \times (45) \times 6 \times 60}{0.05} = 104976\,J$$

$$But\ \theta = m'L$$

$$\therefore m' = \frac{\theta}{L} = \frac{104976}{335 \times 10^3} = 0.313\ kg$$

Mass of ice left = 4 – 0.313 = 3.687 kg
Hence, the amount of ice remaining after 6 h is 3.687 kg.

19.

Base area of the boiler, A = 0.15 m^2
Thickness of the boiler, $l = 1.0\ cm = 0.01\ m$
Boiling rate of water, $R = 6.0\frac{kg}{min}$ss, $m = 6\ kg$
Time, t = 1 min = 60 s
Thermal conductivity of brass,
$K = 109$ J $s^{-1}m^{-1}k^{-1}$
Heat of vaporization, L = 2256 × 10^3Jkg^{-1}
The amount of heat flowing into water through the brass base of the boiler is given by:

$$\theta = \frac{KA(T_1 - T_2)t}{l} \text{ ... (i)}$$

Where,
T_1 = Temperature of the flame in contact with the boiler
T_2 = Boiling point of water = 100°C
Heat required for boiling the water
θ = mL … (ii)
Equating equations (i) and (ii), we get:

$$\therefore mL = \frac{KA(T_1 - T_2)t}{l}$$

$$T_1 - T_2 = \frac{mLl}{KAt}$$

$$= \frac{6 \times 2256 \times 10^3 \times 0.01}{109 \times 0.15 \times 60} = 137.98°C$$

Therefore, the temperature of the part of the flame in contact with the boiler is 237.98°C.

20.

(a) A body with a large reflectivity is a poor absorber of light radiations. A poor absorber will in turn be a poor emitter of radiations. Hence, a body with a large reflectivity is a poor emitter.

(b) Brass is a good conductor of heat. When one touches a brass tumbler, heat is conducted from the body to the brass tumbler easily. Hence, the temperature of the body reduces to a lower value and one feels cooler.

Wood is a poor conductor of heat. When one touches a wooden tray, very little heat is conducted from the body to the wooden tray. Hence, there is only a negligible drop in the temperature of the body and one does not feel cool.

Wood is a poor conductor of heat. When one touches a wooden tray, very little heat is conducted from the body to the wooden tray. Hence, there is only a negligible drop in the temperature of the body and one does not feel cool.

Thus, a brass tumbler feels colder than a wooden tray on a chilly day.

© An optical pyrometer calibrated for an ideal black body radiation gives too low a value for temperature of a red-hot iron piece kept in the open.

Black body radiation equation is given by:

$$E = \sigma(T^4 - T_0^4)$$

Where,
E = Energy radiation
T = Temperature of optical
pyrometer T_0 = Temperature of open
space σ = Constant
Hence, an increase in the temperature of open space reduces the radiation energy.

(d) When the same piece of iron is placed in a furnace, the radiation energy, E = σ T^4
Without its atmosphere, earth would be inhospitably cold. In the absence of atmospheric gases, no extra heat will be trapped. All the heat would be radiated back from earth's surface.

(e)A heating system based on the circulation of steam is more efficient in warming a building than that based on the circulation of hot water. This is because steam contains surplus heat in the form of latent heat ($540\frac{cal}{g}$).

21.

According to Newton’s law of cooling, we have:

$$-\frac{dT}{dt} = K(T - T_0)$$

$$\frac{dT}{K(T - T_0)} = -Kdt \text{ ... (i)}$$

Where,

Temperature of the body $= T$

Temperature of the surroundings = T_0 = 20°C

K is a constant

Temperature of the body falls from 80°C to 50°C in time, $t = 5 \text{ min} = 300\ s$

Integrating equation (i), we get:

$$\int_{50}^{80} \frac{dT}{K(T - T_0)} = -\int_{0}^{300} Kdt$$

$$[log_e(T - T_0)]_{50}^{80} = -K[t]_0^{300}$$

$$\frac{2.3026}{K} log_{10} \frac{80 - 20}{50 - 20} = -300$$

$$\frac{2.3026}{K} log_{10} 2 = -300$$

$$\frac{-2.3026}{300} log_{10} 2 = K \text{ ... (ii)}$$

The temperature of the body falls from 60°C to 30°C in time = t

Hence, we get:

$$\frac{2.3026}{K} log_{10} \frac{60 - 20}{30 - 20} = -t$$

$$\frac{-2.3026}{K} log_{10} 4 = K \text{ ... (iii)}$$

Equating equations (ii) and (iii), we get

$$\frac{2.3026}{K} log_{10} 4 = \frac{-2.3026}{300} log_{10} 2$$

$$\therefore t = 300 \times 2 = 600\ s = 10\ min$$

Therefore, the time taken to cool the body from 60°C to 30°C is 10 minutes.

Solutions: Expert Test - I

1.

(c) Due to change in temperature, the thermal strain produced in a rod of length L is given by

$$\frac{\Delta L}{L} = \alpha \Delta T$$

ΔL = L α ΔT

Where l = original length of rod and α = coefficient of liner expansion of solid rod

As the change in length(Δl) of the given two rods of copper and aluminium are independent of temperature change, i.e. ΔT is same for both copper and aluminium.

$L_{Cu}\ \alpha_{Cu} = L_{Al}\alpha_{Al}$ … (i)

Here, $\alpha_{Cu} = 1.7 \times 10^{-5}\ K^{-1}$

$\alpha_{Al} = 2.2 \times 10^{-5}\ K^{-1}$

and L_{Cu} = 88 cm

Substituting the given values in Eq. (i), we get

$$L_{Al} = \frac{L_{Cu}\alpha_{Cu}}{\alpha_{Al}} = \frac{1.7 \times 10^{-5} \times 88}{2.2 \times 10^{-5}} \simeq 68 \text{ cm}$$

2.

(c) According to question,

Coefficient of linear expression of brass = α_1

Coefficient of linear expression of steel = α_2

Length of brass and steel rods are l_1 and l_2 respectively.

As given difference increase in length $(l_2' - l_1')$ is same for all temperature.

So, $l_2' - l_1' = l_2 - l_1$

$$l_2(1 + \alpha_2 \Delta t) - l_1(1 + \alpha_1 \Delta t) = l_2 - l_1$$

$$l_2 \alpha_2 = l_1 \alpha_1$$

3.

(d) Since, heat required, Q = mcΔT

$$= \left(\frac{4}{3}\pi r^3 \cdot \rho\right) c\Delta T$$

[∵ m = V_{sphere} ρ]

Since, π, ρ, c and T are constants.

$$Q \propto r^3 \text{ or } \frac{Q_1}{Q_2} = \frac{r_1^3}{r_2^3}$$

$$= \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1.5 r_2}{r_2}\right)^3 = \frac{27}{8}$$

Hence, correct option is (d).

4.

(b) According to question as conservation of energy, energy gained by the ice during its fall from height his given by

E = mgh

As given, only one quarter of its energy is absorbed by the ice.

So, $\frac{mgh}{4} = mL_f \Rightarrow h = \frac{mL_f \times 4}{mg}$

$$= \frac{L_f \times 4}{g} = \frac{3.4 \times 10^5 \times 4}{10} = 13.6 \times 10^4 = 136000$$

m = 136 km

5.

(b) Heat lost by Ist body = heat gained by IInd body. Body at 100°C temperature has greater heat capacity than body at 0°C so final temperature will be closer to 100°C. So, $T_c > 50°$ C.

6.

(b) In first conditions;

Given, the initial temperature of the cup of coffee, $T_i = 90°C$

The final temperature of the cup of coffee, $T_f = 80°C$

The time taken to drop the temperature 90°C to 80°C is t.

The temperature of the surrounding, $T_0 = 20°C$

Using the Newton's law of cooling,

Rate of cooling = $\frac{dT}{dt} = K\left[\frac{T_i + T_f}{2} - T_0\right]$

Substituting the values in the above equation, we get

$$\frac{90-80}{t} = K\left[\frac{90+80}{2} - 20\right]$$

$$\frac{10}{t} = K[65] \Rightarrow K = \frac{2}{13t}$$

In second conditions;

The initial temperature of the cup of coffee, T_i' = 80°C

The final temperature of the cup of coffee, T_f' = 60°C.

Using the Newton's law of cooling,

Rate of cooling = $\frac{dT}{dt} = K\left[\frac{T_i' + T_f'}{2} - T_0\right]$

Substituting the values in the above equation, we get

$$\frac{80-60}{t_1} = \frac{2}{13t}\left[\frac{60+80}{2} - 20\right]$$

$$\frac{20}{t_1} = \frac{2}{13t}[50] \Rightarrow t_1 = \frac{13}{5}t$$

7.

(d) According to Wein's displacement law,

$$\lambda = \frac{b}{T}$$

i.e., $\lambda \propto \frac{1}{T}$ ... (i)

We know that,

$\lambda_{\text{bluish}} < \lambda_{\text{yellowish}} < \lambda_{\text{reddish}}$

Hence, using Eq.(i), we have

$T_A > T_C > T_B$

8.

(d) From Newton's law of cooling, the time taken (t) by a body to cool from T_1 to T_2 when placed in a medium of temperature T_0 can be calculated from relation

$$\frac{T_1 - T_2}{t} = \frac{1}{K}\left(\frac{T_1 + T_2}{2} - T_0\right)$$

When the object cool from 80°C to 70°C in 12 minutes, then from Newton's law of cooling,

$$\frac{80-70}{12} = \frac{1}{K}\left(\frac{80+70}{2} - 25\right)$$

[∵ $T_0 = 25$ ℃]

$\frac{5}{6} = \frac{1}{K}50$ … (i)

Similarly, when object cool from 70°C to 60°C we get

$$\frac{70-60}{t} = \frac{1}{K}\left(\frac{70+60}{2} - 25\right)$$

$\frac{10}{t} = \frac{1}{K}40$ … (ii)

Divide Eq. (i) and (ii), we get

$$\frac{5}{6} \times \frac{t}{10} = \frac{50}{40}$$

$$t = \frac{5}{4} \times 12$$

= 15 minutes

9.

(c) If area of cross-section of a surface is not uniform or if the steady state condition is not reached, the heat flow equation can be applied to a thin layer of material perpendicular to direction of heat flow.

The rate of heat flow by conduction for growth of ice is given by,

$$\frac{d\theta}{dt}=\frac{KA(\theta_0-\theta_1)}{x}$$

where, dθ = ρAdxL, $\theta_0 = 0$ and $\theta_1 = -\theta$

Given, $\theta_0 = 0°C$, $\theta_1 = -26°C$

The rate of increase of thickness can be calculated from Eq.

$$\frac{d\theta}{dt}=\frac{KA(\theta_0-\theta_1)}{x}$$

$$\frac{\rho AdxL}{dt}=\frac{KA(\theta_0-\theta_1)}{x}$$

$$\frac{dx}{dt}=\frac{KA(\theta_0-\theta_1)}{\rho AxL}$$

$$=\frac{K[0-(-26)]}{\rho xL}=\frac{26K}{\rho xL}$$

10.

(a) According to Wien's law,

$$\lambda_{\max}\propto\frac{1}{T}$$

i.e. $\lambda_{\max} T$ = constant

where, $\lambda_{\max}$ is the maximum wavelength of the radiation emitted at temperature T.

$$\therefore \quad \lambda_{\max_1}T_1=\lambda_{\max_2}T_2$$

$$\text{or}\quad \frac{T_1}{T_2}=\frac{\lambda_{\max_2}}{\lambda_{\max_1}} \qquad \dots\text{(i)}$$

Here, $\lambda_{\max_1}=\lambda_0$ and $\lambda_{\max_2}=\frac{3}{4}\lambda_0$

Substituting the above values in Eq. (i), we get

$$\frac{T_1}{T_2}=\frac{\frac{3}{4}\lambda_0}{\lambda_0}=\frac{3}{4}$$

$$\text{or}\quad \frac{T_1}{T_2}=\frac{3}{4} \qquad \dots\text{(ii)}$$

As we know that, from Stefan's law, the power radiated by a body at temperature T is given as

$$P = \sigma AeT^4$$

i.e. $P \propto T^4$

(∵ the quantity σAe is constant for a body)

$$\frac{P_1}{P_2}=\frac{T_1^4}{T_2^4}=\left(\frac{T_1}{T_2}\right)^4$$

From Eq. (i), we get

$$\frac{P_1}{P_2}=\left(\frac{3}{4}\right)^4=\frac{81}{256}$$

Given, $P_1 = P$ and $P_2 = nP$

$$\frac{P_1}{P_2}=\frac{P}{nP}=\frac{81}{256}$$

$$\text{or}\quad n=\frac{256}{81}$$

11.

(a) In parallel arrangement of n rods Equivalent thermal conductivity is given by

$$K_{eq}=\frac{K_1A_1+K_2A_2+\ldots+K_nA_n}{A_1+A_2+\ldots+A_n}$$

If rods are of same area, then

$$K_{eq}=\frac{K_1+K_2+\ldots+K_n}{n}$$

Now, in the question, it is not given that rods are of same area. But we can judge that from given diagram.

∴ Equivalent thermal conductivity of the system of two rods

$$K_{eq}=\frac{K_1+K_2}{2}$$

12.

(d) Radiated power of a black body,

$P = \sigma AT^4$

where, A = surface area of the body

T = temperature of the body

and σ = Stefan's constant

When radius of the sphere is halved, new area,

$$A'=\frac{A}{4}$$

∴ Power radiated,

$P' = \sigma \frac{A}{4}(2T)^4 = 16/4\,(\sigma A T^4)$

= 4P = 4 × 450 = 1800 watts

13.

(c) Given, temperature, $T_1 = 5760$ K

Since, it is given that energy of radiation emitted by the body at wavelength 250 nm in U_1, at wavelength 500 nm is U_2 and that at 1000 nm is U_3.

∵ According to Wien's law, we get

$\lambda_m T = b$

where, b =Wien's constant = 2.88×10^6 nmk

$$\lambda_m=\frac{b}{T}$$

$$\lambda_m=\frac{2.88\times10^6\,nmK}{5760K}$$

$\lambda_m = 500$ nm

$\because \lambda_m$ = wavelength corresponding to maximum energy, so, $U_2 > U_1$.

14.

(b) According to Newton's law of cooling,

$\Delta T = \Delta T_0\, e^{-\lambda t}$

$3T - 2T = (3T - T)\, e^{-\lambda \times 10}$ ... (i)

Again for next 10 minutes

$T' - T = \left(2T\right) \times e^{-\lambda(20)}$ … (ii)

From Eqs. (i) and (ii), we get

$$T' - T = \left(2T\right)\left(e^{-\lambda \times 10}\right)^2 = \left(2T\right)\left(\frac{1}{2}\right)^2$$

$$= \frac{T}{2}$$

$$\therefore \quad T' = T + \frac{T}{2} = \frac{3T}{2}$$

15.

(b) Equation of Wien's displacement law is given by $\lambda_m T$ = constant

---

Solutions: Expert Test - II

1.

(a) Let L_1' and L_2' be the lengths of the wire when temperature is changed by ΔT°C.

At T°C,

$L_{eq} = L_1 + L_2$

At T = Δ°C

$L'_{eq} = L'_1 + L'_2$

$$\therefore\ L_{eq}\left(1 + \alpha_{eq}\Delta T\right) = L_1\left(1 + \alpha_1 \Delta T\right) + L_2\left(1 + \alpha_2 \Delta T\right)$$

$$\left[\because L' = L\left(1 + \alpha \Delta T\right)\right]$$

$$\left(L_1 + L_2\right)\left(1 + \alpha_{eq}\Delta T\right) = L_1 + L_2 + L_1\alpha_1\Delta T + L_2\alpha_2\Delta T$$

$$\alpha_{eq} = \frac{\alpha_1 L_1 + \alpha_2 L_2}{L_1 + L_2}$$

2.

(a) Change in length of the metal wire (Δl) when its temperature is changed by ΔT is given by

$\Delta l = l\alpha\Delta T$

Here, α = Coefficient of linear expansion

Here, Δl = 0.02%, ΔT = 10°C

$$\therefore\ \alpha = \frac{\Delta l}{l\Delta T} = \frac{0.02}{100 \times 10}$$

$\alpha = 2 \times 10^{-5}$

Volume coefficient of expansion,

$\gamma = 3\alpha = 6 \times 10^{-5}$

$$\because \rho = \frac{M}{V}$$

$$\frac{\Delta V}{V} \times 100 = \gamma \Delta T = \left(6 \times 10^{-5} \times 10 \times 100\right)$$

$= 6 \times 10^{-2}$

Volume increase by 0.06% therefore, density decreases by 0.06%.

3.

(Bonus)

Load = Stress × *Area*

$$Mg = \frac{\Delta L\, Y}{L} A$$

and $A = \pi r^2 = \pi\left(10^{-3}\right)^2 = \pi \times 10^{-6}$

$$L\alpha\Delta T = \frac{FL}{AY}$$

or $\quad 0.2 \times 10^{-5} \times 20 = \dfrac{F \times 0.2}{\left(\pi \times 10^{-6}\right) \times 10^{11}}$

∴ F = 20π N

$$\therefore\ m = \frac{f}{g} = 2\pi \ = 6.28 \text{ kg}$$

4.

(a) Change in length in both rods are same i.e.

$\Delta l_1 = \Delta l_2$

$l\alpha_1 \Delta\theta_1 = l\alpha_2 \Delta\theta_2$

$$\frac{\alpha_1}{\alpha_2} = \frac{\Delta\theta_2}{\Delta\theta_1} \qquad \left[\because \frac{\alpha_1}{\alpha_2} = \frac{4}{3}\right]$$

$$\frac{4}{3} = \frac{\theta - 30}{180 - 30}$$

θ = 230 °C

5.

(a) Young's modulus Y = $\dfrac{stress}{strain} = \dfrac{F/A}{A\left(\Delta l / l\right)}$

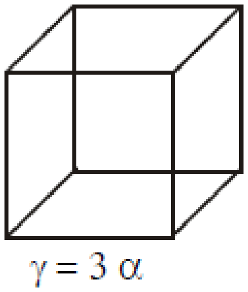
Using, coefficient of linear expansion,

$$\alpha = \frac{\Delta l}{l\Delta T} \Rightarrow \frac{\Delta l}{l} = \alpha \Delta T$$

$$\therefore\ Y = \frac{F}{A\left(\alpha \Delta T\right)}$$

6.

(c) As we know, Bulk modulus



$$K=\frac{\Delta P}{\left(\frac{-\Delta V}{V}\right)}\Rightarrow\frac{\Delta V}{V}=\frac{P}{K}$$

$$V=V_0\left(1+\gamma\Delta t\right)$$

$$\frac{\Delta V}{V_0}=\gamma\Delta t$$

$$\therefore\ \frac{P}{K}=\gamma\Delta t\Rightarrow\Delta t=\frac{P}{\gamma K}=\frac{P}{3\alpha K}$$

7.

(c) Due to thermal exp., change in length

$(\Delta l)=l\,\alpha\,\Delta T$ ... (i)

Young's modulus (Y) = $\frac{Normal\ stress}{Longitudinal\ strain}$

$$Y=\frac{F/A}{\Delta l/l}\Rightarrow\frac{\Delta l}{l}=\frac{F}{AY}$$

$$\Delta l=\frac{Fl}{AY}$$

From eqn (i), $\frac{Fl}{AY}=l\,\alpha\,\Delta T$

8.

(c) $\frac{Reading\ on\ any\ scale-LFP}{UFP-LFP}$

= constant for all scales

$$\frac{340-273}{373-273}=\frac{°Y-(-160)}{-50-(-160)}$$

$$\frac{67}{100}=\frac{y+160}{110}$$

∴ Y = – 86.3° Y

9.

Volume capacity of beaker, V_0 = 500 cc

$V_b = V_0 + V_0\,\gamma_{beaker}\,\Delta T$

When beaker is partially filled with V_m volume of mercury,

$$V_b^1=V_m+V_m\gamma_m\Delta T$$

Unfilled volume ($V_b - V_b'$) = $V_0 - V_m$

For the volume to remain constant at all temp

Ex of liquid = Exp of solid

$$V_0\gamma_{beaker}=V_m\gamma_M$$

$$\therefore\ V_m=\frac{V_0\gamma_{beaker}}{\gamma_M}$$

or, $V_m=\frac{500\times6\times10^{-6}}{1.5\times10^{-4}}$ = 20 cc.

10.

Volume, V = lbh

$$\therefore\ \gamma=\frac{\Delta V}{V}=\frac{\Delta l}{l}+\frac{\Delta b}{b}+\frac{\Delta h}{h}$$

(γ = coefficient of volume expansion)

$\gamma = 5\times10^{-5}+5\times10^{-6}+5\times10^{-6}$

$= 60\times10^{-6}$/°C

∴ Value of C = 60.00

11.

(a) As the rods are identical, so they have same length (l) and area of cross-section (A). They are connected in series. So, heat current will be same for all rods.

Heat current = $\left(\frac{\Delta Q}{\Delta t}\right)_{AB}=\left(\frac{\Delta Q}{\Delta t}\right)_{BC}=\left(\frac{\Delta Q}{\Delta t}\right)_{CD}$

$$\frac{(100-70)K_1A}{l}=\frac{(70-20)K_2A}{l}=\frac{(20-0)K_3A}{l}$$

$K_1(100 - 70) = K_2(70 - 20) = K_3(20 - 0)$

$K_1(30) = K_2(50) = K_3(20)$

$$\frac{K_1}{10}=\frac{K_2}{6}=\frac{K_3}{15}$$

$K_1 : K_2 : K_3$ = 10 : 6 : 15

$K_1 : K_3$ = 2 : 3.

12.

(a) According to question, one half of its kinetic energy is converted into heat in the wood.

$$\frac{1}{2}mv^2\times\frac{1}{2}=ms\Delta T$$

$\Delta T=\frac{v^2}{4\times s}=\frac{210\times210}{4\times4.2\times0.3\times1000}$ = 87.5 °C

13.

(a) Here ice melts due to water.

Let the amount of ice melts = m_{ice}

$m_w s_w \Delta\theta = m_{ice} L_{ice}$

$$\therefore\ m_{ice} = \frac{m_w s_w \Delta\theta}{L_{ice}}$$

$$= \frac{0.2 \times 4200 \times 25}{3.4 \times 10^5} = 0.0617 \text{ kg} = 61.7 \text{ g}$$

14.

(a) Heat given by water = $m_w C_w (T_{mix} - T_w)$

= 200 × 1 × (31 – 25)

Heat taken by steam = $m\,L_{steam} + m\,C_w(T_s - T_{mix})$

= m × 540 + m(1) × (100 – 31)

= m × 540 + m(1) × (69)

From the principal of calorimeter,

Heat lost = Heat gained

∴ (200)(31 – 25) = m × 540 + m(1)(69)

1200 = m(609)

m ≈ 2.

15.

Using the principal of calorimetry

$M_{ice}\,L_f + m_{ice}\,(40 - 0)\,C_w = m_{stream}\,L_v + m_{stream}\,(100 - 40)\,C_w$

M(540) + M × 1 × (100 – 40) = 200 × 80 + 200 × 1 × 40

600 M = 24000

M = 40g

16.

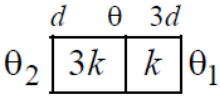
(a) $M_1 C_{ice} \times (10) + M_1 L = M_2 C_\omega\,(50)$

or $M_1 \times 0.5) \times 10 + M_1 L = M_2 \times 1 \times 50$

$$L = \frac{50 M_2}{M_1} - 5$$

17.

(a) $H_1 = H_2$

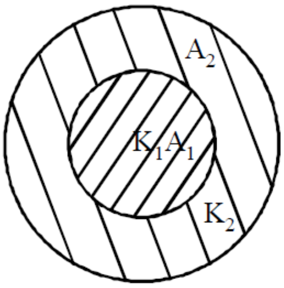


$$\text{or } (3k)A\left(\frac{\theta_2 - \theta}{d}\right) = kA\left(\frac{\theta - \theta_1}{3d}\right)$$

$$\text{or } \theta = \left(\frac{\theta_1 + 9\theta_2}{10}\right)$$

18.

(d) Effective thermal conductivity of system



$$K_{eq} = \frac{K_1 A_1 + K_2 A_2}{A_1 + A_2}$$

$$= \frac{K_1 \pi R^2 + K_2\left[\pi(2R)^2 - \pi R^2\right]}{\pi(2R)^2}$$

$$= \frac{K_1(\pi R^2) + K_2(3\pi R^2)}{4\pi R^2} = \frac{K_1 + 3K_2}{4}$$

19.

(d) Let m gram of ice is added.

From principal of calorimeter

Heat gained (be ice) = heat lost (by water)

∴ 20 × 2.1 × m + (m – 20) × 334 = 50 × 4.2 × 40

376 m = 8400 + 6680

m = 40.1

20.

(c) Heat loss = Heat gain = mSΔθ

So, $m_A S_A \Delta\theta_A = m_B S_B \Delta\theta_B$

$100 \times S_A \times (100 - 90) = 50 \times S_B \times (90 - 75)$

$2S_A = 1.5S_B$

$$S_A = \frac{3}{4} S_B$$

Now, $100 \times S_A \times (100 - \theta) = 50 \times S_B \times (\theta - 50)$

$$2 \times \left(\frac{3}{4}\right) \times (100 - \theta) = (\theta - 50)$$

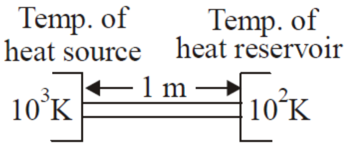
300 – 3θ = 2θ – 100

400 = 5θ

θ = 80 °C

21.

(a)



$$\left(\frac{dQ}{dt}\right) = \frac{kA\Delta T}{l}$$

Energy flux, $\frac{1}{A}\left(\frac{dQ}{dt}\right) = \frac{k\Delta T}{l}$

$= \frac{(0.1)(900)}{1}$ = 90 W/m^2

22.

(c) Let specific heat of unknown metal be ‘s’

According to principal of calorimetry, Heat lost = Heat gain

$m \times s\Delta\theta = m_1 s_{brass} (\Delta\theta_1) + m_2\ s_{water}\ \Delta\theta_2$

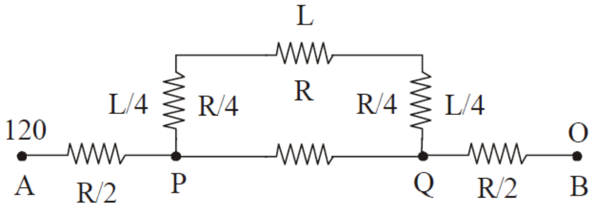
192 × S × (100 – 21.5) = 128 × 394 × (21.5 – 8.4)

+ 240 × 4200 × (21.5 – 8.4)

S = 916 Jkg^{-1}k^{-1}

23.

(a) $\frac{\Delta T_{AB}}{R_{AB}} = \frac{120}{\frac{8}{5}R} = \frac{120\times5}{8R}$



In steady state temperature difference between P and Q,

$\Delta T_{PQ} = \frac{120\times5}{8R}\times\frac{3}{5}R = \frac{360}{8}$

= 45°C

24.

(d) According to principle of calorimetry,

$Q_{given} = Q_{used}$

0.2 × S × (150 – 40) = 150 × 1 × (40 – 27) + 25 × (40 – 27)

0.2 × S × 110 = 150 × 13 + 25 × 13

Specific heat of aluminium

$S = \frac{13\times25\times7}{0.2\times110}$

= 434 J/kg-°C

25.

(d) According to principle of calorimetry,

Heat lost = Heat gain

100 × 0.1(T – 75) = 100 × 0.1 × 45 + 170 × 1 × 45

10T – 750 = 450 + 7650 = 8100

T – 75 = 810

T = 885 °C

26.

(b) As Pt = mCΔT

So, P × 10 × 60 = mC 100 ... (i)

and P × 55 × 60 = mL ... (ii)

Dividing equation (i) by (ii) we get

$\frac{10}{55} = \frac{C\times100}{L}$

∴ L = 550 cal./g.

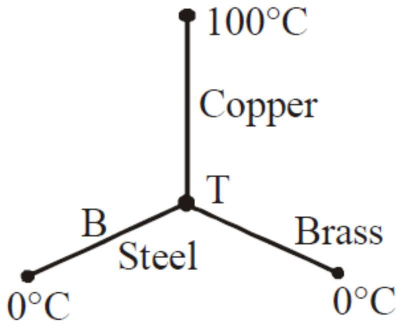
27.

(c) Rate of heat flow is given by,

$Q = \frac{KA(\theta_1 - \theta_2)}{l}$

Where, K = coefficient of thermal conductivity

l = length of rod and A = area of cross-section of rod



If the junction temperature is T, then

$Q_{Copper} = Q_{Brass} + Q_{Steel}$

$$\frac{0.95\times4(100-T)}{46}$$

$$= \frac{0.26\times4\times(T-0)}{13} + \frac{0.12\times4\times(T-0)}{12}$$

200 – 2T = 2T + T

T = 40 °C

∴ $Q_{Copper} = \frac{0.92\times4\times60}{46}$ = 4.8 cal/s

28.

(c) In the given problem, fall in temperature of sphere,

$dT = (3T_0 - 2T_0) = T_0$

Temperature of surrounding, $T_{surr} = T_0$

Initial temperature of sphere, $T_{initial} = 3T_0$

Specific heat of the material of the sphere varies as,

$c = \alpha T^3$ per unit mass (α = a constant)

Applying formula,

$$\frac{dT}{dt}=\frac{\sigma A}{McJ}\left(T^4-T_{surr}^4\right)$$

$$\frac{T_0}{dt}=\frac{\sigma 4\pi R^2}{M\alpha\left(3T_0\right)^3 J}\left[\left(3T_0\right)^4-\left(T_0\right)^4\right]$$

$$dt=\frac{\mathrm{M}\alpha 27T_0^4 J}{\sigma 4\pi R^2\times 80T_0^4}$$

Solving we get,

Time taken for the sphere to cool down temperature 2T₀,

$$t=\frac{M\alpha}{16\pi R^2\sigma}\ln\left(\frac{16}{3}\right)$$

29.

(d) When radius is decrease by ΔR,

$$4\pi R^2\Delta R\rho L=4\pi T\left[R^2-\left(R-\Delta R\right)^2\right]$$

$$\rho R^2\Delta RL=T\left[R^2-R^2+2R\Delta R-\Delta R^2\right]$$

$$\rho R^2\Delta RL=T2R\Delta R \quad [\Delta R \text{ is very small}]$$

$$R=\frac{2T}{\rho L}$$

30.

(d) As the surrounding is identical, vessel is identical time taken to cool both water and liquid (from 30°C to 25°C) is same 2 minutes, therefore

$$\left(\frac{dQ}{dt}\right)_{water}=\left(\frac{dQ}{dt}\right)_{liquid}$$

or, $\frac{\left(m_w C_w+W\right)\Delta T}{t}=\frac{\left(m_l C_l+W\right)\Delta T}{t}$

(W = water equivalent of the vessel)

or, $m_w C_w = m_l C_l$

∴ Specific heat of liquid, $C_l=\frac{m_W C_W}{m_l}$

$=\frac{50\times 1}{100}$ = 0.5 kcal/kg

31.

(b) From Newton's Law of cooling,

$$\frac{T_1-T_2}{t}=K\left[\frac{T_1+T_2}{2}-T_0\right]$$

Here, T_1 = 50°C, T_2 = 40°C

and T_0 = 20°C, t = 600S = 5 minutes

$$\frac{50-40}{5Min}=K\left(\frac{50+40}{2}-20\right) \quad \text{... (i)}$$

Let T be the temperature of sphere after next 5 minutes. Then

$$\frac{40-T}{5}=K\left(\frac{40+T}{2}-20\right) \quad \text{... (ii)}$$

Dividing eqn. (ii) by (i), we get

$$\frac{40-T}{10}=\frac{40+T-40}{50+40-40}=\frac{T}{50}$$

40-T=T/5 , 200 = 6T , T= 33°C∴ $T=\frac{200}{6}$ = 33.3 °C

32.

(b) Rate of Heat loss = $mS\left(\frac{dT}{dt}\right)=e\sigma AT^4$

$$-\frac{dT}{dt}=\frac{e\sigma\times A\times T^4}{\rho\times Vol.\times S}\Rightarrow -\frac{dT}{dt}\propto\frac{1}{\rho S}$$

$$\frac{\left(-\frac{dT}{dt}\right)_A}{\left(-\frac{dT}{dt}\right)_B}=\frac{\rho_B}{\rho_A}\times\frac{S_B}{S_A}=\frac{10^9}{8\times 10^2}\times\frac{4000}{2000}$$

$$\left(-\frac{dT}{dt}\right)_A>\left(-\frac{dT}{dt}\right)_B$$

So, A cools down at faster rate.

33.

(a) According to Newton's law of cooling,

$$\left(\frac{\theta_1-\theta_2}{t}\right)=K\left(\frac{\theta_1+\theta_2}{2}-\theta_0\right)$$

$$\left(\frac{60-50}{10}\right)=K\left(\frac{60+50}{2}-25\right) \quad \text{... (i)}$$

and, $\left(\frac{50-\theta}{10}\right)=K\left(\frac{50+\theta}{2}-25\right)$ ... (ii)

Dividing eq. (i) by (ii),

$$\frac{10}{\left(50-\theta\right)}=\frac{60}{\theta}$$

θ = 42.85°C ≅ 43°C

34.

(b) By Newton's law of cooling

$$\frac{\theta_1-\theta_2}{t}=-K\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$$

Where θ_0 is the temperature of surrounding.

Now, hot water cools from 60°C to 50°C in 10 minutes,

$$\left(\frac{60-50}{10}\right)=K\left(\frac{60+50}{2}-\theta_0\right) \quad \text{... (i)}$$

Again, it cools from 50°C to 42°C in next 10 minutes.

$$\left(\frac{50-42}{10}\right)=-K\left(\frac{50+42}{2}-\theta_0\right) \quad \ldots$$

(ii)

Dividing equations (i) by (ii) we get

$$\frac{1}{0.8}=\frac{55-\theta_0}{46-\theta_0}=\frac{10}{8}=\frac{55-\theta_0}{46-\theta_0}$$

$460 - 10\theta_0 = 440 - 8\theta_0$

$2\theta_0 = 20$, $\theta_0 = 10°C$

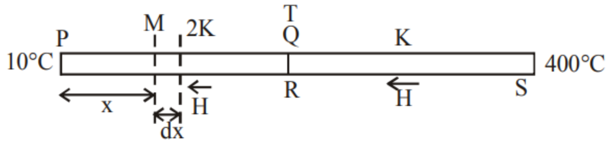
35.

(c) According to Newton's law of cooling the temperature goes on decreasing with time non-linearly.

Solutions: Pro Test - I

1.

At steady state, heat flow rate is same from *P* to *Q* and from *Q* to *S*.



$$\therefore \frac{\mathrm{KA}(400-\mathrm{T})}{\ell}=\frac{2\mathrm{KA}(\mathrm{T}-10)}{\ell} \Rightarrow \mathrm{T}=140°\mathrm{C}$$

The temperature gradient

$$\frac{\mathrm{dT}}{\mathrm{dx}}=\frac{140-10}{1} \qquad \therefore \mathrm{dt}=130\,\mathrm{dx}$$

Therefore, change temperature at a cross-section M distant 'x' from P is

$\Delta\mathrm{T} = 130\mathrm{x}$

Extension in a small elemental length 'dx' is

$dl = dx\alpha\,\Delta\mathrm{T} = dx\,\alpha\,(130\mathrm{x})$

$$\therefore \int dl = 130\alpha \int_0^1 \mathrm{x}\,\mathrm{dx}$$

$$\therefore \Delta l = 130 \times 1.2 \times 10^{-5} \times \frac{1}{2} = 78 \times 10^{-5}$$

2.

$$\mathrm{P}_{\text{heater}} - \mathrm{P}_{\text{cooler}} = \frac{mc\Delta T}{t} = \frac{V\rho c\Delta T}{t}$$

$$\therefore (3000 - \mathrm{P}_{\text{cooler}}) = \frac{0.12\times1000\times4.2\times10^3\times20}{3\times60\times60}$$

$\therefore \mathrm{P}_{\text{cooler}} = 2067\mathrm{W}$

3.

(a) Let *T* be the final steady state temperature of the black body.

In steady state.

Energy lost = Energy gained

$\sigma(T^4 - T_0^4) \times 4\pi R^2 = I(\pi R^2)$

$\therefore 5.7 \times 10^{-8}[T^4 - (300)^4] \times 4 = 912$

$\therefore T = 330\ \mathrm{K}$

4.

(a) Equivalent thermal resistance in configuration-I

$$R_s = R_1 + R_2 = \frac{L}{KA} + \frac{L}{2KA} = \frac{2}{3}\frac{L}{KA}$$

Equivalent thermal resistance in configuration-II

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{KA}{l} + \frac{2KA}{l} \text{ or, } R_P = \frac{l}{3KA} = \frac{R_S}{4.5}$$

i.e., Thermal resistance in configuration-II, R_p is 4.5 times less than thermal resistance in configuration-I R_s

$$\therefore 4.5t_p = t_s \Rightarrow t_p = \frac{t_s}{4.5} = \frac{9}{4.5}s = 2s$$

5.

(9) According to stefan's law, $P \propto T^4$ or $P = P_0T^4$

$\therefore \log_2 P = \log_2 P_0 + \log_2 T^4 \quad \therefore \log_2 \frac{P}{P_0} = 4\log_2 T$

At $\mathrm{T} = 487°\mathrm{C} = 760\mathrm{K}, \log_2 \frac{P}{P_0} = 4\log_2 760 = 1 \ldots$ (i)

At $\mathrm{T} = 2767°\mathrm{C} = 3040\mathrm{K}$,

$\log_e \frac{\mathrm{p}}{\mathrm{p}_0} = 4\log_2 3040 = 4\log_2(760 \times 4)$

$= 4[\log_2 760 + \log_2 2^2] = 4\log_2 760 + 8 = 1 + 8 = 9$

6.

(2) From (i) Stefan- Boltzmann law, $P = \sigma AT^4$ and (ii) Wein's displacement law $= \lambda_m \times T =$ constant

$$\frac{P_A}{P_B} = \frac{A_A}{A_B}\frac{T_A^4}{T_B^4} = \frac{A_A}{A_B} \times \frac{\lambda_B^4}{\lambda_A^4}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = \left[\frac{A_A}{A_B} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}} = \left[\frac{R_A^2}{R_B^2} \times \frac{P_B}{P_A}\right]^{\frac{1}{4}}$$

$$= \left[\frac{400 \times 400}{10^4}\right]^{\frac{1}{4}}$$

$$\therefore \frac{\lambda_A}{\lambda_B} = 2$$

7.

(270°C) Let C be the specific heat capacity of liquid and L be the latent heat of vaporization.

From principal of calorimetry,

Heat = Loss

$m_C S_C\,\Delta T = mC\,\Delta T + mL$

Or

$m_C S_C\,(110 - 80) = 5C\,(80 - 30) + 5L \dots\dots.\dots$ (i)

Where, m_C = mass of calorimeter

S_C = specific heat of calorimeter

Again, when 80g liquid is poured and equilibrium temperature is 50°C

$m_C S_C\,(80 - 50) = 80C(50 - 30) \dots\dots\dots\dots$ (ii)

From eq. (i) & (ii)

1600 C = 250C + 5L

$$\therefore \frac{L}{C} = \frac{1350}{5} = 270℃$$

8.

(4.00)

Rate of the heat flow will be same,

Rate of the heat flow $\frac{dQ}{dt} = \frac{\text{temp. difference}}{\text{thermal resistance}} = \frac{1}{R}(T_2 - T_1)$

Where $R = \frac{L}{KA}$

$$\frac{300 - 200}{R_1} = \frac{200 - 100}{R_2} \text{ or } R_1 = R_2$$

$$\frac{L_1}{K_1 A_1} = \frac{L_2}{K_2 A_2} \therefore \frac{K_1}{K_2} = \frac{A_2}{A_1} = \frac{\pi(2r)^2}{\pi r^2} = 4$$

$[\because L_1 = L_2 = L]$

9.

(d) Power,

$$P = \frac{dQ}{dt} = \frac{d}{dt}(mc)T = (H)\frac{dT}{dt}$$

$$= (H)\frac{d}{dt}\left[T_0\left(1 + \beta t^{\frac{1}{4}}\right)\right]$$

$$P = (H)T_0$$

$$= \frac{\beta t^{-\frac{3}{4}}}{4} \text{ where } H = \text{heat capacity}$$

$$\therefore (H) = \frac{4\mathrm{P}t^{\frac{3}{4}}}{T_0\beta} \qquad \dots (i)$$

But $t^{\frac{1}{4}} = \frac{T(t) - T_0}{\beta T_0}$

$$\therefore t^{\frac{3}{4}} = \frac{[T(t) - T_0]^3}{\beta^3 T_0^3} \qquad \dots (ii)$$

From eq. (i) & (ii),

$$(H) = \frac{4P}{T_0\beta}\frac{[T(t) - T_0]^3}{\beta^3 T_0^3}$$

$$= \frac{4P[T(t) - T_0]^3}{\beta^4 T_0^4}$$

10.

(8.33)

Rate of loss of heat,

$$\frac{\mathrm{dQ}}{\Delta\mathrm{dt}} = \mathrm{e}\sigma\mathrm{A}(\mathrm{T}^4 - T_0^4) \qquad \dots (i$$

$$\Rightarrow \frac{\mathrm{dQ}}{\Delta\mathrm{dt}} = \mathrm{e}\sigma\mathrm{A}(\,T_0 - \Delta T)^{\ 4}$$

$$= \sigma T_0^4\left[\left(1 + \frac{\Delta\mathrm{T}}{T_0}\right)^4 - 1\right]$$

$$\Rightarrow \mathrm{e}\sigma T_0^4\left[\left(1 + 4\frac{\Delta\mathrm{T}}{T_0}\right) - 1\right]$$

$$\frac{\mathrm{dQ}}{\mathrm{Adt}}$$

$$= \sigma\mathrm{e}T_0^3 \cdot 4\Delta\mathrm{T} \qquad \dots (ii)$$

Now from eq. (i)

$$\mathrm{ms} = \frac{\mathrm{dT}}{\mathrm{dt}} = \sigma\mathrm{eA}(\mathrm{T}^4 - T_0^4) \quad [\because \mathrm{Q} = \mathrm{ms}\Delta\mathrm{T}]$$

$$\Rightarrow \frac{\mathrm{dT}}{\mathrm{dt}} = \frac{\sigma\mathrm{eA}}{\mathrm{ms}}[(\mathrm{T_0} + \Delta\mathrm{T}^4)] - \mathrm{T_0^4}$$

$$= \frac{\sigma\mathrm{eA}}{\mathrm{ms}}\mathrm{T_0^4} \times \left[\left(1 + \frac{\Delta\mathrm{T}}{\mathrm{T_0}}\right)^4 - 1\right]$$

$$\frac{\mathrm{dT}}{\mathrm{dt}} = \frac{\sigma\mathrm{eA}}{\mathrm{ms}}\mathrm{T_0^4} \cdot 4\Delta\mathrm{T}$$

$$\frac{\mathrm{dT}}{\mathrm{dt}} = \mathrm{K}\Delta\mathrm{T};$$

$$\left(\mathrm{K}\frac{4\sigma\mathrm{eAT_0^3}}{\mathrm{ms}}\text{Constant for Newton's law of cooling}\right)$$

$$\Rightarrow 4\sigma\mathrm{eAT_0^3} = \frac{\mathrm{K}}{\mathrm{A}}(\mathrm{ms})$$

From eq. (i)

$$\frac{\mathrm{dQ}}{\mathrm{Adt}} = \ \mathrm{e}\sigma T_0^3 \cdot 4\Delta\mathrm{T}$$

Since, rate of loss of heat = heat received per second

$$700 = \left(\frac{\mathrm{K}}{\mathrm{A}}\right)(\mathrm{ms})\Delta\mathrm{T}$$

$$[\mathrm{K} \times \mathrm{ms} = 4200 \times 10^{-30}]$$

$$\Rightarrow \Delta\mathrm{T} = \frac{700 \times \mathrm{A}}{\mathrm{K} \times \mathrm{ms}} = \frac{700 \times 5 \times 10^{-2}}{10^{-3} \times 4200}$$

$$= \frac{50}{6} = \frac{25}{3}$$

$$\therefore \Delta\mathrm{T} = 8.33$$

11.

(c) Energy radiated by the body $= \sigma A(\mathrm{T}^4 - T_0^4)\mathrm{t}$

[For a black body e = 1]

$= \sigma A[(\mathrm{T}_0 - 10)^4 - T_0^4]\mathrm{t}$

$= \sigma A T_0^4 \left[\frac{40}{T_0}\right] \times t$

$= 460 \times 1 \times \frac{40}{300} \times 1 = 61.33J$

$P = \frac{\text{Energy radiated}}{\text{time}}$

$= \sigma A T^4 - \sigma A T_0^4$

$\therefore \left|\frac{dp}{d\mathrm{T}_0}\right| = \sigma A(4T_0^3)$

$\therefore |dp| = \sigma(4T_0^3)d\mathrm{T}_0$

$\therefore |\Delta P| = 4\sigma A T_0^3$

Here as human body is not a black body. So, option (a) and (b) are incorrect.

Energy radiated ∝ A where A is the surface area of the body. Hence option (c) is correct.

Thermodynamics

Chapter Summary

Zeroth Law of Thermodynamics

If systems A and B are each in thermal equilibrium with a third system C, then A and B are in thermal equilibrium with each other.

Heat (ΔQ): It is the energy that is transferred between a system and its environment because of the temperature

difference between them.

If $\Delta \boldsymbol{Q} > 0$, This means heat is given to the system

If $\Delta \boldsymbol{Q} < 0$, This means heat is taken out from the system

At constant pressure $(\Delta Q)_P = nC_P \Delta T$

At constant volume $(\Delta Q)_V = nC_V \Delta T$

Internal energy (U): Internal energy of a system is the energy possessed by the system due to molecular motion and molecular configuration.

$$U = \frac{f}{2} nRT$$

Change in internal energy *(ΔU)*

$$\Delta U = \frac{f}{2} nR\Delta T$$

Change in internal energy does not depend on the path of the process. So, it is called *a* point function *i.e.* it depends only on the initial and final states of the system,

i.e. $\Delta U = U_f - U_i$

Work (ΔW) Suppose a gas is confined in a cylinder that has a movable piston at one end. If P be the pressure of the gas in the cylinder, then force exerted by the gas on the piston of the cylinder $F = PA$ (A = Area of cross-section of piston)

Total amount of work done

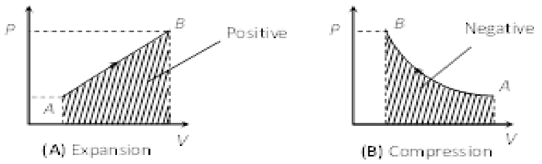
$$W = \int_{Vi}^{V_f} P\,dV = P(V_f - V_i)$$

If we draw P-V diagram, the area bounded by PV-graph and volume axis represents the work done

Work = Area under the graph

If $\Delta \boldsymbol{V} > 0$, This means gas is expanded, ΔW = positive

If $\Delta \boldsymbol{V} < 0$, This means gas is compressed, ΔW = negative



(A) Expansion (B) Compression

First Law of Thermodynamics

According to it heat given to a system (ΔQ) is equal to the sum of increase in its internal energy (ΔU) and the

work done (ΔW) by the system against the surroundings.

$$\boldsymbol{\Delta Q = \Delta U + \Delta W}$$

Isobaric Process

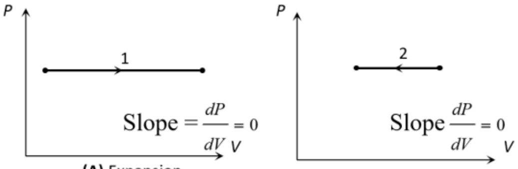
When a thermodynamic system undergoes a physical change in such a way that its pressure remains constant, then the change is known as isobaric process.

P = constant

In this process V and T changes but P remains constant. Hence **Charle's law** is obeyed in this process.

Hence, $V \propto T \Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$

PV Graph



(A) Expansion (B) Compression

Work done in isobaric process

$$\Delta W = \int_{V_i}^{V_f} P\,dV = P\int_{V_i}^{V_f} dV = P[V_f - V_i]$$

[As P = constant]

$$\Rightarrow \Delta W = P(V_f - V_i) = \mu R[T_f - T_i] = \mu R\,\Delta T$$

Isochoric Process

When a thermodynamic process undergoes a physical change in such a way that its volume remains constant, then the change is known as isochoric process.

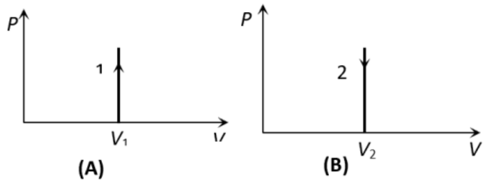
V = constant

In this process P and T changes but V = constant.

Hence **Gay-Lussac's law** is obeyed in this process

$$P \propto T \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2} = \text{constant}$$

PV Graph



(A) (B)

Work done in isochoric process

$$\Delta W = P\Delta V = P[V_f - V_i] = 0$$

Isothermal Process

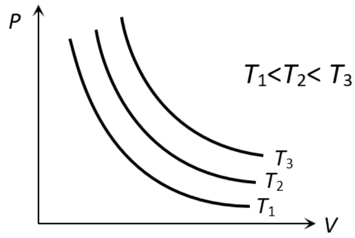
When a thermodynamic system undergoes a physical change in such a way that its temperature remains constant, then the change is known as isothermal changes.

T = constant

In this process, P and V changes but T = constant and **Boyle's law** is obeyed

$$PV = \text{constant} \Rightarrow P_1V_1 = P_2V_2$$

PV Graph



Work done in isothermal process

$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{nRT}{V}dV \qquad [\text{As } PV = nRT]$$

$$W = nRT\log_e\left(\frac{P_i}{P_f}\right) = 2.303nRT\log_{10}\left(\frac{P_i}{P_f}\right)$$

Adiabatic Process

When a thermodynamic system undergoes a change in such a way that no exchange of heat takes place between System and surroundings, the process is known as adiabatic process.

$$\Delta Q = 0$$

$$PV^{\gamma} = Constant$$

where $\gamma = \frac{C_P}{C_V}$ =Adiabatic constant

$= \gamma = 1 + \frac{2}{f}$

where, f is the degree of freedom

For temperature and volume

$TV^{\gamma-1}$ = constant $\Rightarrow T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ or $T \propto V^{1-\gamma}$

For temperature and pressure

$$\frac{T^{\gamma}}{P^{\gamma-1}} = \text{const.} \Rightarrow T_1^{\gamma}P_1^{1-\gamma} = T_2^{\gamma}P_2^{1-\gamma} \text{ or } T \propto P^{\frac{\gamma-1}{\gamma}}$$

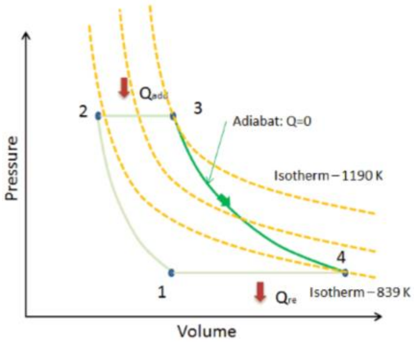
or $P \propto T^{\frac{\gamma}{\gamma-1}}$

PV Graph

(i) Curve obtained on PV graph are called adiabatic curve.

(ii) Slope of adiabatic curve

$$\text{Slope} = \frac{dP}{dV} = -\gamma\frac{PV^{\gamma-1}}{V^{\gamma}} = -\gamma\left(\frac{P}{V}\right)$$



Work done in adiabatic process

$$W = \int_{V_i}^{V_f} P\,dV = \int_{V_i}^{V_f} \frac{K}{V^{\gamma}}dV \Rightarrow$$

$$W = \frac{[P_iV_i - P_fV_f]}{(\gamma-1)} = \frac{\mu R(T_i - T_f)}{(\gamma-1)}$$

Cyclic and Non-Cyclic Process

A cyclic process consists of a series of changes which return the system back to its initial state.

In non-cyclic process the series of changes involved do not return the system back to its initial state.

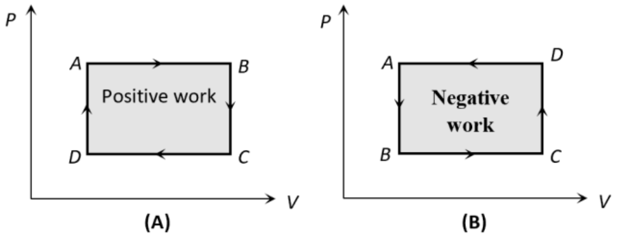
In case of cyclic process as $U_f = U_i$

$$\Delta U = U_f - U_i = 0$$

Change in internal energy for cyclic process is zero and also $\Delta U \propto \Delta T \Rightarrow \boldsymbol{\Delta T = 0}$

i.e. temperature of system remains constant.

If the cycle is clockwise work done is positive and if the cycle is anticlockwise work done is negative.



Second Law of Thermodynamics

1. Kelvin's statement: No one can design an engine which takes heat and converts entire heat into useful work.

Heat Engine is based on this statement.

2. Clausius statement: We cannot withdraw heat from a colder body to a hotter body without doing external work.

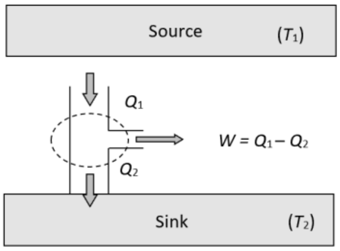
*In other words,*From Clausius statement it is clear that heat cannot flow from a body at low temperature to one at higher temperature unless work is done by an external agent.

Heat Pump is based on this statement.

Working of Heat Engine

Heat engine is a device which converts heat into work continuously through a cyclic process.

1. **Source:** It is a reservoir of heat at high temperature and infinite thermal capacity. Any amount of heat can be extracted from it.
2. **Working substance:** Steam, petrol *etc.*
3. **Sink:** It is a reservoir of heat at low temperature and infinite thermal capacity. Any amount of heat can be given to the sink.



Efficiency of heat engine

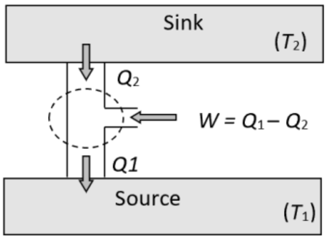
$$\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

Also, $\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

Working of a Heat Pump or Refrigerator

A refrigerator or heat pump is basically a heat engine run in reverse direction.

1. **Source:** At higher temperature T_1.
2. **Working substance:** It is called refrigerant liquid ammonia and freon works as a working substance.
3. **Sink:** At lower temperature T_2.



Coef. of Performance

$$\beta = \frac{\text{Heat extracted}}{\text{Work done}} = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

Relation between coefficient of performance and efficiency of refrigerator

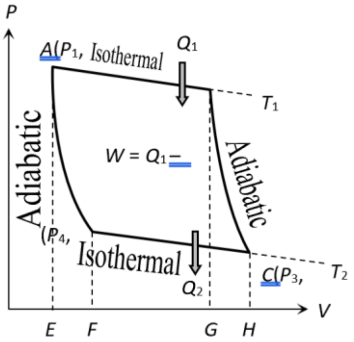
We know $\beta = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2 / Q_1}{1 - Q_2 / Q_1}$ ….. (i)

But the efficiency $\eta = 1 - \frac{Q_2}{Q_1}$ or $\frac{Q_2}{Q_1} = 1 - \eta$ …..(ii)

From (i) and (ii) we get, $\beta = \frac{1-\eta}{\eta}$

Carnot Cycle

As the engine works, the working substance of the engine undergoes a cycle known as Carnot cycle cycle.



(i) First stroke (Isothermal expansion) (curve *AB*) : The cylinder containing ideal gas as working substance allowed to expand slowly at this constant temperature T_1.

Work done = Heat absorbed by the system

$$W_1 = Q_1 = \int_{V_1}^{V_2} P\,dV = RT_1 \log_e\left(\frac{V_2}{V_1}\right) = \text{Area } ABGE$$

(ii) Second stroke (Adiabatic expansion) (curve *BC*) The cylinder is then placed on the non-conducting stand and the gas is allowed to expand adiabatically till the temperature falls from T_1 to T_2.

$$W_2 = \int_{V_2}^{V_3} P\,dV = \frac{R}{(\gamma - 1)}[T_1 - T_2] = \text{Area } BCHG$$

(iii) Third stroke (Isothermal compression) (curve *CD*)

The cylinder is placed on the sink and the gas is compressed at constant temperature T_2.

Work done = Heat released by the system

$$W_3 = Q_2 = -\int_{V_3}^{V_4} P\,dV = -RT_2 \log_e \frac{V_4}{V_3}$$

$$= RT_2 \log_e \frac{V_3}{V_4} = \text{Area } CDFH$$

(iv) Fourth stroke (adiabatic compression) (curve *DA*): Finally the cylinder is again placed on non-conducting stand and the compression is continued so that gas returns to its initial stage.

$$W_4 = -\int_{V_4}^{V_1} P\,dV = -\frac{R}{\gamma-1}(T_2 - T_1)$$

$$= \frac{R}{\gamma-1}(T_1 - T_2) = \text{Area } ADFE$$

Efficiency of Carnot cycle : The efficiency of engine is defined as the ratio of work done to the heat supplied

$$\eta = \frac{\text{Work done}}{\text{Heat input}} = \frac{W}{Q_1}$$

$$\eta = 1 - \frac{T_2}{T_1}$$

Beginner **Ideal Gas Equation**

---

1. The temperature of a gas at pressure P and volume V is 27°C. Keeping its volume constant if its temperature is raised to 927°C, then its pressure will be

(a) 2 P **(b) 3 P**

(c) 4 P **(d) 6 P**

Solution:

The answer is C.

The gas equation states that,

$PV = nRT$

So we can conclude that,

$P \propto T$

Hence,

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Given,

$T_1 = 300K\ , T_2 = 1200, P_1 = P$

$$\therefore P_2 = \frac{P_1 T_2}{T_1}$$

$$\Rightarrow P_2 = \frac{P \times 1200}{300}$$

$\Rightarrow P_2 = 4P\ is\ our\ require$d answer

2. 4 moles of an ideal gas is at 0°C. At constant pressure it is heated to double its volume, then its final temperature will be

(a) 0°C **(b) 273°C**

(c) 546°C **(d) 136.5°C**

Solution:

The answer is B.

Given,

The amount of moles of gas is n =4

The pressure of the gas is P = constant

The initial temperature of the gas is T_1 = 273K

The volume of the gas is doubled.

At constant pressure, using Charles law:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Substitute the values of volume and temperature,

$$\frac{V_o}{273} = \frac{2V_o}{T_2}$$

$T_2 = 546\ K = 273℃$

3. A perfect gas at 27°C is heated at constant pressure so as to triple its volume. The temperature of the gas will be

(a) 81°C **(b) 900°C**

(c) 627°C **(d) 450°C**

Solution:

The answer is C.

$$V \propto T \Rightarrow \frac{V_1}{V_2} = \frac{T_1}{T_2} \Rightarrow \frac{V}{3V} = \frac{(273 + 27)}{T_2}$$

$T_2 = 900K = 627℃$

4. The density of a gas at normal pressure and 27°C temperature is 24. Keeping the pressure constant, the density at 127°C will be

(a) 6 **(b) 12**

(c) 18 **(d) 24**

Solution:

The correct answer is C.

$\rho_1 T_1 = \rho_2 T_2$

$24 \times 300 = \rho_2 \times 400$

$\rho_2 = 18$

5. The volume of a gas at 20°C is 200 ml. If the temperature is reduced to – 20°C at constant pressure, its volume will be

(a) 172.6 ml **(b) 17.26 ml**

(c) 192.7 ml **(d) 19.27 ml**

Solution:

The correct answer is A.

$PV = nRT$

As P is constant

$V \propto T$

$$\left(\frac{V_1}{V_2}\right) = \left(\frac{T_1}{T_2}\right)$$

Given

$V_1 = 200\ ml$

$T_1 = 20\ C$

$T_2 = -20\ C$

$\therefore\ \{200/V_2\} = \{(273 + 20)/(273 - 20)\}$

$\therefore \{200/V_2\} = \{293/253\}$

$\therefore V_2 = 172.6\ ml$

6. If pressure of a gas contained in a closed vessel is increased by 0.4% when heated by 1°C, the initial temperature must be

(a) 250 K **(b) 250°C**
(c) 2500 K **(d) 25°C**

Solution:

The correct answer is A

$we\ know\ that\ \frac{P_1}{T_1}$

$= constant\ for\ constant\ volume.$

Let the initial pressure be P_1 and initial temperature be T_1.

Thus final pressure $P_2 = 1.004\ P_1$ and final temperature $T_2 = T_1 + 1$

Using

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Or

$$T_1 + 1 = 1.004T_1$$

Or

$$\frac{P_1}{T_1} = \frac{1.004P_1}{T_1 + 1}$$

Or

$$T_1 + 1 = 1.004T_1$$

Or

$$o.oo4T_1 = 1$$

$$T_1 = 250K$$

7. To double the volume of a given mass of an ideal gas at 27°C keeping the pressure constant, one must raise the temperature in degree centigrade to

(a) 54° **(b) 270°**
(c) 327° **(d) 600°**

Solution:

The correct answer is C.

$Pressure\ is\ constant\ \&\ PV = nRT$

$i.e\ V = (nRT/P)$

$V \propto T$

Volume is doubled i.e$V_2 = 2V_1$

$(V_2/V_1) = (T_2/T_1)$

$2 = (T_2/T_1)$

$T_2 = 2T_1$

$T_2 = 2(273 + 27)$

$T_2 = 600\ k$

$T_2 = 600 - 273 = 327°C$

8. Volume of gas become four times if

(a) Temperature become four times at constant pressure

(b) Temperature become one fourth at constant pressure

(c) Temperature becomes two times at constant pressure

(d) Temperature becomes half at constant pressure

Solution:

The correct answer is A.

$$V_{rms} = \sqrt{\frac{{v_1}^2 + {v_2}^2 + {v_3}^2 + {v_4}^2 + {v_5}^2}{5}}$$

$$= \sqrt{\frac{2^2 + 3^2 + 4^2 + 5^2 + 6^2}{5}}$$

$$= \sqrt{\frac{100}{5}} = \sqrt{20} = 4.24$$

9. On 0°C pressure measured by barometer is 760 mm. What will be pressure on 100°C

(a) 760 mm **(b) 730 mm**
(c) 780 mm **(d) None of these**

Solution:

The correct answer is B.

$$\frac{C_2}{C_1} = \sqrt{\frac{T_2}{T_1}\frac{M_1}{M_2}}$$

$$= \sqrt{\frac{900}{300} \times \frac{2}{32}} = \sqrt{\frac{3}{16}}$$

$$C_2 = \frac{\sqrt{3}}{4} \times C_1 = \frac{\sqrt{3}}{4} \times 19 = 836\ m/s$$

10. At 100 K and 0.1 atmospheric pressure, the volume of helium gas is 10 litres. If volume and pressure are doubled, its temperature will change to

(a) 400 K **(b) 127 K**
(c) 200 K **(d) 25 K**

Solution:

The correct answer is A.

$PV = nRT$

i.e $PV \propto T$

Given P,V both doubled as $PV \propto T$, temperature will become 4 times of original hence Temperature will change to $4 \times 100 = 400K$

11. The number of molecules in a gas at pressure 1.64×10^{-3} atmospheres and temperature 200 K having the volume 1 cc are

(a) 6.02×10^{16} **(b) 2.63×10^{16}**
(c) 3.01×10^{19} **(d) 12.04×10^{19}**

Solution:

The correct answer is A.

No. of molecules per unit volume,

$$d = \frac{P}{KT}$$

$$d = \frac{1.64 \times 10^{-3}}{1.38 \times 10^{-23} \times 200K}$$

$$= \frac{1.64}{2.76} \times 10^{18} = 0.039 \times 10^{18} = 6.02 \times 10^{16}$$

Expert **Ideal Gas Equation**

---

1. The pressure P, volume V and temperature T of a gas in the jar A and the other gas in the jar B at pressure 2P, volume V/4 and temperature 2T, then the ratio of the number of molecules in the jar A and B will be

(a) 1 : 1 **(b)1 : 2**

(c) 2 : 1 **(d) 4 : 1**

Solution:

The correct answer is D.

$We\ have\ i$deal gas equation pV = nRT

$$= \left(\frac{N}{N_A}\right) RT$$

Where, N = number of molecule, N_A = Avogadro number

$$\frac{N_1}{N_2} = \left(\frac{p_1}{p_2}\right)\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right)$$

$$= \left(\frac{p}{2p}\right)\left(\frac{V}{V/4}\right)\left(\frac{2T}{T}\right) = \frac{4}{1}$$

2. The molecular weight of a gas is 44. The volume occupied by 2.2 g of this gas at 0°C and 2 atm. Pressure will be

(a) 0.56 litre **(b) 1.2 litres**

(c) 2.4 litres **(d) 5.6 litres**

Solution:

The correct answer is A.

$$PV = nRT$$

$$V = \frac{nRT}{P} = \frac{2.2}{44} \times \frac{273 \times 8.314}{2 \times 10^5}$$

$$= \frac{1}{20} \times \frac{273}{2} \times 10^{-5} \times 8.314$$

$$= \frac{273 \times 8.314 \times 10^{-5}}{40} = 56.74 \times 10^{-5}$$

$$= 0.5674 \times 10^{-3} m^3 = 0.5674\ lit$$

3. A balloon contains 1500 m^3 of helium at 27°C and 4 atmospheric pressure.The volume of helium at –3°C temperature and 2 atmospheric pressure will be

(a) 1500 m^3 **(b) 1700 m^3**

(c) 1900 m^3 **(d) 2700 m^3**

Solution:

The correct answer is D.

Here,

$$V_1 = 1500m^3, T_1 = 27°\text{C} = 300K$$

$$P_1 = 4atm, T_2 = -3°\text{C} = 270K$$

$$P_2 = 2atm$$

According to ideal gas equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore V_2 = \frac{P_1V_1}{T_1} \times \frac{T_2}{P_2} = \frac{4 \times 1500 \times 270}{300 \times 2}$$

$$= 2700m^3$$

4. 2 gm of O_2 gas is taken at 27°C and pressure 76 cm. Of Hg. Then find out volume of gas (in litre)

(a) 1.53 **(b) 2.44**

(c) 3.08 **(d) 44.2**

Solution:

The correct answer is A.

$$PV = nRT$$

$$\Rightarrow V = \frac{nRT}{P} = \frac{\frac{2}{32} \times 8.314 \times (27 + 273)}{\frac{760}{1000}} L$$

$$= 2.05\ L$$

Pro **Ideal Gas Equation**

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1. Air is pumped into an automobile tube up to a pressure of 200 kPa in the morning when the air temperature is 22°C. During the day, temperature rises to 42°C and the tube expands by 2%.The pressure of the air in the tube at this temperature, will be approximately

(a) 212 kPa **(b) 209 kPa**

(c) 206 kPa **(d) 200 kPa**

Solution:

The answer is B.

The ideal gas law is the equation of state of an ideal gas. This state of an amount of gas is determined by its pressure, volume and temperature. The equation has the form $PV = nRT$

Where p is pressure , V is volume, n is the number of moles, R the gas constant and T is the temperature.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Given

$$P_1 = 200kPa\ , V_1 = V, T_1 = 273 + 22 = 295K$$

$V_2 = V + 0.02, T_2 = 273 + 42 = 315K$

$$\frac{200 \times V}{295} = \frac{P_2 \times 1.02V}{315} \Rightarrow P_2 = \frac{200 \times 315}{295 \times 1.02}$$

$P_2 = 209.37$

2. Two identical containers A and B with frictionless pistons contain the same ideal gas at the same temperature and the same volume V. The mass of gas contained in A is m_A and that in B is m_B . The gas in each cylinder is now allowed to expand isothermally to the same final volume 2V. The change in the pressure in A and B are found to be ΔP and 1.5 ΔP respectively. Then

(a) $4m_A = 9m_B$ (b) $2m_A = 3m_B$

(c) $3m_A = 2m_B$ (d) $9m_A = 4m_B$

Solution:

The answer is C.

The initial pressure in the two containers will be

$$P_A = \frac{n_A RT}{V} = m_A\left(\frac{RT}{MV}\right)$$

$$P_B = \frac{n_B RT}{V} = m_B\left(\frac{RT}{MV}\right)$$

After isothermal expansion, pressure will be

$$P_a = \frac{n_A RT}{2V} = m_A\left(\frac{RT}{2MV}\right)$$

$$P_b = \frac{n_B RT}{2V} = m_B\left(\frac{RT}{2MV}\right)$$

$$\therefore -\Delta P_A = P_a - P_A = m_A\left(\frac{RT}{2MV}\right)$$

$$\therefore -\Delta P_B = P_b - P_B = m_B\left(\frac{RT}{2MV}\right)$$

But

$$\Delta P_A = \Delta P \ and - \Delta P_B = 1.5\Delta P$$

So,

$$\frac{-\Delta P_A}{-\Delta P_B} = \frac{1}{1.5} \Rightarrow \Delta P_A = \frac{-\Delta P_B}{1.5}$$

$$m_A\left(\frac{RT}{2MV}\right) = \frac{m_B}{1.5}\left(\frac{RT}{2MV}\right)$$

Or

$$m_A = \frac{10m_B}{15}$$

$$\therefore 3m_A = 2m_B$$

3. A vessel of volume V = 30 l contains ideal gas at the temperature 0°C. After a portion of the gas has been let out, the pressure in the vessel decreased by Δ p = 0.78 atm (the temperature remaining constant). Find the mass of the released gas. The gas density under the normal conditions ρ = 1.3 g/1.

Solution:

$\rho_0 = 1atm$

$T = 0°C$

Finally:

$$PV = \frac{m}{M}RT_0$$

$$\Delta PV = \frac{\Delta m}{m}RT_0$$

$$\frac{\Delta P}{P} = \frac{\Delta m}{m} \qquad \Rightarrow \Delta m = \frac{\delta V \Delta P}{\rho_0}$$

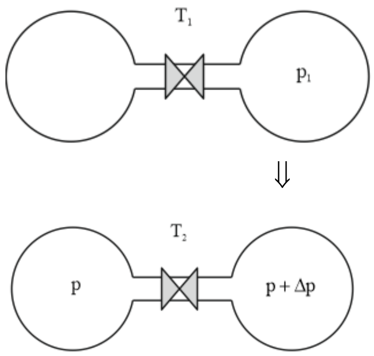
$\therefore$ At normal conditions:

$\rho \to \rho_0$

$m \to \delta V$

4. Two identical vessels are connected by a tube with a valve letting the gas pass from one vessel into the other if the pressure difference $\Delta p \geq$ 1.10 atm. Initially there was a vacuum in one vessel while the other contained ideal gas at a temperature t_1 =27°C and pressure p_1 = 1.00 atm. Then both vessels were heated to a temperature t_2 =107°C. Up to what value will the pressure in the first vessel (which had vacuum initially) increase?

Solution:



Conserving moles:

$$\frac{p_1 v}{T_1} = \frac{pv}{T_2} + \frac{(p + \Delta p)\, v}{T_2}$$

$$\Rightarrow p = \left(\frac{p_1}{T_1} - \frac{\Delta p}{T_2}\right)\frac{T_2}{2}$$

5. A vessel of volume V = 20 l contains a mixture of hydrogen and helium at a temperature t = 20°C and pressure p = 2.0 atm. The mass of the mixture is equal to m = 5.0 g. Find the ratio of the mass of hydrogen to that of helium in the given mixture.

Solution:

Let hydrogen be n_1 moles.

$$pv=\left(n_1+n_2\right)RT$$

$$n_1M_1+n_2M_2=m$$

$$n_1=\frac{\left(m-\frac{pv}{RT}M_2\right)}{M_1-M_2}$$

$$n_2=\frac{\left(m-\frac{pv}{RT}M_1\right)}{M_2-M_1}$$

$\therefore$ mass ratio

$$\frac{m_1}{m_2}=\frac{n_1M_1}{n_2M_2}$$

$$=\frac{\left(\frac{pv}{RT}M_2-m\right)M_1}{\left(m-\frac{pv}{RT}M_1\right)M_2}$$

$$=\frac{\frac{pv}{RT}-\frac{m}{M_2}}{\frac{m}{M_1}-\frac{pv}{RT}}$$

6. A vessel contains a mixture of nitrogen (m_1 = 7.0 g) and carbon dioxide (m_2 = 11g) at a temperature T = 290 K and pressure p_0 = 1.0 atm. Find the density of this mixture, assuming the gases to be ideal.

Solution:

$$pv=\left(\frac{m_1}{M_1}+\frac{m_2}{M_2}\right)RT$$

$$\delta=\frac{m_1+m_2}{v}=\frac{m_1+m_2}{\left(\frac{m_1}{M_1}+\frac{m_2}{M_2}\right)}\frac{P}{RT}$$

7. A vessel of volume V = 7.5 l contains a mixture of ideal gases at a temperature T = 300 K: v_1 =0.10 mole of oxygen, v_2 =0.20 mole of nitrogen, and v_3 = 0.30 mole of carbon dioxide. Assuming the gases to be ideal,

Find:

(a) The pressure of the mixture;

(b) The mean molar mass M of the given mixture which enters its equation of state pV =(m/M) RT, where m is the mass of the mixture.

Solution:

A. $p=\frac{\left(n_1+n_2+n_3\right)RT}{v}$

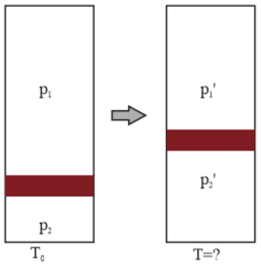
B. $n_1+n_2+n_3=\frac{m_{net}}{M_{net}}$

$$=\frac{n_1M_1+n_2M_2+n_3M_3}{M_{net}}$$

$$\Rightarrow M_{net}=\frac{n_1M_1+n_2M_2+n_3M_3}{n_1+n_2+n_3}$$

8. A vertical cylinder closed from both ends is equipped with an easily moving piston dividing the volume into two parts, each containing one mole of air. In equilibrium at T_0 = 300 K the volume of the upper part is η = 40 times greater than that of the lower part. At what temperature will the ratio of these volumes be equal to η' = 3.0?

Solution:



Piston is at rest in both cases

$$\therefore mg=\left(p_2-p_1\right)A$$

$$=\left(p_2^1-p_1^1\right)A$$

$$\Rightarrow nR\left(\frac{T_0}{v_2}-\frac{T_0}{v_1}\right)=nR\left(\frac{T}{V_2^1}-\frac{T}{V_1^1}\right)$$

$$\Rightarrow T_0\left(\eta+1\right)\left(1-\frac{1}{\eta}\right)$$

$$=T\left(\eta'+1\right)\left(1-\frac{1}{\eta'}\right)$$

$$\Rightarrow T=\left(\frac{\eta^2-1}{\eta'^2-1}\right)\frac{\eta'}{\eta}T_0$$

Let total volume be V

$$v_1\rightarrow\frac{\eta v}{n+1}$$

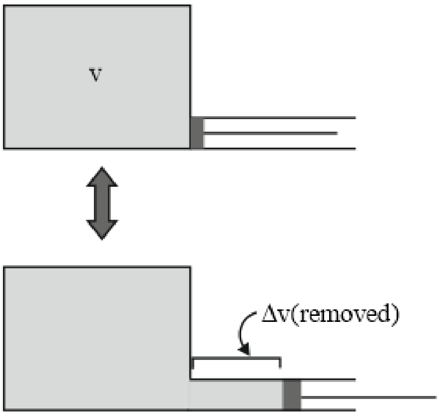
$$v_2\rightarrow\frac{v}{\eta+1}$$

$$v_1\rightarrow\frac{\eta' v}{\eta'+1}$$

$$v_2\rightarrow\frac{v}{\eta'+1}$$

9. A vessel of volume V is evacuated by means of a piston air pump. One piston stroke captures the volume Δ V. How many strokes are needed to reduce the pressure in the vessel η times? The process is assumed to be isothermal, and the gas ideal.

Solution:



After 1st stroke:

$$p_0 v = p_1\left(v+\Delta v\right)$$

$$\Rightarrow p_1 = \frac{p_0 v}{v+\Delta v}$$

$$p_1 v = p_2\left(v+\Delta v\right)$$

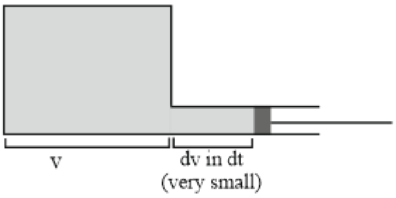
$$\Rightarrow p_2 = p_0\left(\frac{v}{v+\Delta v}\right)^2$$

$$p_n = p_0\left(\frac{v}{v+\Delta v}\right)^{\eta} = \frac{p_0}{\eta}\left(given\right)$$

$$\Rightarrow n = \frac{\ell n\, \eta}{\ell n \dfrac{v+\Delta v}{v}}$$

10. Find the pressure of air in a vessel being evacuated as a function of evacuation time t. The vessel volume is V, the initial pressure is p_0. The process is assumed to be isothermal, and the evacuation rate equal to C and independent of pressure.
Note: The evacuation rate is the gas volume being evacuated per unit time, with that volume being measured under the gas pressure attained by that moment.

Solution:



$$\Rightarrow \frac{dV}{dt} = C$$

At any time t, let the pressure be P.
Between t and t + dt:

$$pv = \left(p+dp\right)\left(v+dv\right)$$

$$\Rightarrow pv = pv + pCdt + vdp + Cdpdt$$

$$\Rightarrow -\frac{dp}{p} = \frac{Cdt}{v}$$

$$\Rightarrow \int_{p_0}^{p} \frac{dp}{p} = -\frac{c}{v}\int_0^t dt$$

$$\Rightarrow p = p_0 e^{-ct/v}$$

11. A chamber of volume V =87 1 is evacuated by a pump whose evacuation rate (see Note to the foregoing problem) equals C = 10 1/s. How soon will the pressure in the chamber decrease by n = 1000 times?

Solution:

$$p = p_0 e^{-ct/v}$$

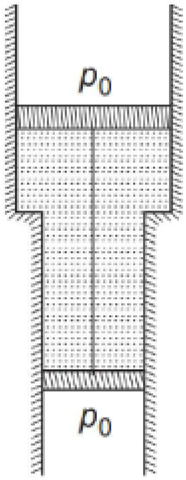
$$\Rightarrow \frac{p_0}{1000} = p_0 e^{-ct/v}$$

$$\Rightarrow 1000 = e^{\frac{10t}{87}}$$

$$\Rightarrow t = 60\sec\ = 1\min$$

(In this realistic situation, we observe that pressure drops quite quickly.)

12. A smooth vertical tube having two different sections is open from both ends and equipped with two pistons of different areas (Fig.). Each piston slides within a respective tube section. One mole of ideal gas is enclosed between the pistons tied with a non-stretchable thread. The cross-sectional area of the upper piston is Δ S = 10 cm^2 greater than that of the lower one. The combined mass of the two pistons is equal to m = 5.0 kg. The outside air pressure is p_0 = 1.0 atm. By how many kelvins must the gas between the pistons be heated to shift the pistons through l = 5.0 cm?



Solution:

When pistons are at rest, $F_{net} = 0$:

$$\therefore mg + (p - p_0)S_2 = (p - p_0)S_1$$

$$\Rightarrow mg = (p - p_0)\Delta S$$

$$\Rightarrow p = p_0 + \frac{mg}{\Delta s}$$

Let initial vol = v

Final vol $= v + S_1\ell - S_2\ell = V + \Delta S\ell$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

$$\Rightarrow \frac{pv}{T} = \frac{p(v + \Delta s\ell)}{T + \Delta T}$$

$$\Rightarrow 1 + \frac{\Delta T}{T} = 1 + \frac{\Delta S\ell}{V}$$

$$\Rightarrow \Delta T = \Delta S\ell\left(\frac{T}{v}\right) = \frac{\Delta S\ell p}{nR}$$

$$= \frac{\Delta S\ell}{nR}\left(p_0 + \frac{mg}{\Delta S}\right)$$

13. Let us assume that air is under standard conditions close to the Earth's surface. Presuming that the temperature and the molar mass of air are independent of height, find the air pressure at the height 5.0 km over the surface and in a mine at the depth 5.0 km below the surface.

Solution:

$$dp = -\delta g dh$$

$$Also, \quad p = \frac{\delta RT}{M}$$

$$Dividing: \frac{dp}{p} = \frac{-Mg}{RT}dh$$

$$\Rightarrow \int_{p_0}^{p} \frac{dp}{p} = -\frac{Mg}{RT}\int_0^h dh$$

$$\Rightarrow p = p_0 e^{-\frac{Mgh}{RT}}$$

$$h = +km$$

$$p_{+5} = 0.5\ atm$$

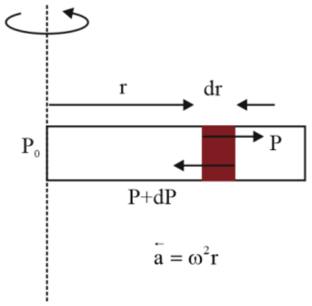
$$h = -5km$$

$$p - 5 = 2\ atm$$

$\therefore$ Pressure increases at faster rate below the surface, than in drops above the surface.

14. A horizontal cylinder closed from one end is rotated with a constant angular velocity ω about a vertical axis passing through the open end of the cylinder. The outside air pressure is equal to p_0, the temperature to T, and the molar mass of air to M. Find the air pressure as a function of the distance r from the rotation axis. The molar mass is assumed to be independent of r.

Solution:



($\because$ this end is open to atmosphere)

F = ma

$$\therefore (p + dp)A - pa = dm\omega^2 r$$

$$\Rightarrow dp.A = \frac{pM}{RT}.A.dr.\omega^2 r$$

$$\Rightarrow \int_{p_0}^{p} \frac{dp}{p} = \frac{m\omega^2}{RT}\int_0^r rdr$$

$$\Rightarrow p = p_0 e^{\frac{M\omega^2 r^2}{2RT}}$$

Beginner Thermodynamic Processes

1. Work done by air when it expands from 50 litres to 150 litres at a constant pressure of 2 atmosphere is

(a) 2 × 10^4 joules

(b) 2 × 100 joules

(c) $2 \times 10^5 \times 100$ joules

(d) $2 \times 10^5 \times 100$ joules

Solution:

The answer is A.

$W = P\Delta V = 2 \times (150 - 50)$

$\Rightarrow 2 \times 100$

$\Rightarrow 200\ litre\ atm$

$\Rightarrow 200 \times 01.33J$=$2 \times 10^4\ J$

2. Work done by 0.1 mole of a gas at 27°C to double its volume at constant pressure is (R= 2 cal mol^{-1} °C^{-1})

(a) 54 cal (b) 600 cal

(c) 60 cal (d) 546 cal

Solution:

The answer is C.

$T = 27℃$

$= 27 + 273 = 300$

From ideal gas equation

$w = -p\text{dv}$

$= -p(2v - v) = -p.v \Rightarrow -nRT$

$= 0.1mol \times 2cal\ mol^{-1}K^{-1} \times 300k$

$= 0.1 \times 2 \times 300 = 60\ cal$

3. Which of the following is correct in terms of increasing work done for the same initial and final state

(a) Adiabatic < Isothermal < Isobaric

(b) Isobaric < Adiabatic < Isothermal

(c) Adiabatic < Isobaric < Isothermal

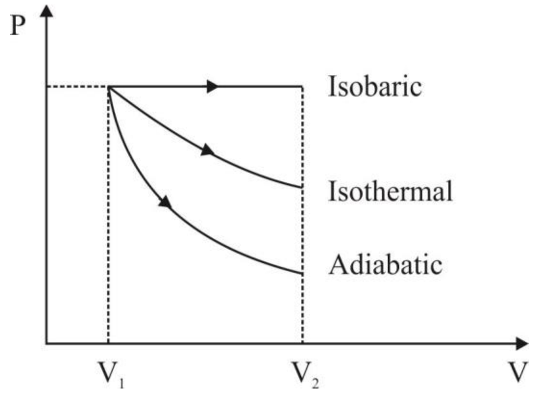
(d) None of these

Solution:

The correct answer is A.

In thermodynamic process, work done is equal to the area covered by the PV curve with volume axis. Hence, according to graph shown

$W_{\text{adiabatic}} < W_{isothermal} < W_{isobaric}$



4. In an isothermal reversible expansion, if the volume of 96 gm of oxygen at 27°C is increased from 70 litres to 140 litres, then the work done by the gas will be

(a) 300 R $\log_{10} 2$ (b) 81 R $\log_{10} 2$

(c) 900 R $\log_{10} 2$ (d) 2.3 × 900R $\log_{10} 2$

Solution:

The correct answer is D.

For the isothermal reversible process

$W = nRT \log_e \frac{V_2}{V_1}$

$\left(\frac{m}{M}\right) RT \log_e \frac{V_2}{V_1} = 2.3$

$\times \frac{96}{32} R(273 + 27) \log_{10} \frac{140}{70}$

$= 2.3 \times 900R \log_{10} 2$

5. A vessel containing 5 litres of a gas at 0.8 m pressure is connected to an evacuated vessel of volume 3 litres. The resultant pressure inside will be (assuming whole system to be isolated)

(a) 4/3 m (b) 0.5 m

(c) 2.0 m (d) 3/4 m

Solution:

The correct answer is B

From the ideal gas equation

$P_1V_1 = P_2V_2$

Given

$V_1 = 5L\ , P_1 = 0.8m\ , P_2 = P\ , V_2 = 5 + 3 = 8L$

$0.8 \times 5 = P \times (3 + 5) \Rightarrow P = 0.5m$

6. When an ideal gas in a cylinder was compressed isothermally by a piston, the work done on the gas was found to be -1.5 × 10^4 joules. During this process about

(a) 3.6 × 10^3 cal of heat flowed out from the gas

(b) 3.6 × 10^3 cal of heat flowed into the gas

(c) 1.5 × 10^4 cal of heat flowed into the gas

(d) 1.5 × 10^4 cal of heat flowed out from the gas

Solution:

The correct answer is A

In isothermal compression, there is always an increases of heat which must flow out the gas.

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta W(\Delta U = 0)$

$$\Rightarrow \Delta Q = -1.5 \times 10^4 J = \frac{-1.5 \times 10^4}{4.18} cal$$
$$= -3.6 \times 10^3 cal$$

The –ve sign represents the heat is flowed out from the gas.

7. One mole of an ideal gas expands at a constant temperature of 300 K from an initial volume of 10 litres to a final volume of 20 litres. The work done in expanding the gas is (R = 8.31 J/mole-K)

(a) 750 joules **(b) 1728 joules**
(c) 1500 joules **(d) 3456 joules**

Solution:

The correct answer is B.

Given

$T = 300K$

$V_1 = 10L$

$V_2 = 20L$

According to the isothermal process, work done is given by

$$W_{iso} = \mu RT \log_e \frac{V_2}{V_1}$$
$$= 1 \times 8.31 \times 300 \log_e \frac{20}{10} = 750J$$

8. The volume of an ideal gas is 1 litre and its pressure is equal to 72cm of mercury column. The volume of gas is made 900 cm^3 by compressing it isothermally. The stress of the gas will be

(a) 8 cm (mercury) **(b) 7 cm (mercury)**
(c) 6 cm (mercury) **(d) 4 cm (mercury)**

Solution:

The correct answer is A.

For isothermal process

$$p_1V_1 = p_2V_2$$
$$\Rightarrow p_2 = \frac{p_1V_1}{V_2} = \frac{72 \times 1000}{900} = 80cm$$
$$Stress\ \Delta p = p_1 - p_2 = 80 - 72 = 8\ cm$$

9. A monoatomic gas (γ = 5/3) is suddenly compressed to $\frac{1}{8}$ of its original volume adiabatically, then the pressure of the gas will change to

(a) $\frac{24}{5}$ **(b) 8**
(c) $\frac{40}{3}$ **(d) 32 times its initial pressure**

Solution:

The correct answer is D.

Given

$$V_2 = \frac{1}{8}V_1$$
$$y = \frac{5}{3}$$

According to the adiabatic compression

$$PV^y = Constant \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^y \Rightarrow P_2$$
$$= (8)^{\frac{5}{3}}P_1$$
$$= 32P_1$$

10. An ideal gas at 27°C is compressed adiabatically to $\frac{8}{27}$ of its original volume. If $\gamma = \frac{5}{3}$, then the rise in temperature is

(a) 450 K **(b) 375 K**
(c) 225 K **(d) 405 K**

Solution:

The correct answer is B.

In an adiabatic process,

$$TV^{y-1} = constant$$
$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{y-1}$$
$$\frac{300}{T_2} = \left(\frac{\frac{8}{27}V}{V}\right)^{\frac{5}{3}-1} = \left(\frac{8}{27}\right)^{\frac{2}{3}} = \frac{4}{9}$$
$$T_2 = 300 \times \frac{9}{4} = 675K$$
$$\therefore raise\ in\ temperatue = 675 - 300 = 375K$$

11. The amount of work done in an adiabatic expansion from temperature T to T$_1$ is

(a)R(T – T$_1$) **(b)** $\frac{R}{\gamma-1}(T-T_1)$
(c)RT **(d)** $R(T-T_1)(\gamma-1)$

Solution:

The correct answer is B.

Work done by adiabatic process is

$$W_{\text{adi}} = \frac{R}{y-1}(T_i - T_f) = \frac{R}{y-1}(T - T_I)$$

12. An ideal gas is expanded adiabatically at an initial temperature of 300 K so that its volume is doubled. The final temperature of the hydrogen gas is (γ = 1.40)

(a) 227.36 K **(b) 500.30 K**

(c) 454.76 K **(d) –47°C**

Solution:

The correct answer is A.

Given

$T_1 = 300K, Y = 1.40, V_2 = 2V_1$

$TV^{Y-1} = constant \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{Y-1} \Rightarrow T_2$

$= T_1\left(\frac{V_1}{V_2}\right)^{Y-1}$

$\Rightarrow T_2 = 300\left(\frac{1}{2}\right)^{0.4} = 227.36K$

13. In an adiabatic expansion of a gas initial and final temperatures are T_1 and T_2 respectively, then the change in internal energy of the gas is

(a) $\frac{R}{\gamma-1}(T_2-T_1)$ **(b)** $\frac{R}{\gamma-1}(T_1-T_2)$

(c) $R(T_1 – T_2)$ **(d) Zero**

Solution:

The correct answer is A.

$T_1 \rightarrow Initial\ temperature$

$T_2 \rightarrow Final\ temperature$

Since it is adiabatic expansion, there will be no heat energy, so $\Delta Q = 0$ (Q = heat generated).

We know

$\Delta Q = \Delta V + \Delta W$ (W = Work done)

$\Rightarrow \Delta W = -\Delta U \quad (U \rightarrow total\ internal\ energy)$

$We\ know\ in\ a$diabotic process $\rightarrow \Delta W$

$= \frac{R(T_1 - T_2)}{Y - 1}$

So,

$\Delta U = -\Delta W = \frac{-R(T_1 - T_2)}{Y - 1}$

$= \frac{R(T_2 - T_1)}{Y - 1}$

14. Two moles of an ideal monoatomic gas at 27°C occupies a volume of V. If the gas is expanded adiabatically to the volume 2V, then the work done by the gas will be [γ = 5/3, R = 8.31 J/mol K]

(a) –2767.23 J **(b) 2767.23 J**

(c) 2500 J **(d) 2500 J**

Solution:

The correct answer is B.

In adiabatic process,

$TV^{Y-1} = constant$

$T_1V_1 = T_2V_2$

$300 \times V^{\frac{5}{3}-1} = T(2V)^{\frac{5}{3}-1}$

$T_2 = \frac{300}{2^{\frac{2}{3}}} = 188.7$

$= 189K$

Now, work done is given by

$W = \frac{nR(T_1 - T_2)}{Y - 1}$

$= \frac{2 \times 8.314 \times (300 - 189) \times 3}{2}$

$= 2768.56J$

15. A gas (γ = 1.3) is enclosed in an insulated vessel fitted with insulating piston at a pressure of 10^5 N/m^2 . On suddenly pressing the piston the volume is reduced to half the initial volume. The final pressure of the gas is

(a) $2^{0.7} \times 10^5$ **(b) $2^{1.3} \times 10^5$**

(c) $2^{1.4} \times 10^5$ **(d) None of these**

Solution:

The correct answer is B.

Given,

$Y = 1.3$

$P_1 = 10^5 N/m^2$

$2V_1 = V_2$

As it is adiabatic process, so using the relation

$\therefore\ PV^Y = k\ (constant)$

$P_1{V_1}^Y = P_2{V_2}^Y$

$P_1\frac{{V_1}^Y}{{V_2}^Y} = P_2$

$P_2 = 2^{1.3} \times 10^5 N/m^2$

16. A vessel contains 1 mole of O_2 gas (molar mass 32) at a temperature T. The pressure of the gas is P. An identical vessel containing one mole of He gas (molar mass 4) at a temperature 2T has a pressure of

(a) P/8 **(b) P**

(c) 2P **(d) 8P**

Solution:

Correct option is C

We know that:

$PV = nRT$

Here,

$P_1 = P, V_1 = V_2 = V$

$(as\ identical\ vessel\ volume\ V\ is\ same\ for\ both\ g)$

$T_1 = T, T_2 = 2T, n_1 = n_2 = 1, P_2 =?$

Thus

$\frac{P_2}{P_1} = \frac{T_2}{T_1}$

Or

$$\frac{P_2}{P}=\frac{2T}{T}\Rightarrow P_2=2P$$

17. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is :

(a) 32 **(b) 326**

(c) 128 **(d) $\frac{1}{32}$**

Solution:

(c) In adiabatic process

PV^{γ} = constant

$$\therefore\ P\left(\frac{m}{\rho}\right)^{\gamma}=\text{constant} \qquad \left(\because V=\frac{m}{\rho}\right)$$

As mass is constant

$$\therefore\ P\propto\rho^{\gamma}$$

If P_i and P_f be the initial and final pressure of the gas and ρ_i and ρ_f be the initial and final density of the gas. Then

$$\frac{P_f}{P_i}=\left(\frac{\rho_f}{\rho_i}\right)^{\gamma}=(32)^{7/5}$$

$$\frac{nP_i}{P_i}=\left(2^5\right)^{7/5}=2^7$$

n = 2^7 = 128.

18. An ideal gas at atmospheric pressure is adiabatically compressed so that its density becomes 32 times of its initial value. If the final pressure of gas is 128 atmospheres, the value of 'γ'of the gas is :

(a) 1.5 **(b) 1.4**

(c) 1.3 **(d) 1.6**

Solution:

(b) Volume of the gas

$v=\frac{m}{d}$ and

Using PV^{γ} = constant

$$\frac{P'}{P}=\frac{V}{V'}=\left(\frac{d'}{d}\right)^{\gamma}$$

Or $128=(32)^{\gamma}$

$$\therefore\ \gamma=\frac{7}{5}=1.4$$

19. Two cylinders A and B of equal capacity are connected to each other via a stop cock. A contains an ideal gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stop cock is suddenly opened. The process is

(a) Adiabatic **(b) Isochoric**

(c) Isobaric **(d) Isothermal**

Solution:

(a) Since, the entire system is thermally insulated. Thus, their will be no transfer of heat between the system and the surrounding. Hence, when the stop cock is suddenly opened, a sudden process of expansion will takes place with no heat transfer. Thus, the given process will be adiabatic, as it is a process that occurs without transferring heat or mass between a thermodynamic system and its surroundings.

Hence, correct option is (a).

Expert Thermodynamic Processes

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1. One mole of an ideal gas with γ = 1.4 , is adiabatically compressed so that its temperature rises from 27°C to 35°C. The change in the internal energy of the gas is (R = 8.3 J/mol K)

(a) –166 J **(b) 166 J**

(c) –168 J **(d) 168 J**

Solution:

The correct option is B

$$\Delta U=C_V\Delta T$$

$$Now, C_P-C_V=R$$

$$or, \frac{C_P}{C_V}-1=\frac{R}{C_V}$$

$$or, C_V=\frac{R}{\gamma-1}, where\ \gamma=\frac{C_P}{C_V}$$

$$Hence, \Delta U=\frac{R\Delta T}{\gamma-1}=\frac{8.314x8}{1.4-1}=166J$$

2. One mole of a perfect gas in a cylinder fitted with a piston has a pressure P, volume V and temperature T. If the temperature is increased by 1 K keeping pressure constant, the increase in volume is

(a) $\frac{2V}{273}$ **(b) $\frac{V}{91}$**

(c) $\frac{V}{273}$ **(d) V**

Solution:

The answer is C.

Given

$T_1 = 273K$

$T_2 = 274K$

$V_1 = V$

$$For\ isobaric\ process\ \frac{V_2}{V_1} = \frac{T_2}{T_1} \Rightarrow V \times \frac{274}{273}$$

$$Increases = \frac{274V}{273} - V = \frac{V}{273}$$

3. In an adiabatic process on a gas with γ = 1.4, the pressure is increased by 0.5%. The volume decreases by about

(a) 0.36% **(b) 0.5%**

(c) 0.7% **(d) 1%**

Solution:

Correct option is A

From the relative $PV^{Y} = k$

Now differentiating we get

$P\gamma V^{Y-1}dV + dP.V^{Y} = 0$

By making proper rearrangements we get,

$$-\frac{dV}{V} = \frac{dP}{\gamma P}$$

In an adiabatic process on a gas with $\gamma = 1.4$,the pressure is increased by 0.5%.

$$the\ volume\ decreases\ by\ about\ -\frac{dV}{V}$$

$$= 0.5/1.4 \times 100 = 0.36\%$$

4. 12gms of gas occupy a volume of 4×10^{-3} m^3 at temperature of 7°C. After the gas is heated at constant pressure its density becomes 6×10^{-4} gm/cc. What is the temperature to which the gas was heated.

(a) 1000K **(b) 1400K**

(c) 1200K **(d) 800K**

Solution:

Correct option is B.

$$PV = nRT = \frac{w}{M}RT$$

Hence,

$$P = \frac{\rho}{M}RT$$

Initially,

$$PV = \frac{w}{M}RT_1 - - - (1)$$

After heating,

$$P = \frac{\rho}{M}RT_2 - - - -(2)$$

From equation (1) and (2)

The temperature to which the gas was heated is

$$T_2 = \frac{wT_1}{\rho V} = \frac{12 \times 280}{6 \times 10^{-4} \times 4 \times 10^{3}} = 1400$$

Pro Thermodynamic Processes

1. One mole of an ideal gas at an initial temperature of T K does 6 R joules of work adiabatically. If the ratio of specific heats of this gas at constant pressure and at constant volume is 5/3, the final temperature of gas will be

(a) (T + 2.4)K **(b) (T – 2.4)K**

(c) (T + 4)K **(d) (T – 4)K**

Solution:

The answer is D.

$$we\ have\ specific\ heat\ ratio = Y = \frac{5}{3}$$

Also, the specific heat at constant volume is given by,

$$C_V = \frac{R}{Y-1} = \frac{3R}{2}$$

In adiabatic process:

$\Rightarrow \mathrm{dQ} = \mathrm{dU} + \mathrm{dW} = 0$

$\Rightarrow \mathrm{dU} = -\mathrm{dW} = -6\mathrm{R}$

$\Rightarrow C_V\mathrm{dT} = -6\mathrm{R}$

$\Rightarrow \mathrm{dT} = -4$

This gives the final temperature as (T – 4) Kelvin.

2. A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes (R-a). For a<<R the magnitude of the work done in the process is given by, where X is a constant and $\gamma = C_p/C_v$ = 41/30. The value of X is ______.

Solution:

$$W = (\Delta P)_{avg}\, x\, 4\pi R^2 a$$

$$= \left|\frac{dP}{2}.4\pi R^2 a\right|$$

$$for\ small\ change\ (\Delta P)_{avg} < P > arthemetic\ mean$$

$$PV^{\gamma} = c$$

$$=> dP = -\gamma \frac{P}{V} dV = -\frac{\gamma P_0}{V} 4\pi R^2 a$$

$$= \frac{\gamma P_0}{2V} x\, 4\pi R^2 a\, x\ 4\pi R^2 a$$

$$= \frac{\gamma P_0}{2x4\pi R^3} x\, 4\pi R^2 a\, x\, 4\pi R^2 a$$

$$= (4pRP\, x\, a^2)\frac{3\gamma}{2}$$

$$\therefore x = 2.05$$

3. Demonstrate that the internal energy U of the air in a room is independent of temperature provided the outside pressure p is constant. Calculate U, if p is equal to the normal atmospheric pressure and the room's volume is equal to V = 40 m³ .

Solution:

A. Assuming air to be ideal gas, its internal energy is:

$$U = nCvT$$

$$= \frac{nRT}{r-1} = \frac{pV}{r-1}$$

=constant, for a room of value V and constant P. hence proved.

B. For a given room at 1 atm

$$U = \frac{p_0 V}{r-1} = \frac{(10)^5 (40)}{(1.4-1)} = 10^7 J$$

$$= \frac{1}{2} 80 (500)^2$$

Kinetic energy of 80kg man running at 500m/s (supersonic)

4. A thermally insulated vessel containing a gas whose molar mass is equal to M and the ratio of specific heats $C_p/C_v = \gamma$ moves with a velocity v. Find the gas temperature increment resulting from the sudden stoppage of the vessel.

Solution:

$$U_i + KE_i = U_f + KE_f$$

Due to movement of vessel

$$\therefore \Delta U = KE_i$$

$$\Rightarrow nC_v \Delta T = \frac{1}{2} mv^2$$

$$\Rightarrow \Delta T = \frac{\frac{1}{2} mv^2}{\frac{m}{M} \frac{R}{r-1}} = \frac{(r-1) mv^2}{2R}$$

5. Two thermally insulated vessels 1 and 2 are filled with air and connected by a short tube equipped with a valve. The volumes of the vessels, the pressures and temperatures of air in them are known (V_1, P_1,T_1 and V_2, p_2, T_2). Find the air temperature and pressure established after the opening of the valve.

Solution:

As vessels are thermally insulated, U is conserved

$$\therefore n_1 C_v T_1 + n_2 C_v T_2$$

$$= (n_1 + n_2) C_v T$$

$$\Rightarrow n_1 T_1 + n_2 T_2 = (n_1 + n_2) T$$

$$\Rightarrow T = \frac{n_1 T_1 + n_2 T_2}{n_1 n_2}$$

$$\Rightarrow T = \frac{p_1 V_1 + p_2 v_2}{\frac{p_1 V_1}{T_1} + \frac{p_2 v_2}{T_2}}$$

N is conserved

$$\therefore n_1 + n_2 = n_{mix}$$

$$\Rightarrow n_1 + n_2 = \frac{p(v_1 + v_2)}{RT}$$

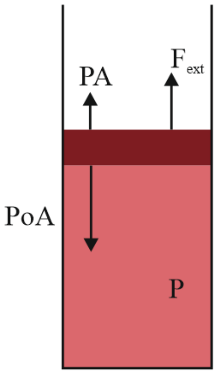
$$\Rightarrow p = \frac{R(n_1 + n_2) T}{v_1 + v_2}$$

$$= \frac{R(n_1 T_1 + n_2 T_2)}{v_1 + v_2}$$

$$= \frac{p_1 v_1 + p_2 v_2}{v_1 + v_2}$$

6. One mole of a certain ideal gas is contained under a weightless piston of a vertical cylinder at a temperature T. The space over the piston opens into the atmosphere. What work has to be performed in order to increase isothermally the gas volume under the piston n times by slowly raising the piston? The friction of the piston against the cylinder walls is negligibly small.

Solution:



$$dw_{ext} = F_{ext}dx = \left(p_0 - p\right)Adx$$

$$= \left(p_0 - p\right)dv$$

$$\Rightarrow w_{ext} = \int_{v_0}^{\eta v_0} \left(p_0 - p\right)dv$$

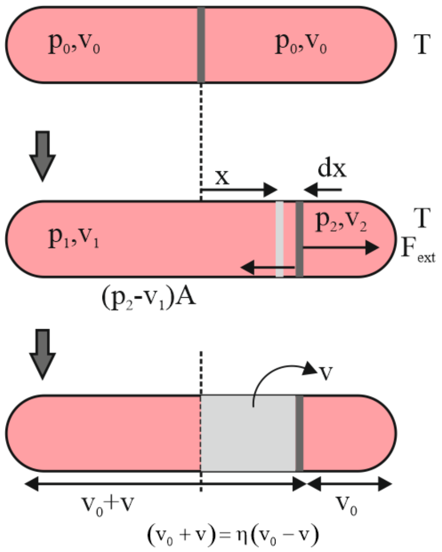
$$= \left(\eta - 1\right)p_0 - v_0 - \eta RT_0 \int_{v_0}^{\eta v_0} \frac{dv}{v}$$

$$\left(\eta - 1\right)\eta RT_0 - \eta RT_0 \ell n\eta$$

$$= \eta RT_0\left(\eta - 1 - \ell n\ \eta\right)$$

7. A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0, in which an ideal gas is contained under the same pressure p_0 and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas η times compared to that of the other by slowly moving the piston?

<u>**Solution:**</u>



$$dw = F_{ext}dx$$

$$= \left(p_2 - p_1\right)Adx$$

$$= \left(\frac{p_0v_0}{v_0 - Ax} - \frac{p_0v_0}{v_0 + Ax}\right)Adx$$

$$= \frac{\left(p_0v_0\right)2Ax.Adx}{v_0^2 - A^2x^2} \qquad Ax \to v, Adx \to dv$$

$$v = \left(\frac{\eta - 1}{\eta + 1}\right)v_0$$

$$\therefore w = 2p_0v_0 \int_0^{v=\left(\frac{\eta-1}{\eta+1}\right)v_0} \frac{vdv}{v_0^2 - v^2} = p_0v_0\ \ell n\frac{\left(\eta + 1\right)^2}{4\eta}$$

Beginner 1st Law of Thermodynamics

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1. The temperature of an ideal gas is kept constant as it expands. The gas does external work. During this process, the internal energy of the gas

(a) Decreases

(b) Increases

(c) Remains constant

(d) Depends on the molecular motion

<u>**Solution:**</u>

The answer is C.

Internal energy changes in an ideal gas may be described solely changes in its kinetic energy. Kinetic energy is simply the internal energy of the perfect gas and internal energy depends only on the temperature of the gas.

2. In a given process for an ideal gas, dW = 0 and dQ < 0. Then for the gas

(a) The temperature will decrease

(b) The volume will increase

(c) The pressure will remain constant

(d) The temperature will increase

Solution:

The correct answer is A.

From first law of thermodynamics,

$\mathrm{dQ} = \mathrm{dU} + \mathrm{Dw}$

we have $\mathrm{dQ} = \mathrm{dU} \quad (as\ \mathrm{dW} = 0)$

But

$\therefore \mathrm{dU} < 0$

$NC_V \Delta T < 0$

Or

$\Delta T < 0$

Hence, the temperature will decrease.

3. A gas is compressed at a constant pressure of 50 N/m^2 from a volume of 10m^3 to a volume of 4m^3. Energy of 100 J then added to the gas by heating. Its internal energy is

(a) Increased by 400 J

(b) Increased by 200 J

(c) Increased by 100 J

(d) Decreased by 200J

Solution:

The answer is A.

$change\ in\ volume\ \Delta V_2 - V_1 = 4 - 10 = 6m^3$

A volume decreases, work is done on the gas and so it is negative.

$W = p\Delta V = 50 \times (-6) = -300J$

Additional heat supplied (G = 100J). So, for the first law of thermodynamics change in external energy is given by

$\Delta U = Q - W = 100 - (-300) = 400J$

Internal energy increases by 400J.

4. Heat given to a system is 35 joules and work done by the system is 15 joules. The change in the internal energy of the system will be

(a)– 50 J **(b) 20 J**

(c) 30 J **(d) 50 J**

Solution:

The answer is B.

Given,

$sW = 15\ joules, Q = 35 joules$

from the first law of thermodynamics

$\Delta U = \Delta Q - W = 35 - 15 = 20J$

5. A perfect gas goes from state A to another state B by absorbing 8 × 10^5 J of heat and doing 6.5 × 10^5 J of external work. It is now transferred between the same two states in another process in which it absorbs 10^5 J of heat. Then in the second process

(a) Work done on the gas is 0.5 × 10^5 J

(b) Work done by gas is 0.5 × 10^5 J

(c) Work done on gas is 10^5 J

(d) Work done by gas is 10^5 J

Solution:

The correct answer is A.

$\mathrm{dU} = \mathrm{dQ} - \mathrm{dW} = (8 \times 10^5 - 6.5 \times 10^5)$

$= 1.5 \times 10^5 J$

$\mathrm{dW} = \mathrm{dQ} - \mathrm{dU} = 10^5 - 1.5 \times 10^5$

$= -0.5 \times 10^5 J$

-ve sign indicates that work done on the gas is $0.5 \times 10^5 J$

6. At 27ºC two moles of an ideal monoatomic gas occupy a volume V. The gas expends adiabatically to a volume 2 V. Calculate

(a) Final temperature of the gas

(b) Change in its internal energy and

(c) The work done by the gas during the process. [R = 8.31 J/mol K]

Solution:

(1) $T_1 = 27 + 273 = 300K$

$\gamma = \frac{5}{3} (f\ or\ monoa \rightarrow micgas)$

$V_1 = V$

$V_2 = 2V$

$T_2 = ?$

$since\ the\ gas\ expands\ adiabatically.$

$T_1 {V_1}^{\gamma-1} = T_2 {V_2}^{\gamma-1}$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$= 300 \left(\frac{V}{2V}\right)^{\frac{5}{3}-1} = 189K$$

(2) $W = \dfrac{-nR(T_2 - T_1)}{\gamma - 1}$

$$= \frac{-2 \times 8.31(189 - 300)}{5/3 - 1}$$

$$= \frac{+8.31 \times 111 \times 3 \times 2}{2} = +2767J$$

Change in internal energy

According to first law of thermodynamics

$\therefore \Delta U = \Delta U + \Delta W \quad But\ \Delta Q = 0$

(As the process is adiabatic)

$\therefore \Delta U = -\Delta W = -2767J$

$\Delta W = 2767J$

7. An ideal gas of one mole is kept in a rigid container of negligible heat capacity. If 25 J of heat is supplied the gas temperature raises by 20 C. Then the gas may be

(a) Helium (b) Argon

(c) Oxygen (d) Carbon dioxide

Solution:

Correct options are A and B.

Given,

$n = 1, dT = 2°\text{C}, dQ = 25\ joule$

We know that,

$dQ = nC_V dT$

$25 = 1 \times C_V \times 2$

$C_V = 12.5$

We also know,

$C_V = \frac{3}{2}R$

$C_V = 1.5 \times 8.314$

$C_V = 12.54$

Since both have equal values gas is mono-atomic,

Since Helium and argon are mono-atomic gases.

8. When an enclosed perfect gas is subjected to an adiabatic process:

(a) Its total internal energy does not change

(b) Its temperature does not change

(c) Its pressure varies inversely as a certain power of its volume

(d) The product of its pressure and volume is directly proportional to its absolute temperature.

Solution:

Correct options are C & D.

Because in adiabatic process, energy is transferred to the surrounding only as work. Hence resulting in total internal energy changes, pressure varies and other parameter also.

9. The internal energy of an ideal gas is equal to negative of the work done by the system, then

(a) The process must be adiabatic

(b) The process must be isothermal

(c) The process must be isobaric

(d) The temperature must decrease

Solution:

Correct answer is A.

If $\Delta U = -W$

Then, $Q = 0$

i.e no heat is exchanged.

The process may be adiabatic.

10. Monoatomic, diatomic and triatomic gases whose initial volume and pressure are same, are compressed till their volume becomes half the initial volume.

(a) If the compression is adiabatic then monoatomic gas will have maximum final pressure.

(b) If the compression is adiabatic then triatomic gas will have maximum final pressure.

(c) If the compression is adiabatic then their final pressure will be same.

(d) If the compression is isothermal then their final pressure will be different.

Solution:

Correct option is A.

For an adiabatic process,

$PV^\gamma = constant$

Or

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^\gamma$$

Or

$P_2 = 2^\gamma P_1$

Now, for a mono-atomic gas, the value of γ is the highest. Thus, for the same change in volume, the mono-atomic gas will have the maximum pressure.

11. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If γ = 1.40, the work done by air is:

($3^{1.4}$ = 4.6555) [Take air to be an ideal gas]

(a) 60.7J (b) 90.5J

(c) 100.8J (d) 48 J

Solution:

(b) Given, V_1 = 1 litre, P_1 = 1 atm

V_2 = 3 litre, γ = 1.40,

Using, PV^r = constant

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$P_2 = P_1 \times \left(\frac{1}{3}\right)^{1.4} = \frac{1}{4.6555}atm$$

$$\therefore \text{Work done, } W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$= \frac{\left(1\times1-\frac{1}{4.6555}\times3\right)1.01325\times10^5\times10^{-3}}{0.4}$$

= 90.1 J

Closest value of W = 90.5 J

12. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J mol^{-1} K^{-1}]

(a) 350J **(b) 374J**

(c) 748J **(d) 700J**

Solution:

(c) As the process is isochoric so,

$$Q = nc_v\Delta T = \frac{67.2}{22.4}\times\frac{3R}{2}\times 20$$

= 90R = 90 × 8.31

$\simeq$ 748 J.

13. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to: (Gas constant R = 8.31 J/mol-K)

(a) 581 J **(b) 291 J**

(c) 146 J **(d) 73 J**

Solution:

(b) Work done,

$$W = P\Delta V = nR\Delta T = \frac{1}{2}\times 8.31\times 70 \simeq 291 \text{ J}$$

14. During an adiabatic compression, 830 J of work is done on 2 moles of a diatomic ideal gas to reduce its volume by 50%. The change in its temperature is nearly: (R = 8.3 JK^{-1}mol^{-1})

(a) 40K **(b) 33K**

(c) 20K **(d) 14K**

Solution:

(c) Given : work done, W = 830 J

No. of moles of gas, $\mu = 2$

For diatomic gas $\gamma = 1.4$

Work done during an adiabatic change

$$W = \frac{\mu R(T_1 - T_2)}{\gamma - 1}$$

$$830 = \frac{2\times 8.3(\Delta T)}{1.4-1} = \frac{2\times 8.3(\Delta T)}{0.4}$$

$$\Delta T = \frac{830\times 0.4}{2\times 8.3} = 20 \text{ K}$$

Expert 1st Law of Thermodynamics

1. 5.6 litre of helium gas at STP is adiabatically compressed to 0.7 litre. Taking the initial temperature to be T$_1$, the work done in the process is

(a) $\frac{9}{8}RT_1$ **(b)** $\frac{3}{2}RT_1$

(c) $\frac{15}{8}RT_1$ **(d)** $\frac{9}{2}RT_1$

Solution:

(a) Initially, Volume V_1 = 5.6l, temperature = T_1

and $\gamma = \frac{5}{3}$ (for monoatomic gas)

The number of moles of gas, $n = \frac{5.6l}{22.4l} = \frac{1}{4}$

Finally (after adiabatic compression)

V_2 = 0.7l

For adiabatic process

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\therefore\ T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1}$$

$= T_1(8)^{2/3} = 4T_1$

Work done in adiabatic process

$$W = \frac{nR\Delta T}{\gamma-1} = \frac{\frac{1}{4}R\left[T_1 - 4T_1\right]}{\left[\frac{5}{3}-1\right]} = \frac{9}{8}RT_1$$

2. An engine takes in 5 mole of air at 20°C and 1 atm, and compresses it adiabaticaly to 1/10th of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be XkJ. The value of X to the nearest integer is _________.

Solution:

For adiabatic process,

$TV^{\gamma-1}$ = constant

or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

T_1 = 20°C + 273 = 293 K,

$V_2 = \frac{V_1}{10}$ and $\gamma = \frac{7}{5}$

$$T_1(V_1)^{\gamma-1} = T_2\left(\frac{V_1}{10}\right)^{\gamma-1}$$

$$293 = T_2\left(\frac{1}{10}\right)^{2/5}$$

$T_2 = 293(10)^{2/5} \simeq 736$ K

ΔT = 736 – 293 = 443 K

During the process, change in internal energy

$$\Delta U = NC_V\Delta T = 5\times\frac{5}{2}\times 8.3\times 443$$

$\simeq 46 \times 10^3$ J = X kJ

$\therefore$ X = 46.

3. Starting at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) ___________.

Solution:

For an adiabatic process,

$TV^{\gamma-1}$ = constant

$\therefore\ T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$$T_2 = (300) \times \left(\frac{V_1}{\frac{V_1}{16}}\right)^{1.4-1} = 300 \times (16)^{0.4}$$

Ideal gas equation, PV = nRT

$\therefore\ V = \frac{nRT}{P}$

V = kT (since pressure is constant for isobaric process)

So, during isobaric process

$V_2 = kT_2$ ... (i)

$2V_2 = kT_f$ ... (ii)

Dividing (i) and (ii)

$$\frac{1}{2} = \frac{T_2}{T_f}$$

$T_f = 2T_2 = 300 \times 2 \times (16)^{0.4} = 1818$ K

4. A gas is compressed isothermally to half its initial volume. The same gas is compressed separately through an adiabatic process until its volume is again reduced to half. Then

(a) Compressing the gas through adiabatic process will require more work to be done.

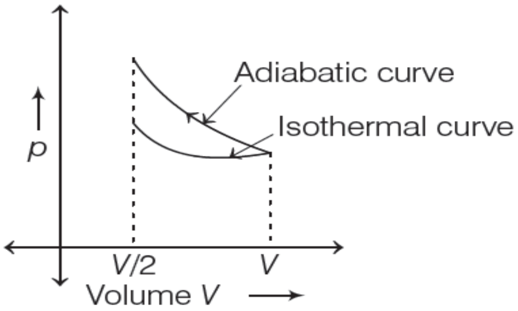
(b) Compressing the gas isothermally or adiabatically will require the same amount of work.

(c) Which of the case (whether compression through isothermal or through adiabatic process) requires more work will depend upon the atomicity of the gas.

(d) Compressing the gas isothermally will require more work to be done.

Solution:

(a) The solution of this question can be understood by plotting a p-V graph for the compression of a gas isothermally and adiabatically simultaneously to half of its initial volume. i.e.



Since, the isothermal curve is less steeper than the adiabatic curve. So, area under the p-V curve for adiabatic process has more magnitude than isothermal curve. Hence, work done in adiabatic process will be more than in isothermal process.

Pro 1st Law of Thermodynamics

1. A container of volume 1m^3 is divided into two equal compartments by a partition. One of these compartments contains an ideal gas at 300 K. The other compartment is vacuum. The whole system is thermally isolated from its surroundings. The partition is removed and the gas expands to occupy the whole volume of the container. Its temperature now would be

(a) 300 K **(b) 239 K**

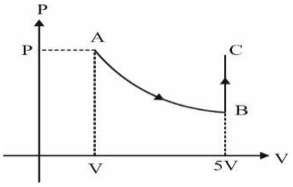
(c) 200 K **(d) 100 K**

Solution:

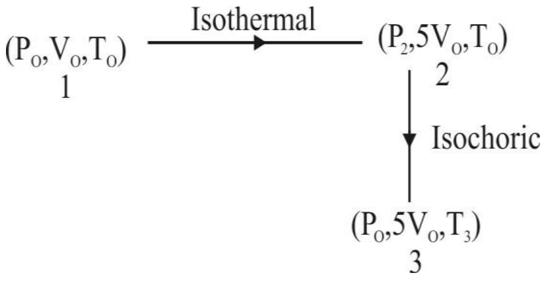
The correct answer is A.

Since, the system is insulated, Q = 0. Other part is vacuum, therefore, work done by the gas W is also zero. Hence, from first law of thermodynamics, $\Delta U = 0$i.e, temperature remains constant.

2. 3 moles of an ideal gas at 300K are isothermally expanded to 5 times its volume and then heated at this constant volume so that its pressure will raise to initial value before expansion. In the whole process 83.14 kJ heat is required. Find the temperature T_C. Also calculate the ratio C_P/C_V of gas. (Given log5=1.61)



Solution:



$n = 3$

$1 \to 2 : \; P_0V_0 = P_2 5V_0 \Rightarrow P_2 = \frac{P_0}{5}$

$2 \to 3 : \; \frac{P_2}{T_0} = \frac{P_0}{T_3} \Rightarrow \frac{\frac{P_0}{5}}{T_0} = \frac{P_0}{T_3} \Rightarrow T_3 = 5T_0$

$T_0 = 300K$

$T_3 = 5 * 300 = 1500\ K$

$1 \to 2\ (Isothermal\ expansion)$

$\Delta Q_{1\to2} = \Delta W_{1\to2} = nRT_0 \ln\left(\frac{5V_0}{V_0}\right)$

$= 3R \times 300 \ln(5)$

$900R \times 1.61 = 1449R$

$2 \to 3 (Isochoric\ heating)$

$\Delta Q_{2\to3} = nC_V\Delta T = 3 \times \frac{R}{\gamma - 1}(5T_0 - T_0)$

$= \frac{1200R \times 3}{\gamma - 1} = \frac{3600R}{\gamma - 1}$

$\Delta Q_{1\to2} = nC_V\Delta T = 3 \times \frac{R}{\gamma - 1}(5T_0 - T_0)$

$= \frac{12R \times 300}{\gamma - 1} = \frac{3600R}{\gamma - 1}$

$\Delta Q_{1\to2} + \Delta Q_{2\to3} = 83.14 \times 10^3 J$

$= 8.314 \times 10^4 = 10^4 R$

$1449R + \frac{3600R}{\gamma - 1} = 10^4 R \Rightarrow \frac{3600}{\gamma - 1} = 8501$

$\gamma - 1 = \frac{3600}{8500} = 0.42 \Rightarrow \gamma = 1.42$

3. Gaseous hydrogen contained initially under standard conditions in a sealed vessel of volume V = 5.0 1 was cooled by Δ T = 55 K. Find how much the internal energy of the gas will change and what amount of heat will be lost by the gas.

Solution:

A. $\Delta U = nC_v\Delta T$

$= \frac{5}{22.4}.5.\frac{(8.3)}{2}.(-55)$

$= -0.25kJ$

B. $Q = \Delta U + \int pdv$

$\Rightarrow Q = \Delta U = -0.25kJ$

$Heat\ lost = -Q = 0.25kJ$

4. What amount of heat is to be transferred to nitrogen in the isobaric heating process for that gas to perform the work A = 2.0 J?

Solution:

$Q = \Delta U + w$

$= nC_v\Delta T$

$= \frac{nR\Delta T}{r-1}$

$= \frac{p\Delta v}{r-1} = \frac{w}{r-1}$

$= \frac{w}{r-1} + w$

$= \frac{r}{r-1}w$

$= \frac{1.4}{(1.4-1)}.2 = 7J$

5. As a result of the isobaric heating by ΔT = 72K one mole of a certain ideal gas obtains an amount of heat Q = 1.60kJ.Find the work performed by the gas, the increment of its internal energy, and the value of γ = C_p/C_v.

Solution:

$A. w = p\Delta v =$

$nR\Delta T = (8.3)(72) = 0.6kJ$

$B. \Delta U = Q - w = 1.6 - 0.6 = 1\ kJ$

$C. \Delta U = \frac{nR\Delta T}{r-1} = 1kJ$

$\Rightarrow r = 1.6$

6. Two moles of a certain ideal gas at a temperature T_0 = 300 K were cooled isochorically so that the gas pressure reduced n = 2.0 times. Then, as a result of the isobaric process, the gas expanded till its temperature got back to the initial value. Find the total amount of heat absorbed by the gas in this process.

Solution:

In phase1: v = const

$\frac{p_0}{T_0} = \frac{p_1}{T_1}$

$\Rightarrow T_1 = T_0\left(\frac{p_1}{p_0}\right) = \frac{T_0}{\eta}$

i.e., temperature got reduced n times.

Also, $Q_1 = \Delta U$

$$= \frac{\eta R\Delta T}{r-1}$$

$$= \frac{nR\left(\frac{T_0}{\eta} - T_0\right)}{r-1}$$

In phase 2: p = constant

$$Q_2 = \Delta U + p\Delta v$$

$$= \frac{nR\Delta T}{r-1} + nR\Delta T$$

$$= \frac{n}{r-1} nR\left(T_0 - \frac{T_0}{\eta}\right)$$

$\therefore$ Net heat: $Q = Q_1 + Q_2$

$$= nRT_0\frac{(\eta-1)}{\eta}$$ (independent of r)

7. Three moles of an ideal gas being initially at a temperature T_0 = 273 K were isothermally expanded n = 5.0 times its initial volume and then isochorically heated so that the pressure in the final state became equal to that in the initial state. The total amount of heat transferred to the gas during the process equals Q = 80 kJ. Find the ratio γ = C_p/C_v for this gas.

Solution:

$$P_0, V, T_0$$

$$\downarrow isothermal$$

$$\frac{p_0}{\eta}, \eta v, T_0$$

$$\downarrow Isochoric$$

$$p_0, \eta v, \eta T_0$$

$\therefore$ In isothermal expansion:

$$Q_1 = nRT_o \ell n\frac{v_2}{v_1} = nRT_0 \ell n\ \eta$$

And, in isochoric process:

$$Q_2 = nC_V\Delta T$$

$$= \frac{nR}{r-1}(\eta-1)T_0$$

$$Q = Q_1 + Q_2$$

$$= nRT_0\left(\ell n\ \eta + \frac{\eta-1}{r-1}\right)$$

$$\Rightarrow r = 1 + \frac{\eta-1}{\frac{Q}{nRT_0} - \ell n\ \eta}$$

8. One mole of oxygen being initially at a temperature T_0 = 290 K is adiabatically compressed to increase its pressure η = 10.0 times. Find:

(a) The gas temperature after the compression;

(b) The work that has been performed on the gas.

Solution:

$$pv^r = const$$

$$\Rightarrow p\left(\frac{T}{p}\right)^r = \text{const}$$

$$\Rightarrow p^{1-r}T^r = \text{const}$$

$$\Rightarrow \text{p}^{\frac{1-r}{r}} T = \text{const}$$

A. for given adiabatic process

$$p\frac{1-r}{r}T_0 = (\eta p)^{\frac{1-r}{r}} T_f$$

$$\Rightarrow T_f = \frac{T_0}{\eta^{\frac{1-r}{r}}} = T_0 n^{\frac{r-1}{r}}$$

B. work done on the gas: $= \Delta U$

$$= \frac{\eta R\Delta T}{r-1} = \frac{\eta RT_0}{r-1}\left(\eta^{\frac{r-1}{r}} - 1\right)$$

9. A certain mass of nitrogen was compressed η = 5.0 times (in terms of volume), first adiabatically, and then isothermally. In both cases the initial state of the gas was the same. Find the ratio of the respective works expended in each compression.

Solution:

Work done by the gas:

$$w_{adi} = -\frac{nR\Delta T}{r-1} = -\frac{nRT_0\left(\eta^{r-1} - 1\right)}{r-1}$$

$$w_{iso} = nRT_0 \ell n\frac{v_2}{v_1} = nRT_0 \ell n\frac{1}{\eta}$$

$\therefore$ Ratio of work done on the gas

$$=\frac{-w_{adi}}{-w_{iso}}=\frac{\left(\eta^{n-1}-1\right)}{(r-1)\ln\eta}$$

$$pv^{r}=const$$

$$\Rightarrow\left(\frac{T}{v}\right)v^{n}=\text{const}$$

$$\Rightarrow Tv^{r-1}=\text{const}$$

$\therefore$ as given:

$$T_0v^{r-1}=T_f\left(\frac{v}{\eta}\right)^{n-1}$$

$$\Rightarrow T_f=T_0\eta^{n-1}$$

$$\Rightarrow \Delta T=T_f-T_0$$

$$=T_0\left(\eta^{n-1}-1\right)$$

10. A certain mass of nitrogen was compressed η = 5.0 times (in terms of volume), first adiabatically, and then isothermally. In both cases the initial state of the gas was the same. Find the ratio of the respective works expended in each compression.

Solution:

Work done by the gas:

$$W_{adi}=\frac{-nR\Delta T}{n-1}=\frac{-nRT_0}{n-1}\left(\eta^{n-1}-1\right)$$

$$W_{iso}=nRT_0\ell n\frac{v_2}{v_1}=nRT_0\ell n\frac{1}{\eta}$$

$\therefore$ Ration of work done on the gas

$$=\frac{-w_{odi}}{-w_{iso}}=\frac{\left(\eta^{n-1}-1\right)}{(n-1)\ell n\eta}$$

$$pv^{n}=const$$

$$\Rightarrow\left(\frac{T}{v}\right)v^{n}=const$$

$$\Rightarrow Tv^{n-1}=const$$

$\therefore$ as given:

$$T_0v^{n-1}=T_f\left(\frac{v}{\eta}\right)^{n-1}$$

$$\Rightarrow T_f=T_o\eta^{n-1}$$

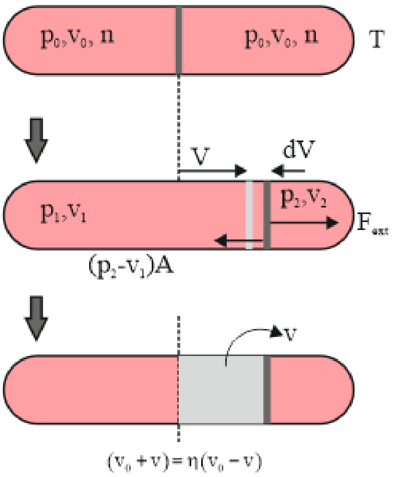
$$\Rightarrow \Delta T=T_f-T_o=T_o\left(\eta^{n-1}-1\right)$$

11. A heat-conducting piston can freely move inside a closed thermally insulated cylinder with an ideal gas. In equilibrium the piston divides the cylinder into two equal parts, the gas temperature being equal to T₀. The piston is slowly displaced. Find the gas temperature as a function of the ratio η of the volumes of the greater and smaller sections. The adiabatic exponent of the gas is equal to γ.

Solution:

Concept: $w_{ext}=\Delta v$

$$\Rightarrow dw_{ext}=dU$$



$$dw=F_{ext}dx=\left(p_2-p_1\right)Adx$$

$$=\left(p_2-p_1\right)dv$$

$$=\left(\frac{nRT}{v_2}-\frac{nRT}{v_1}\right)dv$$

$$=nRT\left(\frac{1}{v_0-v}-\frac{1}{v_0+v}dv\right)$$

$$=2nRT\frac{vdv}{v_0^2-v_2}$$

$$=dU=(2n)\frac{R}{r-1}dT$$

$$\Rightarrow v=\left(\frac{\eta-1}{\eta+1}\right)$$

$$\Rightarrow\int_0^{\left(\frac{\eta-1}{\eta+1}\right)v_0}\frac{vdv}{v_0^2-v_2}=\frac{1}{(r-1)}\int_{T_0}^{T}\frac{dT}{T}$$

$$\Rightarrow T=T_0\left[\frac{(\eta+1)^2}{4\eta}\right]^{\frac{r-1}{2}}$$

12. An ideal gas whose adiabatic exponent equals

γ is expanded so that the amount of heat transferred to the gas is equal to the decrease of its internal energy. Find:
(a) The molar heat capacity of the gas in this process;
(b) The equation of the process in the variables T, V;
(c) The work performed by one mole of the gas when its volume increases η times if the initial temperature of the gas is T_0.

Solution:

For pv^n = const

i. $C = R\left[\frac{1}{r-1} - \frac{1}{n-1}\right]$

ii. $w = \frac{\eta_0 R\Delta T}{n-1}$

Heat = decrease in U

$\therefore nCdt = nC_v dT$

$$C = -\frac{R}{r-1} = R\left[\frac{1}{r-1} - \frac{1}{n-1}\right]$$

$$\Rightarrow 2(n-1) = n-1$$

$$iii. \Rightarrow n = \frac{n+1}{2}$$

iv. Process is : $pv\frac{n+1}{2} = \text{const}$

(a) $C = R\left[\frac{1}{r-1} - \frac{1}{n-1}\right] = -\frac{R}{r-1}$

(b) $PV^{\frac{r+1}{2}} = const$

$$\Rightarrow \left(\frac{T}{V}\right)V^{\frac{r+1}{2}} = const$$

$$\Rightarrow TV^{\frac{\gamma+1}{2}} = const$$

(c) $\left(\frac{T}{V}\right)V^{\frac{r+1}{2}} = const$

$$\Rightarrow T_0 V^{\frac{r+1}{2}} = T_2(\eta v)^{\frac{r+1}{2}}$$

$$\Rightarrow T_2 T_0 \eta^{\frac{1-r}{2}} \qquad \Rightarrow \Delta T = \left(\eta^{\frac{1-r}{2}} - 1\right)$$

$$\therefore w = -\frac{n_0 R\Delta T}{n-1}$$

$$= -\frac{n_0 RT_0\left(\eta^{\frac{1-r}{2}-1}\right)}{\frac{r+1}{2}-1}$$

$$= \frac{2n_0 RT_o\left(1-\eta^{\frac{1-\gamma}{2}}\right)}{\gamma - 1}$$

13. Two insulating cylinders A and B fitted with pistons contain equal amounts of an ideal diatomic gas at temperature 300K. The piston A is free to move, while that of B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in A is 30K. Then the rise in temperature of the gas in B is,

(a) 30K **(b) 18K**
(c) 50K **(d) 42K.**

Solution:

The answer is D.

(d) A is free to move, therefore heat will be supplied at constant pressure

$\therefore \Delta Q_A = nC_P\Delta T_A \quad ----(i)$

B is held fixed , therefore heat will be supplied at constant volume.

$\therefore \Delta Q_B = nC_V\Delta T_B \quad ----(ii)$

But

$\Delta Q_A = \Delta Q_B \ \ (given)$

$\therefore nC_P\Delta T_A = nC_V\Delta T_B \ \ \therefore \Delta T_B = \left(\frac{C_P}{C_V}\right)\Delta_A$

$= \gamma(\Delta T_A)[\gamma = 1.4(dia \to mic)]$

$= (1.4)(30K)$

$\therefore \Delta T_B = 42K$

14. One mole of an ideal gas is compressed from 0.5 lit to 0.25 lit. During the compression, 23.04 × 10^2 J of work is done on the gas and heat is removed to keep the temperature of the gas constant at all times. Find the temperature of the gas. (Take universal gas constant R = 8.31 J mol^{-1} K^{-1})

Solution:

Given: Work of compression $W_{com} = -23.04 \times 10^2 J$

Initial volume $V_1 = 0.5L$

Final volume $V_2 = 0.25L$

Let the temperature of the gas be T

As the temperature is constant all the time, thus all the gas is compressed isothermally.

Work done in isothermal compression

$$W_{cpm} = nRT\ln\left(\frac{V_2}{V_1}\right)$$

$$\therefore\ -2304 = 1\times 8.31\times T\ln\left(\frac{0.25}{0.5}\right)$$

$$-2304 = 1\times 8.31\times T \times(-\ln 2)(\ln 2 = 0.693)$$

$$\Rightarrow T = 400K$$

15. If heat is added at constant volume, 6300 J of heat are required to raise the temperature of an ideal gas by 150 K. If instead, heat is added at constant pressure, 8800 joules are required for the same temperature change. When the temperature of the gas changes by 300 K. Determine the change is the internal energy of the gas.

Solution:

Heat added at constant volume is equal to the change in internal energy of the system.

Change in internal energy

$$\Delta U = nC_V\Delta T$$

$$For\ \ \Delta T = 150K\ ,\Delta U = Q_V = 6300J$$

$$\therefore 6300 = nC_V(150)\ \ ---(1)$$

Let the change in internal energy be ΔU for $\Delta T = 300K$

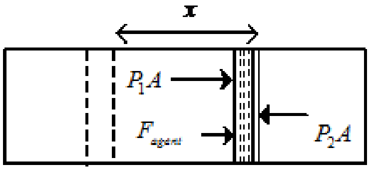
$$\therefore \Delta U = nC_V(300)\ \ ----(2)$$

Dividing (2) and (1) we get

$$\frac{\Delta U}{6300} = \frac{300}{150}$$

$$\Rightarrow \Delta U = 12600J$$

16. A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V$_0$, in which an ideal gas is contained under the same pressure P$_0$ and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas η times compared to that of the other by slowly the piston?



Solution:

Let the agent move the piston as shown.

In equilibrium position,

$$P_1A + F_{agent} = P_2A$$

$$F_{agent} = (P_2 - P_1)A$$

Work done by the agent

$$F_{agent}dx = (P_2 - P_1)A\times dx = (P_2 - P_1)dV$$

Applying PV = constant for two parts, we have

$$P_1(V_0 + Ax) = P_0V_0\ and\ P_2(V_0 - Ax) = P_0V_0$$

$$P_1 = \frac{P_0V_0}{(V_0 + Ax)}\ and\ P_2 = \frac{P_0V_0}{(V_0 - Ax)}$$

$$\therefore P_2 - P_1 = \frac{P_0V_0(2Ax)}{{V_0}^2 - A^2x^2} = \frac{2P_0V_0V}{{V_0}^2 - V^2}$$

When the volume of the left end is η times the volume of right end, we have

$$(V_0 + V) = \eta(V_0 - V)$$

$$V = \left(\frac{\eta - 1}{\eta + 1}\right)V_0\ ---(2)$$

The work done is given by

$$W = \int_0^V (P_2 - P_1)dV = \int_0^V \frac{2P_0V_0V}{({V_0}^2 - V^2)}dV$$

$$= -P_0V_0\left[\ln {V_0}^2 - V^2\right]_0^V$$

$$= -P_0V_0\left[\ln\left({V_0}^2 - V^2\right) - \ln {V_0}^2\right]$$

$$= -P_0V_0\left[\ln\left\{{V_0}^2 - \left(\frac{\eta - 1}{\eta + 1}\right)^2 {V_0}^2\right\} - \ln {V_0}^2\right]$$

$$= -P_0V_0[\ln\{4\eta/(\eta + 1)^2\}] = P_0V_0\left[\frac{(\eta + 1)^2}{4\eta}\right]$$

17. Three moles of an ideal gas being initially at a temperature T$_0$ = 273 K were isothermally expanded η = 5.0 time its initial volume and then isochorically heated so that the pressure in the final state became equal to that in the initial state. The total amount of heat transferred to the gas during the process equals Q = 80 KJ. Find the ratio γ = Cp/Cv for this gas.

Solution:

For the process BC,

$$\Delta V = 0\ \ , thus\ W_{BC} = 0$$

Process AB is isothermal, thus

$$\Delta U_{AB} = 0$$

$$W_{AB} = nRT\ln\frac{V_B}{V_A} = 3(8.314)(273)\ln 5$$

$$= 10958.93\,J$$

$$\Delta U_{BC} = nC_V\Delta T = 3C_V(5T_0 - T_0) = 3276C_V$$

$$Q = \Delta U + W$$

$$80000 = 3276C_V + 10958.93$$

$$\Rightarrow C_V = 21.07$$

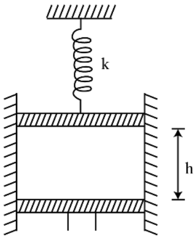
Also,

$C_P - C_V = R = 8.314 \Rightarrow C_P = 29.39$

Now,

$\frac{C_P}{C_V} = 1.4$

18. An ideal gas at NTP is enclosed in a adiabatic vertical cylinder having area of cross section A = 27 cm^2, between two light movable pistons as shown in the figure. Spring with force constant k = 3700 N/m is in a relaxed state initially. Now the lower piston is moved upwards a height h/2, h being the initial length of gas column. It is observed that the upper piston moves up by a distance h/16. Find h taking γ for the gas to be 1.5. Also find the final temperature of the gas.



Solution:

$NTP: T_o = 0°\text{C} = 273K P_o = 1bar = 10^5 Pa$

Final volume,

$$V_f = \left[h + \frac{h}{16} - \frac{h}{2}\right] A = \frac{9}{16} hA = \frac{9}{16} V_o$$

Using $PV^\gamma = K$

$$\frac{P_f}{P_o} = \left(\frac{V_o}{\frac{9}{16} V_o}\right)^{1.5}$$

$P_f = 2.37 P_o$

Now $P_f = P_o + \frac{Kh}{16A}$

$Where\ k = 3700 N/m A = 27 \times 10^{-4} m^2$

$$2.37 P_f = P_o + \frac{3700(h)}{16(27 \times 10^{-4})}$$

$h = 1.6m$

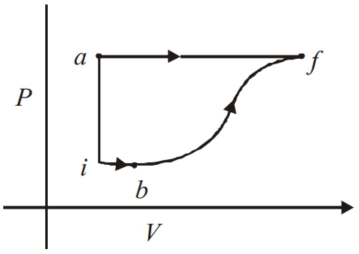
Also,

$TV^{\gamma - 1} = constant$

$$\frac{T_f}{273} = \left(\frac{V_o}{\frac{9}{16} V_o}\right) (\frac{V_o}{\frac{9}{16} V_o})^{0.5}$$

$T_f = 364K$

19. A thermodynamic system is taken from an initial state i with internal energy U_i=100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} =200 j, W_{ib}=50 J and W_{bf}=100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b= 200 J and Q_{iaf}=500 J, The ratio $\frac{Q_{bf}}{Q_{ib}}$ is



Solution:

Applying first law of thermodynamics to path iaf

$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$

$500 = \Delta U_{iaf} + 200$

$\Delta U_{iaf} = 300$ J

Now,$U_{iaf} = \Delta U_{iaf} + W_{ib} + W_{bf} = 300 + 50 + 100$

$Q_{ib} + Q_{bf} = 450$ J

Also $Q_{ib} = \Delta U_{ib} + W_{ib}$

$Q_{ib} = 100 + 5\ 0 + 150$ J ...(ii)

From eq. (i) & (ii) $\frac{Q_{bf}}{Q_{ib}} = \frac{Q_{ibr} - Q_{ib}}{Q_{ib}} = \frac{300}{150} = 2$

20. An ideal monatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperatureT_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperatureT_2, pressure P_2 and volumeV_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statements(s) is (are)

(a) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the energy stored in the spring is 1/4 P_1V_1

(b) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is 3 P_1V_1

(c) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is 7/3 P_1V_1

(d) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied t the gas is 17/6 P_1V_1

Solution:(a, b, c)

Let spring is compressed by x on heating the gas.

(a) As gas is ideal monatomic,

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_1}$

Force on spring by gas = kx

$P_2 = P_1 + \frac{kx}{\text{A}}$ (A = area of cross – section of piston) …(ii)

When $V_2 = 2V_1, T_2 = 3T_1$

$\frac{P_1V_1}{T_1} = \frac{P_2 \times (2V_1)}{3T_1}$ $\qquad P_2 = \frac{3}{2}P_1$

Putting this value of P_2 in eqn. (ii) we get

$\frac{3}{2}P_1 = P_1 + \frac{kx}{\text{A}}$ $\qquad kx = \frac{P_1A}{2}$

$x = \frac{V_2 - V_1}{\text{A}} = \frac{2V_1 - V_1}{\text{A}} = \frac{V_1}{\text{A}}$

Energy stored in the spring

$= \frac{1}{2}kx^2 = \frac{1}{2}(kx)(x) = \frac{P_1V_1}{4}$

(b) Change in internal energy,

$\text{U} = \frac{f}{2}(P_2V_2 - P_1V_1) = \frac{3}{2}(\frac{3}{2}P_1 \times 2V_1 - P_1V_1)$

(c) Again, when $V_2 = 3V_1$ and $T_2 = 4T_1$ then

From eqn. (i),

$\frac{P_1V_1}{T_1} = \frac{P_2\,(3V_1)}{4T_1}$ $\qquad P_2 = \frac{4}{3}P_1$

$x = \frac{V_2 - V_1}{\text{A}} = \frac{2V_1}{\text{A}}$

Form eqn. (ii),

$\frac{4}{3}P_1 = P_1 + \frac{kx}{\text{A}}$ $\qquad kx = \frac{P_1A}{2}$

Work done by gas = Work done by gas on at Energy stored I spring.

$W_8 = P_1\text{A}x + \frac{1}{2}kx^2 = P_1(2V_1) + \frac{1}{2}(\frac{P_1A}{3})(\frac{2V_1}{A})$

$= 2P_1V_1 + \frac{1}{3}P_1V_1 = \frac{7}{3}P_1V_1$

$(d)\ \Delta Q = W_8 + \Delta U$

$= \frac{7}{3}P_1V_1 + \frac{3}{2}(P_2V_2 - P_1V_1)$

$= \frac{7}{3}P_1V_1 + \frac{3}{2}(\frac{4}{3}P_1 \times 3V_1 - P_1V_1)$

21. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volumeV_1. It expands isothermally to volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are $W_{iso}$and W_{adia} , respectively. If the ratio $\frac{W_{iso}}{W_{adia}} = f$ In 2, then f is______.

Solution:(1.77 to 1.78)

For monatomic gas , $\gamma\ \frac{5}{3}$

In adiabatic process

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow \frac{P1}{4}(4V_1)^{5/3} = P_2(32V_1)^{5/3}$

$\Rightarrow P_2 = \frac{P1}{4}(1/8)^{5/3} = \frac{P1}{128}$

And $W_{\text{adi}} = {P_1V_1 - P_2V_2}/{\gamma - 1}$

22. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressureP_0 , volumeV_0, and temperatureT_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are,(Given $2^{1.2}$=2.3; $2^{3.2}$ 9.2; R is gas constant)

(a)The work |W| done during the process is $13RT_0$

(b)The final pressure of the gas m0069ture after compression is in between $9P_0$ and $10P_0$

(c)The average kinetic energy of the gas mixture aft4ee4r compression is in between $18RT_0$ and $19RT_0$

(d)Adiabatic constant of the gas mixture is 1.6

Solution:(a, b, d)

Adiabatic constant of the gas mixture,

$$\gamma_m = \frac{n_1C_{p1} + n_2C_{p2}}{n_1C_{p1} + n_2C_{p2}} = \frac{5\times\frac{5R}{2}+1\times\frac{7R}{2}}{5\times\frac{3R}{2}+1\times\frac{5R}{2}} = 1.6$$

For an adiabatic process, $P_{v\gamma}$ =Constant

$$\therefore P = P_0\left(\frac{V_0}{V}\right)^{1.6} = P_0(4)^{1.6}$$

$$= P_0(2^2)^{1.6} = P_0 2^{3.2} = 9.2P_0$$

Work done during the process,

$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{9.2P_0 \times \left(\frac{V_0}{4}\right) - P_0V_0}{1 - 1.6}$$

$$= -\frac{13P_0V_0}{6}$$

But $P_0V_0 = 6RT_0(as\, n = 5 + 1 = 6)$

$$\therefore W = -\frac{13(6RT_0)}{6} = -13RT_0 \therefore |W|$$

$$= 13RT_0$$

The average K.E. of the gas mixture,

$K.E = nCV_{mi} \times T_2$

From$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

Or, $T_2 = T_1(2)^{\frac{6}{5}} = 23T_0$

$\therefore K.E = nCV_{mi} \times T_2 = 23RT_0$

Beginner Graphs

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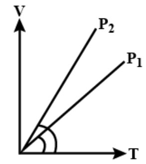
1. In the following V–T diagram, what is the relation between P_1 and P_2?



(a) $P_2 = P_1$ **(b) $P_2 > P_1$**
(c) $P_2 < P_1$ **(d) cannot be predicted**

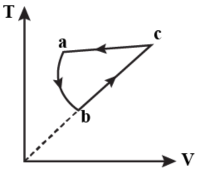
Solution: The answer is C.

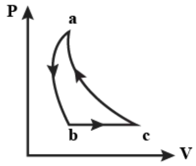
Since PV= nRT

$$\frac{V}{T} = \frac{nR}{P}$$

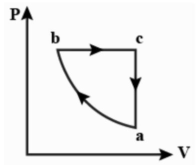
$Slope\ of\ the\ graph \propto \frac{1}{pressure}\ so\ it\ evident\ from\ the\ graph\ that\ P_2\ will\ be\ smaller\ as\ compared\ to\ P_1$

2. PV curve for the process whose VT curve is

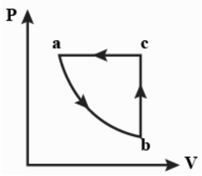




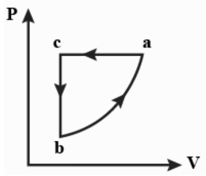
(a)



(b)



(c)



(d)

Solution:

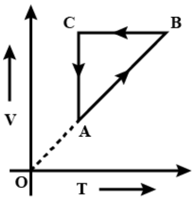
The answer is A.

1) From path b to c, the pressure is constant which is possible only for option A and B. So, option C and D are wrong.

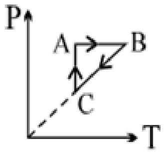
2) From path a to b volume is increasing in the T – V graph which is possible in option A only.

3) From path c to a volume is decreasing which is possible only in option A.

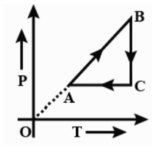
3. An ideal gas undergoes a thermodynamics cycle as shown in figure.



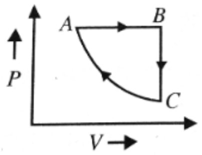
Which of the following graphs represents the same cycle?



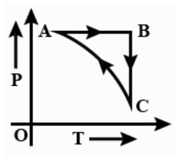
(a)



(b)



(c)



(d)

Solution:

The correct answer is A, C.

From the above problem.

Process to C to B (isochoric process)

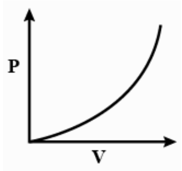
Process A to B

Slope is constant so,

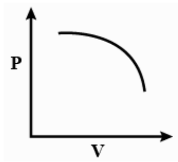
P = constant (isobaric process)

Process c to A (isothermal process)

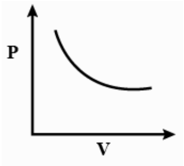
4. An ideal gas follows a process PT = constant. The correct graph between pressure & volume is



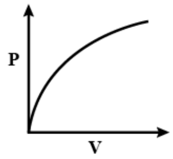
(a)



(b)



(c)



(d)

Solution:

The correct answer is C.

An ideal gas follows a process PT = constant.

The correct graph between pressure and volume is represented by option C.

$$PT = constant \quad ---(1)$$

$$PV = nRT$$

$$P = \frac{nRT}{V} \quad ----(2)$$

Substitute (1) in (2)

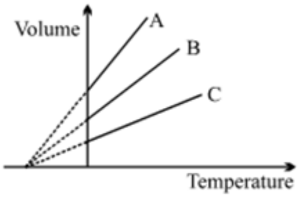
$$P = \frac{nRk}{PV}$$

Thus

$$P^2 \propto \frac{1}{V}$$

5. The expansion of an ideal gas of mass m at a

constant pressure P is given by the straight line B. Then the 5expansion of the same ideal gas of mass 2 m at a pressure 2P is given by the straight line



(a) C **(b) A**

(c) B **(d) none**

Solution:

The correct answer is C.

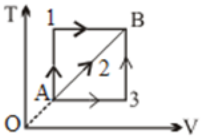
The expansion of an ideal gas of mass m at a constant pressure P is given by the straight line B. Then the expansion of the same ideal gas of mass 2 m at a pressure 2P is given by the straight line B.

The ideal gas equation is PV = nRT or PVM = wRT

Thus, the volume is directly proportional to the mass and inversely proportional to the pressure.

Thus, when the ratio of mass to pressure is held constant, the volume remains unchanged at a given temperature.

6. A given mass of a gas expands from a state A to the state B by three paths 1, 2 and 3 as shown in T-V indicator diagram. If W_1 , W_2 and W_3 respectively be the work done by the gas along the three paths, then



(a) $W_1 > W_2 > W_3$ **(b) $W_1 < W_2 < W_3$**

(c) $W_1 = W_2 = W_3$ **(d) $W_1 < W_2$, $W_1 > W_3$**

Solution: The correct answer is A.

So process 1,

First gas follows constant volume

Path then expand isothermally.

$$(W_{AB})_1 = W_V + W_{iAO}$$

$$= nRT \ln \frac{v_f}{v_i}$$

$$= nRv_f \frac{T_i}{v_i} \ln \frac{v_f}{v_i} \quad (\mathit{can\ be\ written\ from\ process}\ 2)$$

In process 2:

$$T \propto v \quad \Rightarrow P = constant$$

$$W = P(v_f - v_i) = \frac{nRT_i}{v_i}(v_f - v_i)$$

In process 3:

$$(W_{BA}) = nRT_i \ln\left(\frac{v_i}{v_f}\right)$$

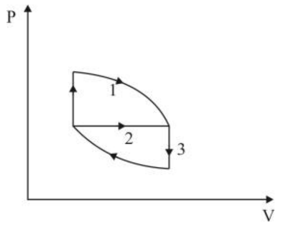
From above we see

$$(W_{AB})_1 > (W_{BA})_3$$

Now

$$Ratio\ of\ \frac{(W_{AB})_1}{(W_{AB})_2} = \frac{v_f \ln(v_f/v_i)}{(v_f - v_i)}$$

We represent all three process in PV curve, it is like



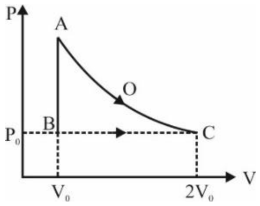
Area under the curve give the work done

Hence,

$$W_1 > W_2 > W_3$$

Expert Graphs

1. An ideal gas is taken from point A to point C on P-V diagram through two process AOC and ABC as shown in the figure. Process AOC is isothermal



(a) Process AOC requires more heat than process ABC.

(b) Process ABC requires more heat than process AOC.

(c) Both process AOC & ABC require same amount of heat.

(d) Data is insufficient for comparison of heat requirement for the two processes.

Solution:

The correct answer is A.

Being a state function, change in internal energy (ΔU) is the same for the two processes AOC and ABC, i.e. $\Delta U_{\mathrm{AOC}} = \Delta U_{\mathrm{ABC}}$

Work done = W = area under P-V diagram

Hence

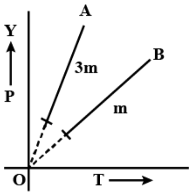
$$W_{AOC} > W_{ABC}$$

$$\Rightarrow \Delta U_{AOC} + W_{AOC} > \Delta U_{ABC} + W_{ABC}$$

$$\Rightarrow \Delta Q_{AOC} > \Delta Q_{ABC}$$

2. Two different masses m and 3m of an ideal gas

are heated separately in a vessel of constant volume, the pressure P and absolute temperature T, graphs for these two cases and shown in the figure as A and B. The ratio of slopes of curves B to A is



(a) 3 : 1 **(b) 1 : 9**
(c) 9 : 1 **(d) 1 : 9**

Solution:

The answer is A.

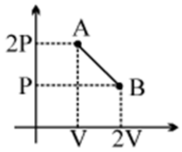
Slope of the passing through origin on P – T diagram

$$Slope\ of\ B\left(\frac{P}{T}\right)_B=\frac{nR}{V}=\frac{\left(\frac{3m}{M_0}\right)R}{V}$$

$$Slope\ of\ B\left(\frac{P}{T}\right)_A=\frac{n'R}{V}=\frac{\left(\frac{m}{M_0}\right)R}{V}$$

$$\frac{\left(\frac{P}{T}\right)_B}{\left(\frac{P}{T}\right)_A}=\frac{3m}{m}=\frac{3}{1}$$

3. The process AB is shown in the diagram. As the gas is taken from A to B, its temperature



(a) initially increases then decreases
(b) initially decreases then increases
(c) remains constant
(d) variation depends on type of gas

Solution:

The correct answer is A.

Let us first find equation of the straight line,

$$\frac{P-2p}{V-v}=\frac{2p-p}{v-2v}$$

$$P-2p=V\times\left(\frac{-p}{v}\right)$$

$$\therefore\quad equation\ of\ the\ line:P=V\times\left(\frac{-p}{v}\right)+2p$$

We know that internal Energy $(U)=PV$

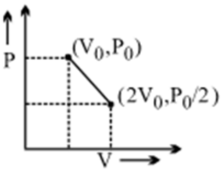
$$U=P\times V=V\times\left(V\times\left(\frac{-p}{v}\right)+2p\right)$$

$$U=-V^2\times\left(\frac{P}{v}\right)+2pV$$

As we go from point A to point B volume increases.

As volume increases internal energy initially increases and then decreases, since the internal energy as a function of volume is a downward parabola.

4. One mole of a gas expands obeying the relation as shown in the P/V diagram. The maximum temperature in this process is equal to



(a) $\frac{P_0V_0}{R}$ **(b)** $\frac{3P_0V_0}{R}$

(c) $\frac{9P_0V_0}{8R}$ **(d) None of these**

Solution:

The correct answer is C.

This question can be solved by maxima and minima.

Since the graph shown here is a straight line, we can write it's equation in the form of : y = mx + c

Here

$$y=P\ ,x=V,m=\frac{P_2-P_1}{V_2-V_1}=\frac{\frac{P_0}{2}-P_0}{2V_0-V_0}$$

$$\Rightarrow m=\frac{-P_0}{2V_0}$$

$$c\ is\ the\ intercept\ and\ c=\frac{3P_0}{2}$$

We can find c by extra piloting the graph

Now

$$P=\frac{-P_0}{2V_0}V+\frac{3P_0}{2}\quad ------(1)$$

Also from equation of state, we know the

PV = nRT

For 1 mole

PV = RT

Or

$$P=\frac{RT}{V}$$

Substituting this in equation (1)

$$\frac{RT}{V}=\frac{-P_0}{2V_0}V+\frac{3P_0}{2}$$

$$T=\frac{-P_0}{2RV_0}V^2+\frac{3P_0}{2R}V$$

To find the minimum temperature, we will differentiate the above equation

i.e

$$\frac{dT}{dV}=\frac{2P_0}{2RV_0}.V+\frac{3P_0}{2R}$$

For

$$\frac{dT}{dV}=0\Rightarrow\frac{P_0}{RV_0}.V=\frac{3P_0}{2R}$$

$$V=\frac{3V_0}{2}$$

This is the critical point.

$$\frac{d^2T}{dV^2}=-\frac{P_0}{RV_0}$$

Which is negative since

Pressure and volume can't be negative.

⇒ pt. of maximum

Now at

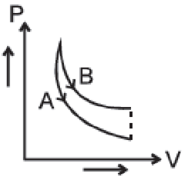
$$V=\frac{3V_0}{2}$$

$$maximum\ temperature=T_{max}$$

$$=\frac{-P_0}{2RV_0}\frac{9{V_0}^2}{4}+\frac{3P_0}{2R}.\frac{3V_0}{2}$$

$$\Rightarrow T_{max}=\frac{9P_0V_0}{8R}$$

5. In figure, A and B are two adiabatic curves for two different gases. Then A and B corresponds to:



(a) Ar and He respectively
(b) He and H_2 respectively
(c) O_2 and H_2 respectively
(d) H_2 and He respectively

Solution:

$$Y=\frac{C_P}{C_V}$$

$\rightarrow increases\ with\ atomicity, atomicity\ of\ B > atomicity\ of\ A.$

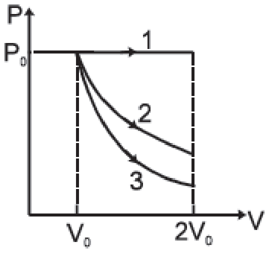
Among the given options,

B is correct option.

$Atomicity\ of\ H_2 > atomicity\ of\ H_e$

$\therefore Y\ of\ H_2 > Y\ of\ H_e$

6. In figure, an ideal gas is expanded from volume V_0 to $2V_0$ under three different processes. Process 1 is isobaric, process 2 is isothermal and process 3 is adiabatic. Let ΔU_1, ΔU_2 and ΔU_3 be the change in internal energy of the gas is these three processes. Then:



(a) $\Delta U_1 > \Delta U_2 > \Delta U_3$ (b) $\Delta U_1 < \Delta U_2 < \Delta U_3$
(c) $\Delta U_2 < \Delta U_1 < \Delta U_3$ (d) $\Delta U_2 < \Delta U_3 < \Delta U_1$

Solution:

Correct option is A

From process (1) : Isobaric

$$\Delta W=P\Delta V$$

$$PV=nRT$$

$$P\Delta V=nR\Delta T$$

$$\Delta T=\frac{\Delta W}{nR}$$

$$\Delta U=\frac{nR}{(r-1)}.\Delta T=\frac{nR}{r-1}.\frac{\Delta W}{nR}$$

$$U_1=\frac{\Delta W}{r-1}$$

From process (2)

Isothermal: $\Delta T=0$

$U_2=0$

From process (3):

Adiabatic :$\Delta Q=0$

$\Delta W=-\Delta u$

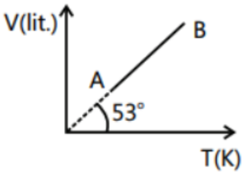
$U_3=-\Delta \mathrm{W}$

$$\frac{nR}{(r-1)}\Delta T=\Delta W$$

$$\Delta T=\frac{\Delta W(r-1)}{nR}$$

So, $U_1 > U_2 > U_3$

7. V-T curve for 2 moles of a gas is straight line as shown in the graph here. Find the pressure of gas at A.



Solution:

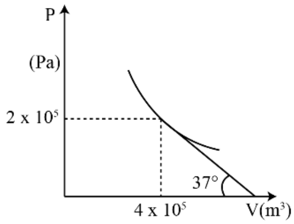
We have, V = (nR/p) T and we know that V – T curve is a straight line.

Pressure is constant and its slope = nR/p= tan 53°

$$\Rightarrow \frac{2\times 8.314\times 10^3 Pa\ L/moleK}{P}=\frac{4}{3}$$

$$\Rightarrow P = 1.25\times 10^4\ Pa$$

8. P-V graph for an ideal gas undergoing polytropic process PV^m = constant is shown here. Find the value of m.



Solution:

Slope of the curve,

$$\frac{dP}{dV}=\tan(180-37)=\tan 143^\circ=-0.75$$

The given polytropic process

$$PV^m = constant$$

Differentiating we get

$$P\times mV^{m-1}dV = V^m dP = 0$$

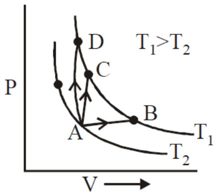
$$\therefore mPV^{m-1}dV = -V^m dP$$

$$\Rightarrow \frac{dP}{dV} = -\frac{mP}{V}$$

$$\therefore -0.75 = \frac{m\times 2\times 10^5}{4\times 10^5}$$

$$\Rightarrow m = 1.5$$

9. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as A → B, A → C and A →D. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the work done as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are:



(a) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$ (b) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$

(c) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$

(d) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$

Solution:

(b)Temperature change ΔT is same for all three processes A → B; A → C and A → D

$\Delta U = nC_v\Delta T$ = same

$E_{AB} = E_{AC} = E_{AD}$

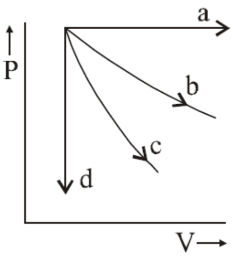
Work done, $W = P\times\Delta V$

AB → volume is increasing ⇒ $W_{AB} > 0$

AD → volume is decreasing ⇒ $W_{AD} < 0$

AC → volume is constant ⇒ $W_{AC} = 0$

10. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by:



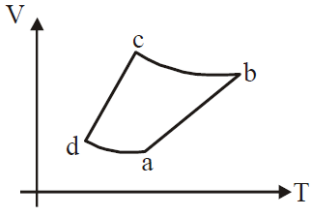
(a) a d b c **(b) d a c b**

(c) a d c b **(d) d a b c**

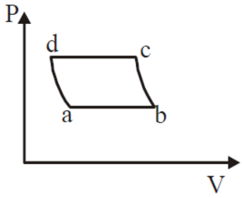
Solution:

(d) a → Isobaric, b → Isothermal, c → Adiabatic, d → Isochoric

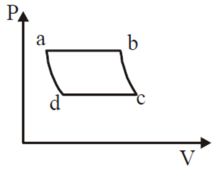
11. An ideal gas goes through a reversible cycle a → b → c → d has the V - T diagram shown below. Process d → a and b → c are adiabatic.



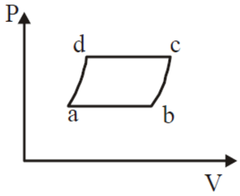
The corresponding P - V diagram for the process is (all figures are schematic and not drawn to scale) :



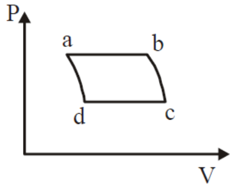
(a)



(b)



(c)



(d)

Solution:

(b) In VT graph

ab-process : Isobaric, temperature increases.

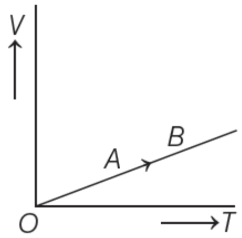
bc process : Adiabatic, pressure decreases.

cd process : Isobaric, volume decreases.

da process : Adiabatic, pressure increases.

The above processes correctly represented in P-V diagram (b).

12. The volume (V) of a monoatomic gas varies with its temperature (T), as shown in the graph. The ratio of work done by the gas, to the heat absorbed by it, when it undergoes a change from state A to state B, is



(a) $\frac{1}{3}$ **(b)** $\frac{2}{3}$

(c) $\frac{2}{5}$ **(d)** $\frac{2}{7}$

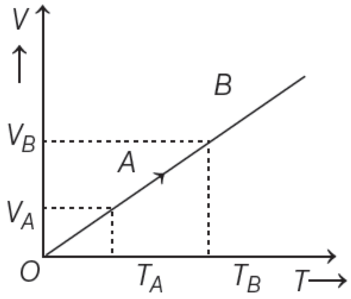
Solution:

(c) According to the given graph,

Volume (V) ∝ Temperature (T)

Or $\frac{V}{T}$ = constant

Thus, the process is isobaric.



∴ Work done, $\Delta W = p\Delta V$

$= nR\Delta T = nR(T_B - T_A)$ ... (i)

Heat absorbed $\Delta Q = nC_p\, \Delta T$

$= nC_p\,(T_B - T_A)$ ...(ii)

As, $C_p = \frac{\gamma R}{\gamma - 1}$, $\left[where,\ \gamma = 1 + \frac{2}{f}\right]$

For a monoatomic gas, f = 3

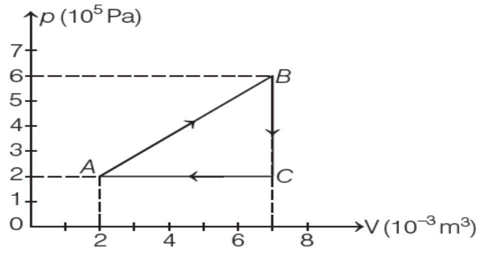
$$C_p = \left(R + \frac{3}{2}R\right) = \frac{5}{2}R$$

Substituting the value of C_p in Eq. (ii), we get

$$\Delta Q = n\left(\frac{5}{2}R\right)(T_B - T_A)$$

Hence, $\frac{\Delta W}{\Delta Q} = \frac{nR(T_B - T_A)}{n\left(\frac{5}{2}R\right)(T_B - T_A)} = \frac{2}{5}$

13. A gas is taken through the cycle A →B→C→A, as shown. What is the net work done by the gas?



(a) 2000 J **(b) 1000 J**

(c) Zero **(d) –2000 J**

Solution:

(b) According to question,

Net work done = Area enclosed in pv curve i.e. ΔABC

Area of ΔABC

$= \frac{1}{2} \times 5 \times 10^{-3} \times 4 \times 10^{5}\ \text{J} = 10^{3}\ \text{J} = 1000\ \text{J}$

Beginner Cyclic Process

1. In a cyclic process, the internal energy of the gas

(a) Increases **(b) Decreases**

(c) Remains constant(d) Becomes zero

Solution:

The answer is C.

Internal energy is a state function. In a cyclic process the system returns to its initial state and so the internal energy is not changed. The heat added to the system equals the work done by the system. A free expansion is an adiabatic change that does not work.

2. In a cyclic process, work done by the system is

(a) Zero

(b) Equal to heat given to the system

(c) More than the heat given to system

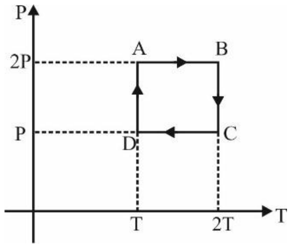
(d) Independent of heat given to the system

Solution:

The correct answer is B.

In a cyclic process, since the system returns to its initial state, the change in internal energy must be zero. Therefore, by first law of thermodynamics the net heat given to the system must be equal to the net work done by the system. Net heat given to the system must be equal to the net work done by the system.

3. U represents the Internal Energy. Which of the following is not true?



(a) $U_A = U_D$ **(b) $U_B < U_A$**

(c) $U_B = U_C$ **(d) $U_C > U_D$**

Solution:

The answer is D.

Internal energy of a gas

$$U = \frac{3}{2}nRT$$

Where R is constant.

Thus, for a given number of moles of the same gas

$$U \propto T$$

From the graphs, we conclude that

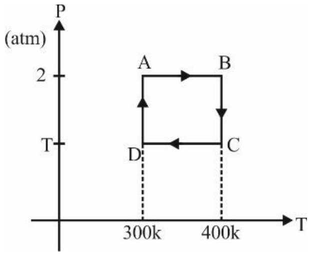
$$U_A = U_D$$

$$U_B = U_C$$

$$U_B > U_A$$

$$U_C > U_D$$

4. Two moles of helium gas undergoes a cyclic process. Calculate the following:



(a) Q_{net} **(b)W_{net}**

(c) ΔU

Solution:

In a cyclic process.

$$\Delta U = 0$$

$$Q_{net} = W_{net}$$

$$Q_{AB} = nC_p\Delta T$$

$$= (2)\left(\frac{5}{2}R\right)(400 - 300) = 500R$$

$$Q_{BC} = nRT_B \ln\left(\frac{p_i}{p_f}\right)$$

$$= (2)(R)(400)\ln\left(\frac{2}{1}\right) = 800R\ln(2)$$

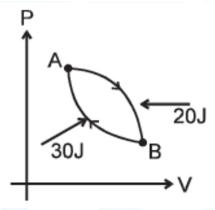
$$Q_{CD} = nRT_D \ln\left(\frac{p_i}{p_f}\right)$$

$$= (2)(R)(300)\ln\left(\frac{2}{1}\right) = -600R\ln(2)$$

$$\therefore Q_{net} = W_{net} = (200R)\ln(2)$$

$$= (200)(8.31)(0.693) \approx 1153J$$

5. In a cyclic process shown in the figure an ideal gas is adiabatically taken from B and A, the work done on the gas during the process B → A is 30 J, when the gas is taken from A → B the heat absorbed by the gas is 20 J. The change in internal energy of the gas in the process A → B is:



(a) 20 J **(b) – 30 J**

(c) 50 J **(d) – 10 J**

Solution:

The correct answer is B.

$$W_{B-A} = -30J$$

Adiabatic process, $Q_{B-A} = 0J$

$$Q_{B-A} = W_{B-A} + U_{B-A}$$

$$U_{B-A} = 30J$$

We know that the internal energy only depends on the final and initial state.

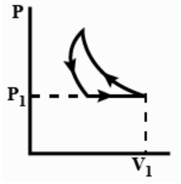
$$U_{B-A} = U_B - U_A$$

$$U_{B-A} = U_B - U_A = -U_{B-A}$$

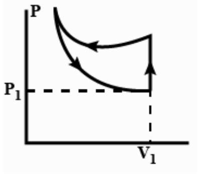
$$U_{A-B} = -30J$$

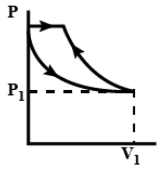
6. A certain mass of an ideal gas is at pressure P_1 and volume V_1. It is compressed isothermally and then allowed to expand adiabatically until its pressure returns to P_1. The gas is then allowed to expand its original volume. Which of the following P-V graphs are these process correctly shown?

(a)

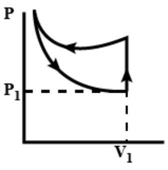


(b)





(c)



(d)

Solution:

The correct answer is A.

For an isothermal process:

$PV = constant$

$PdV = -dPV$

$\frac{dP}{dV} = \frac{-P}{V}$

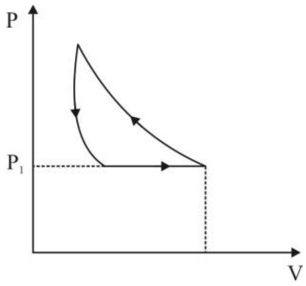
For adiabatic process:

$PV^{\gamma} = constant$

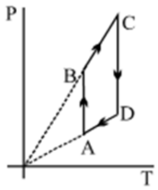
$\frac{dP}{dV} = -\gamma\frac{P}{V}$

A certain mass of an ideal gas is at pressure P_1 and volume V_1. It is compressed isothermally hence volume decreases and pressure increases with hyperbola.Then it is expand adiabatically until its pressure returns to P_1 the graph is little steeper hyperbola than isothermal graph.

The gas is then allowed to expand its original volume represents a straight line with zero slope to pressure axis and at increasing volume best represented by,



7. Pressure versus temperature graph of an ideal gas is shown in figure



(a) During the process AB work done by the gas in positive

(b) During the process CD work done by the gas is negative

(c) During the process BC internal energy of the gas is increasing

(d) None

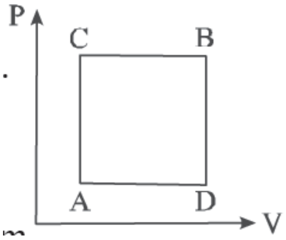
Solution:

The correct answer is C.

Internal energy of an ideal gas is a function of temperature alone, and doesn't depend on any other parameters like volume, or pressure.

In process BC, the temperature increases linearly, hence the internal energy of the gas also increases.

8. A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used 60 J of heat flows into the system and 30J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is:

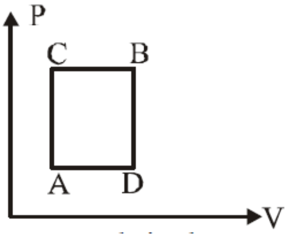


(a) 40J **(b) 80J**

(c) 100 J **(d) 20J**

Solution:

(a)ΔU remains same for both paths ACB and ADB



$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

$60\text{ J} = 30\text{ J} + \Delta U_{ACB}$

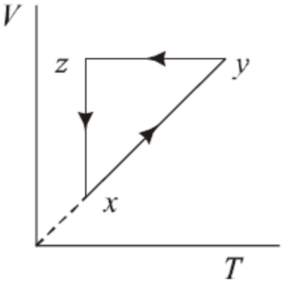
$U_{ACB} = 30\text{ J}$

$\therefore \Delta U_{ADB} = \Delta U_{ACB} = 30\text{ J}$

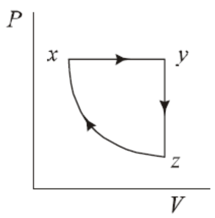
$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB}$

$= 10\text{ J} + 30\text{ J} = 40\text{ J}$

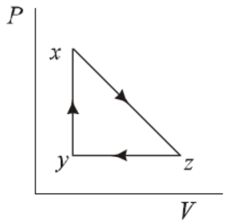
9. A thermodynamic cycle xyzx is shown on a V T diagram.



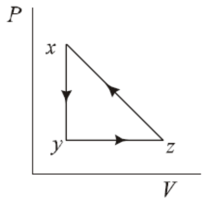
The P-V diagram that best describes this cycle is: (Diagrams are schematic and not to scale)



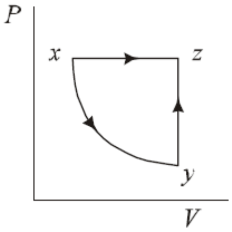
(a)



(b)



(c)



(d)

Solution:

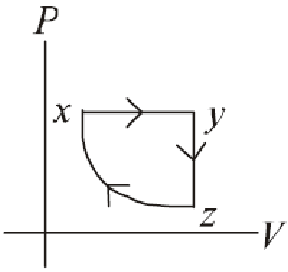
(a) From the corresponding V-T graph given in question,

Process xy → Isobaric expansion,

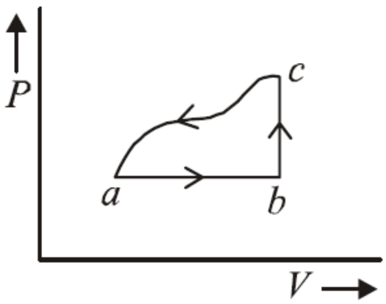
Process yz → Isochoric (Pressure decreases)

Process zx → Isothermal compression

Therefore, corresponding PV graph is as shown in figure



10. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is – 180 J, The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work down by the gas along the path abc is:



(a) 120J **(b) 130J**

(c) 100J **(d) 140J**

Solution:

(b) $\Delta U_{ac} = -(\Delta U_{ca}) = -(-180) = 180$ J

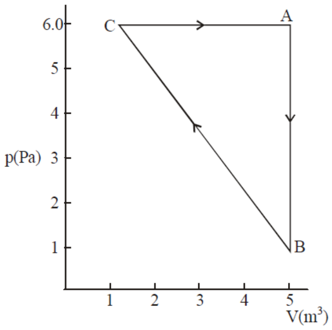
Q = 250 + 60 = 310 J

Now Q = ΔU + W

Or 310 = 180 + W

Or W = 130 J

11. For the given cyclic process CAB as shown for gas, the work done is:



(a) 30J **(b) 10J**

(c) 1J **(d) 5J**

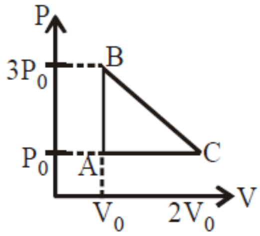
Solution:

(b) Total work done by the gas during the cycle is equal to area of triangle ABC.

$$\therefore\ \Delta W = \frac{1}{2}\times 4\times 5 = 10\text{ J}$$

Expert Cyclic Process

1. One mole of an ideal monoatomic gas is taken along the path ABCA as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given b



(a) $\frac{25}{8}\frac{P_0V_0}{R}$ **(b)** $\frac{25}{4}\frac{P_0V_0}{R}$

(c) $\frac{25}{16}\frac{P_0V_0}{R}$ **(d)** $\frac{5}{8}\frac{P_0V_0}{R}$

Solution:

(a) Equation of the BC

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

Using PV = nRT

$$\text{Temperature, } T = \frac{P_0V - \frac{2P_0V^2}{V_0} + 4P_0V}{1\times R}$$

(∵ n = 1 mole given)

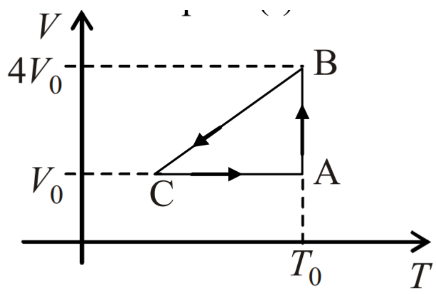
$$T = \frac{P_0}{R}\left[5V - \frac{2V^2}{V_0}\right]$$

$$\frac{dT}{dV}=0\Rightarrow 5-\frac{4V}{V_0}=0$$

$$V=\frac{5}{4}V_0$$

$$T=\frac{P_0}{R}\left[5\times\frac{5V_0}{4}-\frac{2}{V_0}\times\frac{25}{16}V_0^2\right]=\frac{25}{8}\frac{P_0V_0}{R}$$

2. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following



(a)Internal energies at A and B are the same

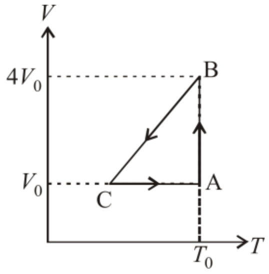
(b)Work done by the gas in process AB is P_0V_0 In 4

(c)Pressure at C is $\frac{P_0}{4}$

(d)Temperature at C is $\frac{T_0}{4}$

Solution:(a, b)

Process A to B



Here, $T_A=T_B \therefore U_A=U_B$

Also $P_0V_0=P_B\times 4V_0$

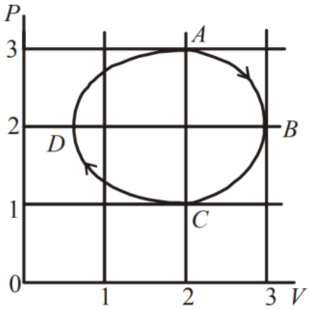
$\rightarrow P_B=\frac{P_0}{4}$

Work done

$$W_{AB}=nRT_0\log_e\frac{4V_0}{V_0}$$

$$=P_0V_0\log_e 4 \qquad [\because P_0V_0=nRT_0]$$

3. The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is an semi circle and CDA is half of an ellipse. Then,



(a) The process during the path $A\rightarrow B$ is isothermal

(b) Heat flows out of the gas during the path $B\rightarrow C\rightarrow D$

(c) Work done during the $A\rightarrow B\rightarrow C$ is zero

(d) Positive work is done by the gas in the cycle ABCDA

Solution:(b, d)

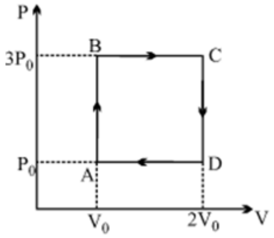
(a) Process A-B is not isothermal. In case of an isothermal process we get a rectangular hyperbola in a P-V diagram.

(b) In process B→C→D, ΔU is negative. PV decreases and volume also decreases; therefore, W is negative. From first law of thermodynamic, Q is negative i.e., there is a heat loss.

(c) $W_{AB}>W_{BC}.Therefore$ work done during path A→B→C is positive.

(d) Work done is clockwise cycle in a PV diagram is positive.

4. An ideal monoatomic gas is carried around the cycle ABCDA as shown in the fig. The efficiency of the gas cycle is



(a) $\frac{4}{21}$ **(b)** $\frac{2}{21}$

(c) $\frac{4}{31}$ **(d)** $\frac{2}{31}$

Solution:

The correct answer is A.

Heat is absorbed only during processes AB and Bc

Gas is mono atomic. Hence

$$C_V=\frac{3}{2}RC_P=\frac{5}{2}R$$

Heat is absorbed during AB,

$$\Delta Q_{AB} = nC_V\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}V\Delta P = 3P_oV_o$$

Heat absorbed during BC,

$$\Delta Q_{BC} = nC_P\Delta T = \frac{5}{2}nR\Delta T = \frac{5}{2}P\Delta V = \frac{15}{2}P_oV_o$$

Hence, net heat absorbed

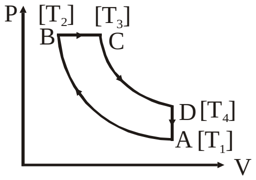
$$= \Delta Q = \Delta Q_{AB} + \Delta Q_{BC} = \frac{21}{2}P_oV_o$$

Also, net work done= ΔW = area under P-V graph= $2P_oV_o$

Hence,

$$efficiency = \frac{\Delta W}{\Delta Q} = \frac{4}{21}$$

5. **A certain mass of a diatomic gas occupies a volume of 100 cm^3 at a pressure of 75 cm of Hg. It is compressed adiabatically until the pressure increased to 80 cm of Hg. Calculate the work done by gas in the process. ($\rho_{Hg} = 13.534 g/cm^3$)**



Solution:

$$P_1{V_1}^r = P_2{V_2}^r$$

$$75 \times (100)^{1.4} = 80 \times (V_2)^{1.4}$$

$$\left(\frac{V_2}{100}\right)^{1.4} = \frac{75}{80}$$

$$\left(\frac{V_2}{100}\right) = (0.9375)^{1/1.4}$$

$$V_2 = 95.43$$

$$W = \frac{P_1V_1 - P_2V_2}{r-1}$$

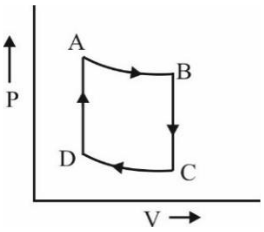
$$= \frac{(75 \times 100 - 80 \times 95.42)}{1.4 - 1} \times g \times 13.539$$

$$= \frac{139.52}{0.4} \times 9.81 \times 13.539$$

$$= 46.32 \times 10^5 J$$

Pro **Cyclic Process**

1. **One mole of monoatomic ideal gas is taken through the cycle shown in figure.**



A → B Adiabatic expansion

B → C Cooling at constant volume

C → D Adiabatic compression

D → A Heating at constant volume

The pressure and temperature at A, B etc., are denoted by P$_A$, T$_A$; P$_B$, T$_B$ etc. respectively. Given T$_A$ = 1000 K, P$_B$ = (2/3)P$_A$ and P$_C$ = (1/3)P$_A$ Calculate

(a)

The work done by the gas in the process A→ B

(b)

The heat lost by the gas in the process B→C and

(c)

Temperature T$_D$ given $(2/3)^{2/5} = 0.85$ and R = 8.31 J/mol K.

Solution:

The answer is A

$$W_{AB} = \frac{nR[T_A - T_B]}{\gamma - 1}$$

Hence

$$n = 1; R = 8.32; T_A = 1000K$$

$$\gamma = \frac{5}{3}$$

To find T$_B$ we use

$$\frac{T_A^\gamma}{P_A^{\gamma-1}} = \frac{T_B^\gamma}{P_B^{\gamma-1}} \Rightarrow \left(\frac{P_A}{P_B}\right)^{\gamma-1} = \left(\frac{T_A}{T_B}\right)^\gamma = (3/2)^{\gamma-1}$$

$$T_B = 850K$$

$$W_{AB} = \frac{1 \times 8.31[1000 - 850]}{\frac{5}{3} - 1} = 1870J$$

b)$Q = n \times C_V\Delta T = n \times C_V\Delta(T_B - T_C)$

$$T_B = 850K$$

To find T$_C$ we use

$$\frac{P_B}{T_B} = \frac{P_C}{T_C} (volume\ constant)$$

Substituting the values we get $T_C = 425K$

$$Q = 1 \times \frac{3}{2} \times 8.31 \times [425 - 850] = -5298J$$

c) Temperature T$_D$

$$T_A = T_D = 1000K$$

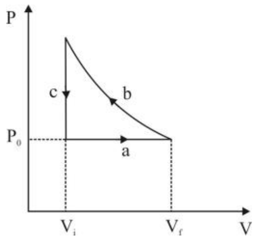
2. The answer is.

For enclosed gas number of moles

$$n = \frac{8}{32} = \frac{1}{4} mole$$

From gas equation for process (a)

$$\frac{P_0V_i}{T_i} = \frac{P_0V_f}{T_f} \Rightarrow \frac{V_i}{V_f} = \frac{300}{400} = \frac{3}{4}$$



(1)Heat absorbed by the gas during operation ‘a’

$$Q = nC_P\Delta T = \frac{1}{4}\left[\frac{670}{1000} \times 32 + \frac{25}{3}\right] \times 100$$

$$= \frac{2233}{3}J = 744.3J$$

(2)Work done by the gas in case ‘a’

$$W = nR\Delta T = \frac{1}{4} \times \frac{25}{3} \times 100 = \frac{625}{3}J$$

(3)Work done by the gas in case ‘b’

$$W = nRT\log_e\left(\frac{V_f}{V_i}\right) = \frac{1}{4}\frac{25}{3} \times 400\ln\left(\frac{4}{3}\right)$$

$$= \frac{2500}{3}\ln(4/3)$$

(4)Heat extracted from gas in case ‘c’

$$Q = nC_V\Delta T = \frac{1}{4} \times \frac{670}{1000} \times 32 \times 100 = 536J$$

2. Calculate the efficiency of a heat engine in a gas (whose ratio of specific heat) while it is being taken through a cycle as shown in the indicator diagram. (Process A to B and C to D is adiabatic)

Solution:

BC is an isobaric process and temperature from T_2 to T_3

Heat absorbed = $Q_1 = C_P(T_3 - T_2)$

CD is an adiabatic process. So no heat is absorbed or rejected.

DA is an isochoric process.

So heat rejected = $Q_2 = C_V(T_4 - T_1)$ and AB is an adiabatic process. So again, no heat is absorbed or rejected during AB.

Net amount of heat converted to work $= Q_2 - Q_1$

$= C_P(T_3 - T_2) - C_V(T_4 - T_1)$

So, efficiency

$$\eta = \frac{work\ done}{heat\ absorbed}$$

$$= \frac{Q_2 - Q_1}{Q_1}$$

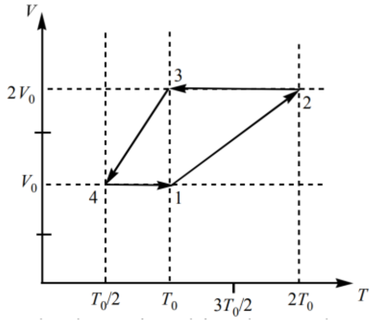
$$= \frac{C_P(T_3 - T_2) - C_V(T_4 - T_1)}{C_P(T_3 - T_2)}$$

$$= 1 - \frac{C_V}{C_P}\frac{(T_4 - T_1)}{T_3 - T_2} = 1 - \frac{1}{\gamma}\frac{(T_4 - T_1)}{T_3 - T_2}$$

3. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram.

The correct statement(s) is/are:

[R is the gas constant]



(a)Work done in this thermodynamic cycle $(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1)$is |W| = ½ RT_0

(b)The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(c)The ratio of heat transfer during processes 1 → 2 and 3 → 4 is $Q_{1\rightarrow2}/Q_{2\rightarrow3}$ = 5/3

(d)The ratio of heat transfer during processes 1 → 2 and 3 → 4 is $Q_{1\rightarrow2}/Q_{2\rightarrow3}$ = 1/2

Solution:

The P-V graph of the given V-T graph is given below.

(a)Work done during cyclic process (1→2→3→4 →1) W

= area enclosed in the loop = $\frac{P^0}{2}V^0$

∴ $P_0V_0 = nRT_0$ ∴ $\frac{P_0V_0}{2} = \frac{nRT_0}{2}$

∴ Work done W = $\frac{nRT_0}{1} = \frac{RT_0}{2}$ [as n = 1]

(b) Process 1→2 is isobaric

Process 2→ 3 is isobaric

Process 3→ 4 is isobaric

Process 4 → 1 is isobaric

Hence no adiabatic process is involved.

(c) $|\Delta Q_{1\rightarrow2}| =$

$|nC_p\Delta T| = |nC_p(2T_0 - T_0)| = |nC_pT_0$

$|\Delta Q_{2\rightarrow3}| = |\Delta U| = |nC_v\Delta T| = |nC_vT_0|$

∴ $\left|\Delta\frac{Q_{1\rightarrow2}}{\Delta Q_{2-3}}\right| = \frac{C_p}{C_v} = \frac{5}{3}$

(d) $\left|\Delta Q_{3\to4}\right| = nC_p\frac{T_0}{2}$

$\left|\Delta\frac{Q_{1\to2}}{\Delta Q_{3-4}}\right| = \frac{nC_pT_0}{nC_p\frac{T_0}{2}} = \frac{2}{1}$

Hence, a, c are correct.

Beginner **Polytropic Process**

1. An ideal gas obeys $T\sqrt{P}$ = constant at STP (273 K, 1 atm). If volume of gas changes from V to 8V, Find the final temp. of gas.

Solution:

$TP^{1/2} = constant$

$T\left(\frac{T}{V}\right)^{1/2} = constant$

$T^{3/2}V^{-1/2} = constant$

${T_1}^{3/2}{V_1}^{-1/2} = {T_2}^{3/2}{V_2}^{-1/2}$

$(273)^{3/2}(V)^{-1/2} = {T_2}^{3/2}\left(8V^{-1/2}\right)$

$(273)^{3/2}(8)^{1/2} = {T_2}^{3/2}$

$273 \times 8^{1/3} = T_2$

$273 \times 2 = T_2$

$T_2 = 546K$

2. During an experiment, an ideal gas is found to obey the equation P^{2}V = constant. The gas is initially at temperature T and volume V. When it expands to a volume 2V, the temperature becomes

(a) $\sqrt{2}T$ **(b) T/2**

(c) $\sqrt{3}T$ **(d) 2T**

Solution:

The answer is C.

$P^2V = constant$

Or

$\left(\frac{RT}{V}\right)^2 V = Constant \quad [\therefore PV = RT]$

Or,

$\frac{T^2}{V} = constant$

Now,

$T_1 = T\,, V_1 = V$

$T_2 = ?\,, V_2 = 2V$

$\therefore \frac{{T_1}^2}{V_1} = \frac{{T_2}^2}{V_2}$

$\therefore {T_2}^2 = \frac{V_2}{V_1} \times {T_1}^2$

$\frac{2V}{V} \times T^2$

$\therefore T_2 = \sqrt{2}T$

3. A perfect gas is found to obey the relation PV$^{3/2}$ = constant, during an adiabatic process. If such a gas, initially at a temperature T, is compressed adiabatically to half its initial volume, then its final temperature will be

(a) 2T **(b) 4T**

(c) $\sqrt{2}T$ **(d)** $2\sqrt{2}T$

Solution:

The answer is C

$pV^{2/3} = Constant$

$\frac{nRT}{V}V^{2/3} = Constant$

$\frac{T}{V^{1/3}} = Constant$

$T = (k)V^{1/3}$

Given, $pV^{\gamma} = constant\,, \gamma = \frac{3}{2}$

$TV^{\gamma-1} = constant$

$\Rightarrow T_f = T_i\left(\frac{V_i}{V_f}\right)^{r-1}$

$\Rightarrow T_f = T\left(\frac{2V_f}{V_f}\right)^{\frac{3}{2}-1}$

$T_f = T(2)^{1/2}$

$\Rightarrow T_f = \sqrt{2}T$

4. Two mole diatomic ideal gas is taken through the process PT = const. Its temperature is increased from T$_0$K to 2T$_0$K . Find work done by the system?

Solution:

pT = constant

$\therefore p(pV) = constant\ (as\ T \propto pV)$

$or\ \ pV^{1/2} = constant$

Comparing with $pV^x = constant$

We have,

$x = \frac{1}{2}$

$W = \frac{nR\Delta T}{1-x} = \frac{2R(2T-T)}{1-\frac{1}{2}}$

$= 4RT$

5. An ideal gas mixture filled inside a balloon expands according to the relation PV$^{2/3}$ = constant. The temperature inside the balloon is

(a) increasing **(b) decreasing**

(c) constant **(d) can't be said**

Solution:

The correct option is A

$PV^{2/3} = constant$

$\frac{nRT}{V} V^{2/3} = constant$

$\frac{T}{V^{1/3}} = constant$

$T = kV^{1/3}$

Hence, Option A is correct.

6. An ideal gas expands in such a way that PV^2 = constant throughout the process.

(a) The graph of the process of T-V diagram is a parabola.

(b) The graph of the process of T-V diagram is a straight line.

(c) Such an expansion is possible only with heating.

(d) Such an expansion is possible only with cooling.

Solution:

Correct option is D.

From ideal gas equation, PV = nRT

⇒ VT = constant

⇒ V is inversely proportional to T

⇒expansion causes increase in V

⇒it leads to decrease in T

⇒gas is cooled

Expert Polytropic Process

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1. During an experiment, an ideal gas is found to obey a condition VP^2 = constant. The gas is initially at a temperature T, pressure P and volume V. The gas expands to volume 4V.

(a)The pressure of gas changes to $\frac{P}{2}$

(b)The temperature of gas changes to 4T

(c)The graph of above process on the P-T diagram is parabola

(d)The graph of above process on the P-T diagram is hyperbola.

Solution:

Correct option is A and D.

During an adiabatic process $PV^{\gamma} = constant$ ... (1)

Given,

$VP^2 = constant$

$\therefore V_1{P_1}^2 = V_2{P_2}^2 = constant$

Given,

$V_2 = 4V_1 \Rightarrow V_1{P_1}^2 = 4V_1{P_2}^2 \Rightarrow P_2 = \frac{P_1}{2}$

$\therefore PV^{0.5} = constant$ ... (2)

Comparing (2) with (1) we get,

$\gamma = \frac{1}{2}$

Option B

From adiabatic process, $TV^{\gamma-1} = constant$

$\therefore T_1{V_1}^{\gamma-1} = T_2{V_2}^{\gamma-1}$

$\therefore \frac{T_2}{T_1} = \left(\frac{1}{4}\right)^{0.5-1} = \left(\frac{1}{4}\right)^{-0.5} = 2^{(-2)\times(0.5)} = 2$

Option A and D.

For adiabatic process,

$\frac{T^{\gamma}}{P^{\gamma-1}} = constant$

$Substituting\ \gamma = \frac{1}{2}, we\ get\ \frac{T^{0.5}}{P^{0.5-1}} = constant$

$\therefore TP = constant$

$\therefore PT\ curve\ is\ not\ a\ parabola, but\ a\ hyperbola$

2. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P$_0$ and V$_0$ are constants. Calculate the charge in the temperature of the gas if its volume changes from V$_0$ to 2V$_0$.

(a) $\frac{1}{2}\frac{P_0V_0}{R}$ **(b)** $\frac{5}{4}\frac{P_0V_0}{R}$

(c) $\frac{3}{4}\frac{P_0V_0}{R}$ **(d)** $\frac{1}{4}\frac{P_0V_0}{R}$

Solution:

(b) We have given,

$$P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$$

When V$_1$ = V$_0$

$$P_1 = P_0\left[1 - \frac{1}{2}\right] = \frac{P_0}{2}$$

When V$_2$ = 2V$_0$

$$P_2 = P_0\left[1 - \frac{1}{2}\left(\frac{1}{4}\right)\right] = \left(\frac{7P_0}{8}\right)$$

$$\Delta T = T_2 - T_1 = \left|\frac{P_1V_1}{nR} - \frac{P_2V_2}{nR}\right| \quad \left[\because T = \frac{PV}{nR}\right]$$

$$\Delta T = \left|\left(\frac{1}{nR}\right)(P_1V_1 - P_2V_2)\right|$$

$$= \left(\frac{1}{nR}\right)\left|\left(\frac{P_0V_0}{2} - \frac{7P_0V_0}{4}\right)\right|$$

$$= \frac{5P_0V_0}{4nR} = \frac{5P_0V_0}{4R} \qquad (\because n = 1)$$

3. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is:

(a) $\frac{3}{5}$ **(b)** $\frac{2}{5}$

(c) $\frac{2}{3}$ **(d)** $\frac{5}{3}$

Solution:

(b) Equation of adiabatic change is

TV$^{\gamma-1}$ = constant

Put $\gamma = \frac{7}{5}$, we get: $\gamma - 1 = \frac{7}{5} - 1$

$\therefore\ x = \frac{2}{5}$

Pro Polytropic Process

1. Suppose the pressure P and the density ρ of air are related as P/ρ = constant regardless of height (n is constant here). Find the corresponding temperature gradient. (Take average molecular weight of air = M)

Solution:

We have,

$$\frac{dp}{dh} = -pg \qquad (1)$$

But,

$$from\ p = Cp^n(where\ C\ is\ a\ const)\frac{dp}{dh}$$
$$= Cnp^{n-1}$$

We have from gas low

$$p = \rho\frac{R}{M}T, \qquad so\ using\ (2)$$

$$Cp^n = \rho\frac{R}{M}.T, or\ T = \frac{M}{R}Cp^{n-1}$$

Thus,

$$\frac{dT}{d\rho} = \frac{M}{R}.C(n-1)p^{n-2} \quad (3)$$

But,

$$\frac{dT}{dh} = \frac{dT}{d\rho}.\frac{d\rho}{dp}.\frac{dp}{dh}$$

So,

$$\frac{dT}{dh} = \frac{M}{R}C(n-1)\rho^{n-2}\frac{1}{Cn\rho n - 1}(-\rho g)$$
$$= \frac{-Mg(n-1)}{nR}$$

2. An ideal gas is taken through a process in which the pressure and the volume are changed according to the equation P = KV. Show that the molar heat capacity of the gas for the process is given by C = C$_v$ +R/2.

Solution:

For the gas work done will be given by

$$W = \int -PdV = -\int kVdV = -k\left(\frac{{V_2}^2 - {V_1}^2}{2}\right)$$

Also, since P = kV

$$W = \frac{P_2V_2 - P_1V_1}{2} = \frac{nR\Delta T}{2}$$

Also, internal energy change will be

$$\Delta U = nC_V\Delta T$$

Since

$$\Delta U = Q + W$$

$$Q = nC\Delta T = \Delta U - W = nC_V\Delta T + \frac{R}{2}n\Delta T$$

$$C = C_V + \frac{R}{2}$$

3. The volume of one mole of an ideal gas with the adiabatic exponent y is varied according to the law $V = \frac{\alpha}{T}$, where a is a constant. Find the amount of heat obtained by the gas in this process if the gas temperature increased by Δ T.

Solution:

$$\Delta U = \frac{nR\Delta T}{r-1}$$

$$w = \int pdV \qquad = \int\frac{nRT}{v}dv$$

$$= \int\frac{nRT}{\left(\frac{a}{T}\right)}\frac{(-adT)}{T^2}$$

$$= -nR\int_T^{T+\Delta T} dT = -nR\Delta T$$

$$Q = \Delta U + W = nR\Delta T\left(\frac{1}{r-1} - 1\right) = nR\Delta T.\frac{2-r}{r-1}$$

4. Demonstrate that the process in which the work performed by an ideal gas is proportional to the corresponding increment

of its internal energy is described by the equation pV^n = const, where n is a constant.

Solution:

$$dw = pdv = \frac{nRT}{v}dv \qquad ..(i)$$

$$dU = \frac{nRdT}{r-1} \qquad ....(ii)$$

$$w \propto U \quad (given)$$

$$\Rightarrow dw \propto dU \qquad \Rightarrow dw = k,dU$$

$$\frac{nRTdv}{v} = k_1 \frac{nRdT}{r-1}$$

$$\Rightarrow \int \frac{dv}{v} = k_2 \int \frac{dT}{T}$$

$$\Rightarrow \ell nV = k_2 \ell nT + k_3$$

$$\Rightarrow \ell nV = k_2 \ell n(k_4 T)$$

$$\Rightarrow v = k_4 T^{k_2} \qquad \Rightarrow v = k_4\left(\frac{pv}{nR}\right)^{k_2}$$

$$\Rightarrow v^{1/k_2} = k_4{}^{V/k_2} . \frac{pv}{nR}$$

$$\Rightarrow p = k_5 v^{1/k_2 - 1}$$

$$\Rightarrow pv^{1-\frac{1}{k_2}} = k_5$$

$$\Rightarrow pv^n = \text{constant}$$

5. One mole of argon is expanded polytropically, the polytropic constant being n = 1.50. In the process, the gas temperature changes by AT = -26 K. Find :

(a) The amount of heat obtained by the gas;

(b) The work performed by the gas.

Solution:

(a) $Q = n_0 C\Delta T$

$$= n_0 R\left(\frac{1}{r-1} - \frac{1}{n-1}\right)\Delta T$$

(b) $w = -\dfrac{n_0 R\Delta T}{n-1}$

For a polytrophic process,

$$pv^n = const$$

$$C = R\left[\frac{1}{r-1} - \frac{1}{n-1}\right](derived\ in\, 2.45)$$

$$w = -\frac{n_0 R\Delta T}{n-1}$$

6. An ideal gas whose adiabatic exponent equals y is expanded according to the law $p = \alpha V$, where a is a constant. The initial volume of the gas is equal to V$_0$. As a result of expansion the volume increases η times. Find:

(a) the increment of the internal energy of the gas;

(b) the work performed by the gas;

(c) the molar heat capacity of the gas in the process.

Solution:

For PV^n = const

$$C = R\left[\frac{1}{r-1} - \frac{1}{n-1}\right]$$

$$w = -\frac{n_0 R\Delta T}{n-1}$$

Polytropic process:

$$pv^{-1} = \alpha \equiv pv^n = const$$

$$finding\ \Delta T$$

$$\left(\frac{nRT}{v}\right)v^{-1} = \alpha$$

$$\Rightarrow \frac{T_0}{v_0^2} = \frac{T_2}{(\eta v_0)^2}$$

$$\Rightarrow T_2 = \eta^2 T_0$$

$$\Delta T = T_2 - T_0 = (\eta^2 \ -1)T_0$$

$$= (\eta^2 - 1)\frac{p_0 v_0}{\eta_0 R} = \frac{(\eta^2 - 1)\alpha v_0^2}{\eta_0 R}$$

(a) $\Delta U = \dfrac{n_0 R\Delta T}{r-1}$

$$= \frac{(\eta^2 - 1)\alpha v_0^2}{r-1}$$

(b) $w = \dfrac{-n_0 R\Delta T}{n-1}$

$$= -\frac{(\eta^2 - 1)\alpha v_0^2}{n-1}$$

(c) $c = R\left[\dfrac{1}{r-1} - \dfrac{1}{n-1}\right]$

7. One mole of an ideal gas whose adiabatic exponent equals γ undergoes a process in which the gas pressure relates to the temperature as $p = \alpha T^{\alpha}$, where a and α are constants. Find:

(a) The work performed by the gas if its temperature gets an increment ΔT ;
(b) The molar heat capacity of the gas in this process; at what value of α will the heat capacity be negative?

Solution:

For PV^n = const

$$C = R\left[\frac{1}{r-1} - \frac{1}{n-1}\right]$$

$$w = -\frac{n_0 R\Delta T}{n-1}$$

$$p = aT^{\alpha}$$

$$\Rightarrow p = a\left(\frac{pv}{nR}\right)^{\alpha}$$

$\therefore$ process is:

$$pv^{\frac{\alpha}{\alpha-1}} = const$$

$$\Rightarrow n = \frac{\alpha}{\alpha-1}$$

(a) $w = -\dfrac{n_o R\Delta T}{n-1}$

$$= -\frac{n_o R\Delta T}{\frac{\alpha}{\alpha-1} - 1} = (1-\alpha)n_0 R\Delta T$$

(b) $c = \dfrac{R}{r-1} - \dfrac{R}{\frac{\alpha}{\alpha-1} - 1}$

$$= \frac{R}{r-1}(\alpha-1)R$$

$$c < 0 \; if, < \alpha - 1$$

$$\Rightarrow \alpha < \frac{r}{r-1}$$

8. An ideal gas with the adiabatic exponent γ undergoes a process in which its internal energy relates to the volume as $U = aV^{\alpha}$, where a and α are constants. Find:
(a) The work performed by the gas and the amount of heat to be transferred to this gas to increase its internal energy by ΔU ;
(b) The molar heat capacity of the gas in this process.

Solution:

For PV^n = const

$$C = R\left[\frac{1}{r-1} - \frac{1}{n-1}\right]$$

$$w = -\frac{n_0 R\Delta T}{n-1}$$

$$U = av^{\alpha}$$

$$\frac{nRT}{r-1} = av^{\alpha}$$

$$\frac{pv}{r-1} = av^{\alpha}$$

$\therefore$ process is :

$$pv^{1-\alpha} = const$$

$$n = 1-\alpha$$

(a) $w = -\dfrac{n_o R\Delta T}{n-1}$

$$= -\left(\frac{n_o R\Delta T}{\alpha-1}\right) = \frac{(r-1)}{n-1} = \Delta U\frac{(1-r)}{n-1}$$

$$= \Delta U\frac{(1-r)}{(1-\alpha-1)} = \frac{(r-1)\Delta U}{\alpha}$$

(b) $c = R\left[\dfrac{1}{r-1} - \dfrac{1}{\alpha-1}\right]$

9. An ideal gas has an adiabatic exponent γ . In some process its molar heat capacity varies as $C = \dfrac{\alpha}{T}$, where a is a constant, Find :
(a) The work performed by one mole of the gas during its heating from the temperature T_0 to the temperature n times higher;
(b) The equation of the process in the variables p, V.

Solution:

$$n_0 CdT = n_0 C_v dT + pdV$$

$$A)C = C_v + \frac{pdV}{n_0 dT}$$

$$\Rightarrow \frac{\alpha}{T} = C_V + \frac{pdV}{dT}$$

$$\Rightarrow \int pdv = \int_{T_0}^{\eta T_0}\left(\frac{\alpha}{T} - C_V\right)dT$$

$$\Rightarrow w = \alpha\ell n\eta - \frac{R}{r-1}T_0(\eta-1)$$

B. $\frac{\alpha}{T} = C_v + \frac{RT}{V}\frac{dV}{dT}$

$$\int \frac{1}{T}\left(\frac{\alpha}{T} - C_v\right) dT = \int R\frac{dv}{v}$$

$$\Rightarrow -\frac{\alpha}{T} - C_V \ell nT = R\ln nV + k$$

$$\Rightarrow -\frac{\alpha R}{pv} - \frac{R}{r-1}\ell n\frac{pv}{R}$$

$$= R\ell nk'V$$

$$\Rightarrow \ell ne^{-\frac{\alpha}{pv}} + \ell n\left(\frac{pv}{R}\right)^{\frac{1}{1-r}}$$

$$= \ell n\left(k'v\right)$$

$$\Rightarrow \frac{e\left(p-1\right)}{e^{pv}}\frac{r}{pv} = const$$

10. One mole of an ideal gas whose pressure changes with volume as P = αV, where $\frac{P}{n}$ is a constant, is expanded so that its volume increase η times. Find the change in internal energy and heat capacity of the gas.

Solution:

Let V be the initial volume of the gas, it expanded to a volume µV. The work done in this process is given by

$$W = \int_v^{\eta v} PdV = \int_v^{\eta v} \alpha VdV = \alpha\left[\frac{V_2}{2}\right]_v^{\eta v}$$

$$\left[\frac{\alpha V_2}{2}\eta^2 - 1\right]$$

The pressure of the gas varies volume as

$P = \alpha V$

So, the initial and final pressure will be $\alpha V$and $\eta\alpha V$. The change in internal energy is given by

$$dU = nC_V dT = \frac{R(T_f - T)}{\gamma - 1} = \frac{P_f V_f - P_i V_i}{\gamma - 1}$$

$$= \frac{\eta^2\alpha V^2 - \alpha V^2}{\gamma - 1} = \frac{\alpha V^2}{\gamma - 1}(\eta^2 - 1)$$

The heat exchange in this process is given by

$Q = U + W$

$$\frac{\alpha V^2}{\gamma - 1}(\eta^2 - 1) + \frac{\alpha V^2}{2}(\eta^2 - 1)$$

$$= \frac{\alpha V^2}{2}(\eta^2 - 1)\left[\frac{\gamma + 1}{\gamma - 1}\right]$$

Hence,

$$T_i = \frac{P_i V_i}{nR} = \frac{\alpha V^2}{nR}$$

And

$$T_f = \frac{P_f V_f}{nR} = \frac{\mu^2\alpha V^2}{nR}$$

Now heat capacity,

$$C = \frac{Q}{T_f - T_i}$$

$$C = \frac{1}{T_f - T_i}\left[\frac{\alpha V^2}{2}(\eta^2 - 1)\left\{\frac{\gamma + 1}{\gamma - 1}\right\}\right]$$

$$= \frac{nR}{\alpha V^2(\mu^2 - 1)}\left[\frac{\alpha V^2}{2}(\eta^2 - 1)\left\{\frac{\gamma + 1}{\gamma - 1}\right\}\right]$$

$$= \frac{nR}{2}\left[\frac{\gamma + 1}{\gamma - 1}\right]$$

Hence, n = 1

11. A gas undergoes a process in which the pressure and volume are related by VP^n = constant. Find the bulk modulus of the gas.

Solution:

The correct answer is.

$$Bulk\ modulus\ \beta = -\frac{\Delta P}{\Delta V/V}$$

$$For\ this\ process = VP^n = k$$

$$PV^{\frac{1}{n}} = k$$

Differentiating this we get,

$$P\frac{1}{n}V^{\frac{1}{n}-1} + V^{\frac{1}{n}} = 0$$

$$V.\frac{dP}{dV} = \frac{-1}{n}P$$

$$\left|V.\frac{dP}{dV}\right| = \frac{P}{n}$$

12. An ideal gas has a molar heat capacity C$_V$ at constant volume. Find the molar heat capacity of this gas as a function of volume, if the gas undergoes the process: T = T$_0$e$^{\alpha V}$.

Solution:

The process is given as $T = T_o e^{\alpha V}$

Differentiating we get $dT = T_o e^{\alpha V} dV$

$$\Rightarrow \frac{dT}{dV} = \propto T \qquad (1)$$

Using 1st law of thermodynamics,

$dQ = dU + dW$

Where

$$\Delta U = C_V\Delta T \quad and\ dW = PdV \quad and\ \Delta Q = C\Delta T$$

$$\therefore C\Delta T = C_V\Delta T + PdV$$

Or

$$C = C_V + P\frac{dV}{dT}$$

$$\Rightarrow C = C_V + P \times \frac{1}{\alpha T} \qquad (1)$$

From ideal gas equation:

$$\frac{P}{T} = \frac{R}{V} \quad in\ equation\ , we\ get$$

$$C = C_V + \frac{R}{\alpha V}$$

13. An ideal gas, whose adiabatic exponent is equal to γ, is expanded so that the amount of heat transferred to the gas is equal to the decrease of internal energy. Find

(a) The molar heat capacity of the gas in this process

(b) The equation of the process in the variables T,V;

(c) The work performed by one mole of the gas when its volume increases times if the initial temperature of the gas is T_0.

Solution:(a)

$$\Delta U = \frac{vR}{\gamma - 1}\Delta T\ and\ Q = VC_n\Delta T$$

Where C_n is the molar heat capacity in the process. It is given that

$$Q = -\Delta U$$

So,

$$C_n\Delta T = \frac{R}{\gamma - 1}\Delta T\ or\ C_n = \frac{R}{\gamma - 1}$$

(b)

By the first law of thermodynamics,

$$dQ = dU + dA,$$

$$or, 2dQ = dA(as\ dQ = dU)$$

$$2VC_ndT = pdV\ , or\ \frac{2Rv}{\gamma - 1}dT + pdV = 0$$

So,

$$\frac{dT}{T} + \frac{\gamma - 1}{2}\frac{dV}{V} = 0\ , or\ TV^{(\gamma-1)/2} = constant$$

(c)

We know,

$$C_n = \frac{(n-\gamma)R}{(n-1)(\gamma-1)}$$

But from part (a), we have

$$C_n = -\frac{R}{\gamma - 1}$$

Thus

$$-\frac{R}{\gamma - 1} = \frac{(n-\gamma)R}{(n-1)(\gamma-1)}$$

Which yields

$$n = \frac{1+\gamma}{2}$$

From part (b) we know

$$TV^{(\gamma-1)/2} = constant$$

So,

$$\frac{T_0}{T} = \left(\frac{V}{V_0}\right)^{(\gamma-1)/2} = \eta^{(\gamma-1)/2}$$

$(Where\ T\ is\ the\ final\ temperature)$

Work done by the gas for one mole is given by

$$A = R\frac{(T_0 - T)}{n-1} = \frac{2RT_0\left[1 - \eta^{(\gamma-1)/2}\right]}{\gamma - 1}$$

14. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure P_i = 10^5 Pa and volume V_i = $10^{-3}m^3$ changes to a final state at P_f = (1/32) ×10^5 Pa and V_f = 8× $10^{-3}m^3$ in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumeV_f. The amount of heat supplied to the system in the two-step process is approximately

(a) 112 J **(b) 294 J**

(c) 588 J **(d) 813 J**

Solution:

(c) ∴ PV^r = constant

$P^3V^5 = constant \rightarrow PV^{5/3}$ = constant ∴ ɣ = 5/3

For the two step process

$$W = P_i\left(V_f - V_i\right) = 10^5(7 \times 10^{-3}) = 7 \times 10^2 J$$

$$\Delta U = \frac{f}{2}\left(P_fV_f - P_iV_i\right) = \frac{1}{ɣ-1}\left(\frac{1}{4} \times 10^2 - 10^2\right)$$

$$\Delta U = \frac{3}{2}.\frac{3}{4} \times 10^2 = -\frac{9}{8}10^2\ \text{J}$$

From first law of threnody mimics

$$Q - W = \Delta U$$

$$\therefore\ Q = 7 \times 10^2 - \frac{9}{8} \times 10^2 = \frac{47}{8} \times 10^2\ \text{J}$$

$$= 588\ \text{J}$$

Beginner Molar Heat Capacities

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1. For an ideal gas, the heat capacity at constant pressure is larger than that at constant volume because

(a) Positive work is done during expansion of the gas by the external pressure

(b) Positive work is done during expansion by the gas against external pressure

(c) Positive work is done during expansion by the gas against intermolecular forces of attraction

(d) More collisions occur per unit time when volume is kept constant.

Solution:

Correct option is B

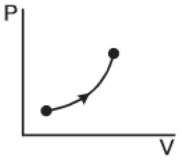
When heat is supplied at constant volume, temperature increases accordingly to the ideal gas equation.

$$P = \frac{nRT}{V}$$

As V is constant and T is increasing, pressure will also increase.

Than at constant pressure as temperature is increase volume increases, resulting in expansion of the gas, resulting in positive work, Hence the heat given is used up for expansion and then to increase the internal energy, the heat capacity at constant pressure is larger.

2. If molar heat capacity of the given process (as shown in figure) is C, then



(a) $C < C_V$ **(b) $C = 0$**

(c) $C > C_V$ **(d) $C = C_V$**

Solution:

Correct option is C.

$\boldsymbol{C} - C_V = R[Mayer's formula]$

So,

$C = R + C_V$

Therefore,$C > C_V$

3. If 32 gm of 02 at 27°C is mixed with 64 gm of 02 at 327°C in an adiabatic vessel, then the final temperature of the mixture will be:

(a) 200°C **(b) 227°C**

(c) 314.5°C **(d) 235.5°C**

Solution: (b)

$$\frac{n_1 fRT_1}{2} + \frac{n_2 fRT_2}{2} = \frac{(n_1 + n_2) fRT_3}{2}$$

$$\frac{1\times 300}{2} + \frac{2\times 600}{2} = \frac{3\times T}{2}$$

$T = 500\text{ K}$

4. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its temperature. The ratio of C_P/C_V for the gas is equal to:

(a) $\frac{4}{3}$ **(b) 2**

(c) $\frac{5}{3}$ **(d)** $\frac{3}{2}$

Solution:

As per the question;

$\Rightarrow P \propto (PV)^3$

$\Rightarrow P^2V^3 = \text{costant}$

$\Rightarrow PV^{3/2} = \text{costant}$

$\Rightarrow \gamma = \frac{3}{2} \Rightarrow \frac{C_P}{C_V} = \frac{3}{2}$

5. The molar heat capacity in case of a diatomic gas if it does a work of $\frac{Q}{4}$ when heat Q is supplied to it is:

(a) $\frac{2}{5}R$ **(b)** $\frac{5}{2}R$

(c) $\frac{10}{3}R$ **(d)** $\frac{6}{7}R$

Solution:

$dU = C_V dT = \left(\frac{5}{2}R\right)dT$ or $dT = \frac{2(dU)}{5R}$ …….. (i)

From the first law of thermodynamics,

$dU = dQ = dW$

$= Q - \frac{Q}{4} = \frac{3Q}{4}$

Now molar heat capacity,

$$C = \frac{dQ}{dT} = \frac{Q}{\frac{2(dU)}{5R}} = \frac{5RQ}{2\left(\frac{3Q}{4}\right)} = \frac{10}{3}R$$

6. P-V diagram of a diatomic gas is a straight line passing through origin. The molar heat capacity of the gas in the process will be -

(a) 4R **(b) 2.5 R**

(c) 3 R **(d)** $\frac{4R}{3}$

Solution: (c)

P-V diagram of the gas is a straight line passing through origin.

Hence, $P \propto V$ or $PV^{-1} = \text{constant}$

Molar heat capacity in the process PV^z = constant is

$C = \frac{R}{\gamma - 1} + \frac{R}{1 - x}$; Here $\gamma = 1.4$ (For diatomic gas)

$\Rightarrow C = \frac{R}{1.4-1} + \frac{R}{1+1} \Rightarrow C = 3R$

7. The molar specific heat at a constant pressure of an ideal gas is $\frac{7}{2}R$. The ratio of specific heat at constant pressure to that at constant volume is:

(a) $\frac{7}{5}$ **(b)** $\frac{8}{7}$

(c) $\frac{5}{7}$ **(d)** $\frac{9}{7}$

Solution:

Hint: Use Mayer's formula. Step 1: Find the value of C_v.

We have given molar specific heat at constant pressure, $C_p = \frac{7R}{2}$

Mayor's relation can be written as:

$C_p - C_v = R$

$\Rightarrow C_v = C_p - R$

$= \frac{7R}{2} - R = \frac{5R}{2}$

Step 2: Find the ratio of C_p and C_v.

Hence, the required ratio, $\gamma = \frac{C_p}{Cv} = \frac{7}{5}$

8. Equal moles of monoatomic and diatomic gases are mixed, γ for mixture is $\left(\gamma = \frac{C_p}{C_v}\right)$

(a) 1.5 **(b) 1**

(c) 2 **(d) 1.67**

Solution:

$$C_v = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{(n_1 + n_2)}$$

$$C_p = \frac{n_1 C_{P_1} + n_2 C_{P_2}}{(n_1 + n_2)}$$

$$\gamma = \frac{C_p}{C_v} = 1.5$$

9. An ideal gas expands according to the law $PV^2 =$ const. The molar heat capacity C is

(a) $C_V + R$ **(b)** $C_V - R$

(c) $C_V + 2R$ **(d)** $C_V - 3R$

Solution: $PV^n = \text{constt.}$

$\gamma = \frac{C_p}{C_v}$

$C_P - C_v = R$

$\gamma C_v - C_v = R$

$C_v = \frac{R}{\gamma - 1}$

$C = \frac{R}{\gamma - 1} + \frac{R}{1-n}$

$C = C_v + \frac{R}{1-2} \quad \{\because PV^2 = \text{constt.}\}$

$C = C_v - R$

10. The pressure of a monoatomic gas increases linearly from $4 \times 10^5 \text{ N/m}^2$ to $8 \times 10^5 \text{ N/m}^2$ when its volume increases from 0.2 m^3 to 0.5 m^3. The molar heat capacity of the gas is: [R = 8.31 J/mol k]

(a) 20.1 J/mol K **(b) 17.14 J/mol K**

(c) 18.14 J/mol K **(d) 20.14 J/mol K**

Solution:

Molar heat capacity is defined as:

$$C = \frac{\Delta Q}{n\Delta T} = \frac{\Delta Q \times R}{P_F V_F - P_I V_I}$$

$$= \frac{6.6 \times 10^5 \times 8.31}{10^5 (8 \times 0.5 - 4 \times 0.2)}$$

i.e. $C = \frac{54.846}{3.2} = 17.14 \text{ J/mol K}$

11. A reversible adiabatic path on a P-V diagram for an ideal gas passes through state A where P=0.7×10^5 N/ m^2 and v = 0.0049 m3. The ratio of specific heat of the gas is 1.4. The slope of path at A is :

(a) 2.0 × 107 Nm–5 **(b) 1.0 × 107 Nm–5**

(c) – 2.0 × 107 Nm–5 **(d) –1.0 × 107 Nm–5**

Solution:

Correct option is C

For reversible adiabat,

$$Pv^{\gamma} = constant \Rightarrow vdP + P\gamma dv = 0 \Rightarrow \frac{dP}{dv} = -\frac{\gamma P}{v}$$

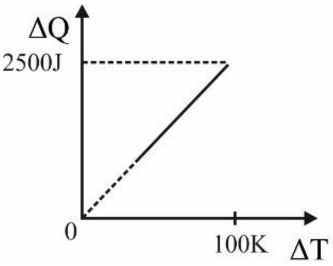
For

$P = 0.7 \times 10^5 Nm^{-2}, v = 0.0049 m^3, \gamma = 1.4$

$$required\ slope = -\frac{1.4 \times 0.7 \times 10^5 Nm^{-2}}{0.0049 m^3}$$

$= -2 \times 10^7 Nm^{-5}$

12. One mole of a gas mixture is heated under constant pressure, and heat required ΔQ is plotted against temperature difference acquired. Find the value of γ for mixture.



Solution:

$$Q = \Delta U + W = n\frac{f}{2}R\Delta T + nR\Delta T$$

$$Q = \left(1+\frac{f}{2}\right)Rn\Delta T \Rightarrow \frac{2500}{100} = \left(\frac{1+f}{2}\right)R \Rightarrow f = 4$$

$$\gamma = 1 + \frac{2}{f} = \frac{3}{2}$$

13. Ideal diatomic gas is taken through a process ΔQ = 2ΔU. Find the molar heat capacity for the process (where ΔQ is the heat supplied and ΔU is change in internal energy)

Solution:

Let C be the molar heat capacity of this gas.

$\therefore \Delta Q = nC\Delta T$

And we know

$\Delta U = nC_V\Delta T$

$\therefore \Delta Q = 2\Delta U$

$\therefore nC\Delta T = 2nC_V\Delta T$

$\Rightarrow C = 2C_V = 2\left(\frac{5}{2}R\right) = 5R$

Expert **Molar Heat Capacities**

1. One mole of a monoatomic ideal gas is mixed with one mole of a diatomic ideal gas. The molar specific heat of the mixture at constant volume is:

(a) 8 **(b)** $\frac{3R}{2}$

(c) 2R **(d) 2.5 R**

Solution:

Hint: The molar specific heat of the mixture at constant volume is given as $C_{v(\text{mixture})} = \frac{n_1C_1 + n_2C_2}{n_1 + n_2}$

Step 1: Use the above relation.

$$C_{v(\text{mixture})} = \frac{n_1C_1 + n_2C_2}{n_1 + n_2}$$

Step 2: Calculate C_v of individual gases

For monoatomic gas,

$$\gamma = \frac{5}{3}$$

$$C_v = \frac{R}{\gamma - 1}$$

$$C_{v_1} = \frac{3R}{2}$$

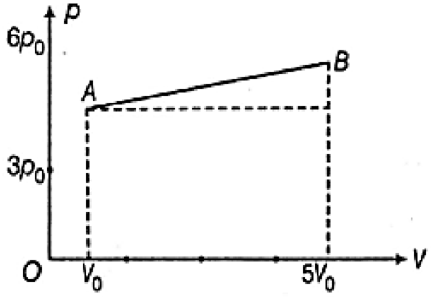
$$C_{v_2} = \frac{5R}{2}$$

Step 3 : Calculate C_v, mix.

Hence

$$C_v,\text{mix} = \frac{\frac{3R}{2} + \frac{5R}{2}}{2} = 2R$$

2. One mole of a monoatomic ideal gas undergoes the process $A \to B$ **in the given** $_{p-}$V **diagram. The molar heat capacity for this process is**



(a) $\frac{3R}{2}$ **(b)** $\frac{13R}{6}$

(c) $\frac{5R}{2}$ **(d) 2R**

Solution: $W_{A\to B}$ = Area under p – v diagram

$$= 4V_0 \times 3p_0 + \frac{1}{2}4v_0 \times 3p_0 = 18P_0V_0$$

$$\therefore \Delta U = nC_v\Delta T$$

$$= 1.\frac{3}{2}R\left(T_B - T_A\right)$$

$$= \frac{3}{2}R\left(\frac{30P_0V_0}{R} - \frac{3P_0V_0}{R}\right) = \frac{81}{2}P_0V_0$$

Thus, $\Delta Q_{A\to B} = \Delta W_{A\to B} + \Delta U_{A\to B}$

$$= 18P_0V_0 = \frac{117}{2}P_0V_0$$

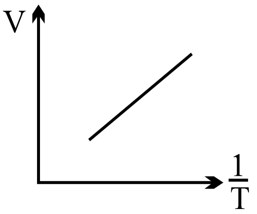
Molar heat capacity, $C = \frac{\Delta Q}{n\Delta t}$

$$= \frac{117P_0V_0/2}{\frac{30P_0V_0}{R} - \frac{3P_0V_0}{R}}$$

$$= \frac{117P_0V_0/2}{\frac{27P_0V_0}{R}} = \frac{13R}{6}$$

3. One mole of an ideal monoatomic gas

undergoes a process as shown in the figure. Find the molar specific heat of the gas in the process.



Solution:

From the graph shown,

$TV = K = constant --- (1)$

For a polytropic process,

$PV^m = constant$

$\frac{nRT}{V}V^m = constant$

$TV^{m-1} = constant \; ----(2)$

On computing (1) and (2)

$m - 1 = 1$

$m = 2$

For monoatomic gas,

$\gamma = \frac{5}{3}$

Molar specific heat capacity of the gas in polytropic process is given as

$$C = \frac{R}{\gamma - 1} + \frac{R}{1 - m}$$

$$\therefore C = \frac{R}{\frac{5}{3} - 1} + \frac{R}{1 - 2} = \frac{R}{2}$$

Pro **Molar Heat Capacities**

1. An ideal monoatomic gas undergoes a process where its pressure is inversely proportional to its temperature.

(a) Calculate the specific heat for the process.

(b) Find the work done by two moles of gas if the temperature changes from T_1 to T_2.

Solution:

(a)

$p \propto \frac{1}{T}$ $or \; pT = constant$

$\therefore p(pV) = constant$

$or \; pV^{1/2} = constant$

In the process, $pV^x = constant$

Molar heat capacity is

$$C = C_V + \frac{R}{1 - x}$$

$$or \; C = \frac{3}{2}R + \frac{R}{1 - \frac{1}{2}}$$

$$= \frac{3}{2}R + 2R = \frac{7}{2}R$$

(b)

$W = Q - \Delta U = nC\Delta T - nC_V\Delta T$

$= n(C - C_V)\Delta T$

$= 2\left[\frac{7}{2}R - \frac{3}{2}R\right](T_2 - T_1)$

$= 4R(T_2 - T_1)$

2. A mixture of 4 gm helium and 28 gm of nitrogen in enclosed in a vessel of constant volume 300°K. Find the quantity of heat absorbed by the mixture to double the root mean velocity of its molecules. (R = Universal gas constant)

Solution:

Given,

$T = 300K$

$V = 2V$

Root mean square velocity

$$V = \sqrt{\frac{3RT}{M}}$$

$$\frac{V'}{V} = \sqrt{\frac{T'}{T}}$$

$$\therefore \frac{2V}{V} = \sqrt{\frac{T'}{300}}$$

$T' = 1200K$

Number of moles He,

$n_1 = \frac{4}{4} = 1$

Number of moles N_2,

$n_2 = \frac{28}{28} = 1$

Total number of moles

$n = 1 + 1 = 2$

For Helium,

$C_V = \frac{3}{2}R$

For Nitrogen,

$C_V = \frac{5}{2}R$

Specific heat capacity of the mixture

$$C_V = \frac{n_1 \times C_{V1} + n_2 \times C_{V2}}{n_1 + n_2}$$

$C_V = 2R$

Net heat absorbed at constant volume

$Q_V = nC_V(T' - T)$

$\therefore Q_V = 2 \times 2R(1200 - 300) = 3600R$

3. Calculate the value of γ = Cp/Cv for a gaseous mixture consisting of v_1 = 2.0 moles of oxygen and v_2 =3.0 moles of carbon dioxide. The gases are assumed to be ideal.

Solution:

Method 1:

$$U_{mix} = U_1 + U_2$$

$$(n_1 + n_2)\frac{RT}{r-1}$$

$$= \frac{n_1RT}{r_1-1} + \frac{n_2RT}{r_2-1}$$

$$\Rightarrow r = \frac{n_1r_1(r_2-1)+n_2r_2(r_1-1)}{n_1(r_2-1)+n_2(r_1-1)}$$

Method 2:

$$C_v = \frac{n_1Cv_1 + n_2Cv_2}{n}$$

$$= \frac{\frac{n_1R}{r_1-1}+\frac{n_2R}{r_2-1}}{n}$$

$$C_p = \frac{n_1Cp_1 + n_2Cp_2}{n}$$

$$= \frac{\frac{n_1r_1R}{r_1-1}+\frac{n_2r_2R}{r_2-1}}{n}$$

$$\therefore r = \frac{C_p}{C_v} = \frac{\frac{n_1r_1}{r_1-1}+\frac{n_2r_2}{r_2-1}}{\frac{n_1}{r_1-1}+\frac{n_2}{r_2-1}}$$

$$= \frac{n_1r_1(r_{2-1})+n_2r_2(r_{1-1})}{n_1(r_{2-1})+n_2(r_{1-1})}$$

4. Find the specific heat capacities c_v and c_p for a gaseous mixture consisting of 7.0 g of nitrogen and 20 g of argon. The gases are assumed to be ideal.

Solution:

$$C_v = \frac{\frac{n_1R}{r_1-1}+\frac{n_2R}{r_2-1}}{n_1+n_2}$$

$$= 15.2J / mol - k$$

$$C_p = \frac{\frac{n_1r_1R}{r_1-1}+\frac{n_2r_2R}{r_2-1}}{n_1+n_2}$$

$$= 23.8J / mol - k$$

Specific heat capacities:

$$C_V^{'} = \frac{C_v}{M_{mix}} = 0.42\ J/g-k$$

$$C_p^{'} = \frac{C_p}{M_{mix}} = 0.66\ J/g-k$$

$$M_{mix} = \frac{m_1+m_2}{n_1+n_2}$$

$$= \frac{7+20}{\frac{7}{28}+\frac{20}{40}} = \frac{27}{\frac{1}{4}+\frac{1}{2}} = 36$$

5. Find the molar heat capacity of an ideal gas in a polytropic process pV^n = const if the adiabatic exponent of the gas is equal to γ . At what values of the polytropic constant n will the heat capacity of the gas be negative?

Solution:

A. For a polytropic process of, pV^n = const

$$w = \frac{n_0R(T_1-T_2)}{n-1}$$

$$also,\ \Delta U = \frac{n_0R(T_2-T_1)}{r-1}$$

$$\therefore Q = \Delta U + w$$

$$\Rightarrow n_0C(T_2-T_1)$$

$$= \frac{n_0R(T_2-T_1)}{r-1} + \frac{-n_0R(T_2-T_1)}{n-1}$$

$$\Rightarrow C = R\left[\frac{1}{r-1}-\frac{1}{n-1}\right]$$

$$= \frac{R(n-r)}{(r-1)(n-1)} \quad ....(i)$$

B. From 1:

$$C < O,\ if\ \frac{n-r}{n-1} < 0$$

$$\Rightarrow 1 < n < \gamma$$

6. In a certain polytropic process the volume of argon was increased α = 4.0 times. Simultaneously, the pressure decreased β = 8.0 times. Find the molar heat capacity of argon in this process, assuming the gas to be ideal.

Solution:

$$v \to \alpha v$$

$$p \to \frac{p}{\beta}$$

Let the process be:

Pv^n = constant

$$\Rightarrow pv^n = \left(\frac{p}{\beta}\right)(\alpha v)^n$$

$$\Rightarrow n = \frac{\ell n\ \beta}{\ell n\ \alpha}$$

We derived ,

$$C = R\left[\frac{1}{r-1} - \frac{1}{n-1}\right]$$

$$\therefore C = R\left[\frac{1}{r-1} - \frac{\ell n\ \alpha}{\ell n\ \beta - \ell n\ \alpha}\right]$$

7. An ideal gas has a molar heat capacity C_v at constant volume. Find the molar heat capacity of this gas as a function of its volume V, if the gas undergoes the following process:

(a) $T = T_0 e^{\alpha V}$

(b) $p = p_0 e^{\alpha V}$

where T_0, p_0, and a are constants,

Solution:

$$dQ = dU + dw$$

$$\Rightarrow n_0 Cdt = n_0 C_v dT + pdV$$

$$\Rightarrow C = C_v + \frac{pdV}{n_0 dT}$$

$$A.\ T = T_0 e^{\alpha V}$$

$$\frac{dT}{dv} = T_0 \alpha e^{\alpha V}$$

$$\therefore C = C_v + \left(\frac{n_0 RT}{v}\right)\frac{dV}{n_0 dT}$$

$$= C_v + \frac{RT_0 e^{\alpha v}}{V\ T_0 \alpha e^{\alpha v}} = C_v + \frac{R}{\alpha V}$$

8. One mole of an ideal gas whose adiabatic exponent equals γ undergoes a process p = p_0 + α /V, where p_0 and α are positive constants. Find:

(a) Heat capacity of the gas as a function of its volume;

(b) The internal energy increment of the gas, the work performed by it, and the amount of heat transferred to the gas, if its volume increased from V_1 to V_2.

Solution:

$$dQ = dU + dw$$

$$\Rightarrow n_0 CdT = \frac{n_0 RdT}{r-1} + pdV$$

$$\Rightarrow C = \frac{R}{r-1} + \frac{pdV}{n_0 dT}$$

$$A.\ p = p_0 + \frac{\alpha}{v}$$

$$\Rightarrow n_0 RT = \left(p_0 + \frac{\alpha}{v}\right)v$$

$$\Rightarrow n_0 RdT = p_0 dV$$

$$\therefore C = \frac{R}{r-1} + \left(p_0 \frac{\alpha}{v}\right)\frac{R}{p_0}$$

$$= \frac{R}{r-1} + R\left(1 + \frac{\alpha}{p_0 v}\right)$$

Hence, $C = \dfrac{R}{\gamma - 1} + R\left(1 + \dfrac{\alpha}{p_0 V}\right)$

Finding, ΔT

$$p = p_0 + \frac{\alpha}{v}$$

$$n_0 RT = p_0 V + \alpha$$

$$n_0 R\Delta T = p_0 \Delta v$$

$$\Delta T = \frac{p_0 (v_2 - v_1)}{n_0 R}$$

$$B.\ \Delta U = n_0 C_v \Delta T = \frac{p_0 (v_2 - v_1)}{r-1}$$

$$w = \int pdv = \int_{v_1}^{v_2} \left(p_0 + \frac{\alpha}{v}\right)dv$$

$$= p_0 (v_2 - v_1) + \alpha \ell n \frac{v_2}{v_1}$$

$$Q = \Delta U + w$$

9. One mole of an ideal gas with heat capacity at constant pressure C_P undergoes the process T = T_0 + α V, where T_0 and a are constants. Find:

(a) Heat capacity of the gas as a function of its volume;

(b) The amount of heat transferred to the gas, if its volume increased from V_1 to V_2.

Solution:

$$A.\ n_0CdT = n_0C_vdT + pdv$$

$$C = Cp - R + \frac{pdv}{n_0dT}$$

$$\Rightarrow C = Cp - R + \frac{n_0R\left(T_0 + \alpha v\right)}{vn_0\alpha}$$

$$= Cp - R + R\left(\frac{T_0}{\alpha v} + 1\right)$$

$$= Cp + \frac{RT_0}{\alpha v}$$

$$B.\ dQ = n_0C\ dT$$

$$dQ = n_0\left(Cp\right) + \frac{RT_0}{\alpha v}$$

$$Q = n_0\alpha\int_{v_1}^{v_2}\left(Cp + \frac{RT_0}{\alpha v}\right)dv$$

$$= n_0Cp\alpha\left(v_2 - v_1\right) + n_0RT_0\ell n\frac{v_2}{v_1}$$

10. For the case of an ideal gas find the equation of the process (in the variables T, V) in which the molar heat capacity varies as:

(a) $C = C_v + \alpha T$ **;**

(b) $C = C_v + \beta T$ **;**

(c) $C = C_v + ap$

Where α, β **and a are constants.**

Solution:

$$n_0CdT = n_0CvdT + pdV$$

$$\Rightarrow C = C_v + \frac{pdV}{n_0dT}$$

$$A.\ \frac{pdv}{n_0dT} = \alpha T$$

$$\Rightarrow\left(\frac{n_0RT}{v}\right)\frac{dv}{n_0dT} = \alpha T$$

$$R\int\frac{dv}{v} = \alpha\int dT$$

$$\Rightarrow n\,RV = \alpha T + k\ \left(\text{constant}\right)$$

$$B.\ \frac{pdv}{n_0dT} = \beta v$$

$$\Rightarrow\left(\frac{n_0RT}{v}\right)\frac{dv}{n_0dT} = \beta v$$

$$\Rightarrow R\int\frac{dv}{v^2} = \beta\int\frac{dT}{T}$$

$$\Rightarrow -\frac{R}{\beta V} = \ell nT + k$$

$$C.\ \frac{pdv}{n_0dT} = ap$$

$$\int dv = an_0\int dT$$

$$V = an_0T + k$$

11. The pressure of an ideal gas changes with volumes as P = aV where 'a' is a constant. One moles of this gas is expanded to 3 times its original volume V₀. Find

(a)The heat transferred in the process

(b)The heat capacity of the gas.

Solution:

$$\Delta W = \int_{V_i}^{V_f} PdV = \int_{V_i}^{V_f} aVdV$$

$$= \left[\frac{aV^2}{2}\right]_{V_i}^{V_f} = \frac{aV_f^{\ 2} - aV_i^{\ 2}}{2}$$

$$= \frac{\left(\frac{P_f}{V_f}\right)V_f^{\ 2} - \left(\frac{P_i}{V_i}\right)i^2}{2}$$

$$\left(a = \frac{P_f}{V_f} = \frac{P_i}{V_i}\right)$$

$$\Delta W = \frac{P_fV_f - P_iV_i}{2} = \frac{nR\Delta T}{2}$$

$$\Delta U = nC_V\Delta T = \frac{nR}{\gamma - 1}\Delta T$$

$$\Delta Q = \Delta W + \Delta U$$

$$= \frac{nR\Delta T}{2} + \frac{nR}{\gamma - 1}\Delta T = nR\Delta T\left[\frac{1}{2} + \frac{1}{\gamma - 1}\right]$$

$$= \frac{nR\Delta T}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right)\quad ----(1)$$

We know P = Av

$$\frac{nRT}{V} = aV \Rightarrow T = \frac{aV^2}{nR}$$

$$\therefore T_i = \frac{aV_0^{\ 2}}{R}\quad and\ T_f = \frac{9aV_0^{\ 2}}{R}$$

$$\therefore \Delta T = \frac{8aV_0^{\ 2}}{R} ----(2)$$

Substituting (2) in (1)

Thus

$$\Delta Q = \frac{R}{2}\times\frac{8aV_0^{\ 2}}{R}\left(\frac{\gamma + 1}{\gamma - 1}\right) = \left(\frac{\gamma + 1}{\gamma - 1}\right)4aV_0^{\ 2}$$

(b) C →heat capacity of gas

So,

$$\Delta Q = nC\Delta T$$

Using (i) we get

$$nC\Delta T = \frac{nR\Delta T}{2}\left(\frac{\gamma+1}{\gamma-1}\right)$$

$$C = \left(\frac{\gamma+1}{\gamma-1}\right)\frac{R}{2}$$

Beginner **2nd Law – Heat Engine**

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1. A cannot engine whose low temperature reservoir is at 7°C has an efficiency of 50%. It is desired to increase its efficiency to 70%. By how many degrees should the temperature of high temp. reservoir be increased?

(a) 840 k **(b) 560 k**

(c) 280 k **(d) 380 k**

Solution:

The answer is D.

Initially

$$\eta = \frac{T_1 - T_2}{T_1} \Rightarrow 0.5 = \frac{T_1 - (273+7)}{T_1}$$

$$\Rightarrow \frac{1}{2} = \frac{T_1 - 280}{T_1} = T_1 = 560K$$

Finally

$$\eta' = \frac{T_1{}' - T_2}{T_1{}'} \Rightarrow 0.7 = \frac{T_1{}' - (273+7)}{T_1{}'}$$

$$\Rightarrow T_1{}' = 933K$$

Increase in temperature

$$= 933 - 560 = 373K = 380K$$

2. An ideal gas heat engine operates in a Carnot's cycle between 227°C and 127°C . It absorbs 6 × 10^4 J at high temperature. The amount of heat converted into work is.

(a) 4.8 × 10^4 J **(b) 3.5 × 10^4 J**

(c) 1.6 × 10^4 J **(d) 1.2 × 10^4 J**

Solution:

The answer is D.

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{500} = \frac{1}{5} \therefore \eta = \frac{W}{Q}$$

$$\Rightarrow \frac{1}{5} = \frac{W}{Q}$$

$$\Rightarrow W = \frac{Q}{5} = \frac{6}{5} \times 10^4 = 1.2 \times 10^4 J$$

3. An ideal heat engine exhausting heat at 77°C is to have a 30% efficiency. It must take heat at

(a) 127°C **(b) 227°C**

(c) 327°C **(d) 673°C**

Solution:

The answer is B.

We know,

$$\eta = 1 - \frac{T_1}{T_2}$$

$$\Rightarrow \frac{30}{100} = 1 - \frac{77+273}{T_2}$$

$$\Rightarrow \frac{350}{T_2} = 1 - \frac{3}{10}$$

$$\Rightarrow T_2 = \frac{350 \times 10}{7} = 500K$$

Or, it must take heat at $500 - 273 = 227°C$

4. Efficiency of Carnot engine is 100% if

(a) T2 = 273 K **(b) T2 = 0 k**

(c) T$_1$ = 273 K **(d) T$_1$ = 0 K**

Solution:

The correct answer is B.

$$\eta = 1 - \frac{T_2}{T_1} \; for\ 100\%\ efficiency$$

$$\eta = 1 \; which\ gives\ T_2 = 0K$$

5. A Carnot's engine used first an ideal monoatomic gas then an ideal diatomic gas. If the source and sink temperature are 411°C and 69°C respectively and the engine extracts 1000 J of heat in each cycle, then area enclosed by the PV diagram is

(a) 100 J **(b) 300 J**

(c) 500 J **(d) 700 J**

Solution:

The correct answer is C.

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \left(\frac{273+69}{273+411}\right) = 0.5$$

$$work\ done = \eta \times Q = 0.5 \times 1000 = 500J$$

6. A Carnot engine absorbs an amount Q of heat from a reservoir at an absolute temperature T and rejects heat to a sink at a temperature of T/3. The amount of heat rejected is

(a) Q / 4 **(b) Q / 3**

(c) Q / 2 **(d) 2Q / 3**

Solution:

The correct answer is B.

$$\therefore \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1}$$

Where Q$_1$ = heat absorbed , Q$_2$ = heat rejected

$$\Rightarrow 1 - \frac{T/3}{T} = \frac{W}{Q_1}$$

$$\Rightarrow \frac{2}{3} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$\Rightarrow \frac{2}{3} = 1 - \frac{Q_2}{Q_1} \Rightarrow \frac{Q_2}{Q_1} = \frac{1}{3} \Rightarrow Q_2 = \frac{Q_1}{3} = \frac{Q}{3}$$

7. The temperature of sink of Carnot engine is 27°C . Efficiency of engine is 25%. Then temperature of source is

(a) 227°C **(b) 327°C**
(c) 127°C **(d) 27°C**

Solution:

The correct answer is C.

$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{25}{100} = 1 - \frac{300}{T_1} \Rightarrow \frac{1}{4} - \frac{300}{T_1}$$

$$T_1 = 400K = 127°\mathrm{C}$$

8. The temperature of reservoir of Carnot's engine operating with an efficiency of 70% is 1000K. The temperature of its sink is

(a) 300 K **(b) 400 K**
(c) 500 K **(d) 700 K**

Solution:

The correct answer is A.

$$\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{70}{100} = 1 - \frac{T_2}{1000} \Rightarrow T_2 = 300K$$

9. A thermodynamic cycle takes in heat energy at a high temperature and rejects energy at a lower temperature. If the amount of energy rejected at the low temperature is 3 times the amount of work done by the cycle, the efficiency of the cycle is

(a) 0.25 **(b) 0.33**
(c) 0.67 **(d) 0.9**

Solution:

Correct option is A.

Let heat taken be Q₁, heat rejected be Q₂, and work done be W

Then

$Q_3 = 3W$

Also,

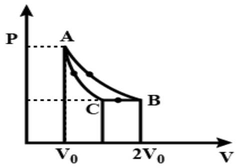
$Q_1 = Q_2 + W$

$Q_1 = 4W$

$$Efficiency\ of\ cycle$$
$$= work\ done/heat\ taken$$

$$\frac{W}{4W} = \frac{1}{4} = 0.25$$

10. In a cycle ABCA consisting of isothermal expansion AB, isobaric compression BC and adiabatic compression CA, find the efficiency of cycle (Given: TA = TB = 400 K, γ = 1.5)



Solution:

$V_A = V_0, T_A = 400K$

$$\therefore P_A = \frac{400nR}{V_0}$$

At B

$$V_B = 2V_0\,, T_B = 400K, P_B = \frac{400nR}{2V_0}$$

At C

$$V_C = ?\,, T_C = ?\,, P_C = \frac{400nR}{2V_0}$$

AC is adiabatic process

PV^{γ} = constant

$$V_C = \left(\frac{P_A}{P_C}\right)^{\frac{1}{\gamma}} V_A = (2)^{\frac{1}{1.5}} V_0 \Rightarrow V_C = 2^{2/3} V_0$$

$$T_C = \frac{P_C V_C}{nR} = 400 \times (2)^{-1/3}$$

From process AB

$\Delta U = 0$

$$\Delta Q = \Delta W = nR \times (400) \times \ln\left(\frac{2V_0}{V_0}\right)$$

$= 400nR \ln 2$

For process BC

$$\Delta U = nC_V \Delta T = \frac{nR}{\gamma - 1} \times \left[400(2)^{-1/3} - 400\right]$$

$$= -2 \times 400nR\left(1 - 2^{-1/3}\right)$$

$$\Delta Q = nC_P \Delta T = -\frac{n\gamma R}{\gamma - 1} \times 400\left[1 - 2^{-1/3}\right]$$

$$= -3 \times 400nR\left[1 - 2^{-1/3}\right]$$

$$\Delta W = -400nR\left[1 - 2^{-1/3}\right]$$

For process AC

$\Delta Q = 0$

$$\Delta U = nC_V \Delta T = 2 \times 400nR\left[1 - 2^{-1/3}\right]$$

$$\Delta W = -\Delta U = -2 \times 400nR\left[1 - 2^{-1/3}\right]$$

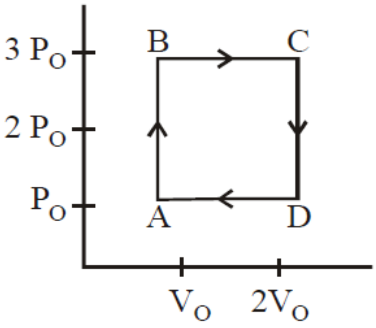
$$Efficiency\ \ (e) = 1 - \frac{Q_{released}}{Q_{absorbed}}$$

$$= 1 - \frac{3 \times 400nR\left[1 - 2^{-1/3}\right]}{400nR \ln 2}$$

$$= 1 - \frac{3\left(1 - 2^{-1/3}\right)}{\ln 2}$$

11. An engine operates by taking a monatomic

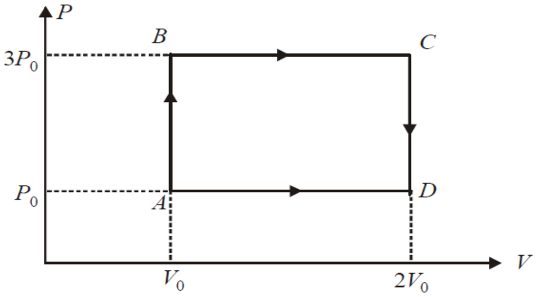
ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to



(a) 19 **(b) 16**
(c) 8 **(d) 4**

Solution:

(a)



From the figure,

Work, $W = 2P_0V_0$

Heat given, $Q_{in} = W_{AB} + W_{BC}$

$$= n \cdot C_V \Delta T_{AB} + nC_P \Delta T_{BC}$$

$$= n\frac{3R}{2}(T_B - T_A) + \frac{n5R}{2}(T_C - T_B)$$

$$\left(\because C_v = \frac{3R}{2} and\, C_P = \frac{5R}{2}\right)$$

$$= \frac{3}{2}(P_BV_B - P_AV_A) + \frac{5}{2}(P_CV_C - P_BV_B)$$

$$= \frac{3}{2}\times[3P_0V_0 - P_0V_0] + \frac{5}{2}[6P_0V_0 - 3P_0V_0]$$

$$= 3P_0V_0 + \frac{15}{2}P_0V_0 = \frac{21}{2}P_0V_0$$

Efficiency, $\eta = \frac{W}{Q_{in}} = \frac{2P_0V_0}{\frac{21}{2}P_0V_0} = \frac{4}{21}$

$$\eta\% = \frac{400}{21} \approx 19.$$

12. A heat engine is involved with exchange of heat of 1915 J, – 40 J, +125 J and – Q J, during one cycle achieving an efficiency of 50.0%. The value of Q is:

(a) 640J **(b) 40J**
(c) 980J **(d) 400J**

Solution:

(c) Efficiency, $\eta = \frac{Work\, done}{Heat\, absorbed} = \frac{W}{\sum Q}$

$$= \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3} = 0.5$$

Here, $Q_1 = 1915$ J, $Q_2 = -40$ J and $Q_3 = 125$ J

$$\therefore \frac{1915 - 40 + 125 + Q_4}{1915 + 125} = 0.5$$

$1915 - 40 + 125 + Q_4 = 1020$

$Q_4 = 1020 - 2000$

$Q_4 = -Q = -980$ J

$Q = 980$ J

13. A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is:

(a) 99J **(b) 100J**
(c) 1J **(d) 90J**

Solution:

(d) For carnot refrigerator

Efficiency = $\frac{Q_1 - Q_2}{Q_1}$

Where,

Q_1 = heat lost from surrounding

Q_2 = heat absorbed from reservoir at low temperature.

Also, $\frac{Q_1 - Q_2}{Q_1} = \frac{w}{Q_1}$

$$\frac{1}{10} = \frac{w}{Q_1}$$

$Q_1 = w \times 10 = 100$ J

So, $Q_1 - Q_2 = w$

$Q_2 = Q_1 - w$

$100 - 10 = Q_2 = 90$ J

14. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the

engine to the low temperature reservoir, in a cycle, is _________________.

Solution:

Given;

$T_1 = 900$ K, $T_2 = 300$K, W = 1200 J

Using, $1 - \frac{T_2}{T_1} = \frac{W}{Q_1}$

$$1 - \frac{300}{900} = \frac{1200}{Q_1}$$

$$\frac{2}{3} = \frac{1200}{Q_1}$$

$Q_1 = 1800$

Therefore heat energy delivered by the engine to the low temperature reservoir, $Q_2 = Q_1 - W = 1800 - 1200 = 600$ J

15. The efficiency of a Carnot engine depends upon

(a) The temperature of the sink only

(b) The temperatures of the source and sink

(c) The volume of the cylinder of the engine

(d) The temperature of the source only

Solution:

(b) Efficiency of Carnot engine is given as

$$\eta = 1 - \frac{T_2}{T_1}$$

where, T_2 = temperature of sink

and T_1 = temperature of source.

Hence, η depends upon the temperature of source and sink both.

Expert 2nd Law – Heat Engine

1. The efficiency of an ideal heat engine working between the freezing point and boiling point of water, is

(a) 6.25% (b) 20%

(c) 26.8% (d) 12.5%

Solution:

(c) Efficiency of an ideal heat engine is given as

$$\eta = 1 - \frac{T_2}{T_1}$$

Where, T_1 is the temperature of the source and T_2 is the temperature of the sink.

Here, $T_2 = 100 + 273 = 373$K

$T_2 = 0 + 273 = 273$ K

$$\eta = 1 - \frac{273}{373}$$

$$= \frac{373 - 273}{373} = \frac{100}{373} = 0.268$$

∴ η% = 0.268 × 100 = 26.8%

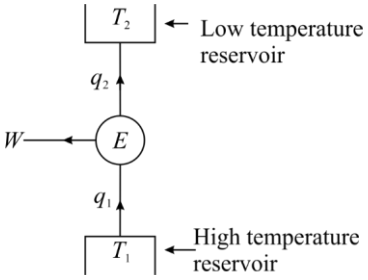
2. A Carnot engine having an efficiency of $\frac{1}{10}$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

(a) 1 J (b) 90 J

(c) 99 J (d) 100 J

Solution:

Consider schematic diagram for a Carnot engine as shown below.



In case of engine,

$$\text{Engine efficiency} = \frac{work}{heat\,absorbed} = \frac{W}{q_1}$$

$$\therefore \frac{W}{q_1} = \frac{1}{10}$$

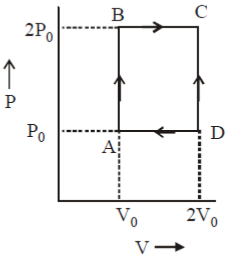
$$\frac{10J}{q_1} = \frac{1}{10} \text{ or } q_1 = 100 \text{ J}$$

When this engine is reversed, it takes in work W and heat q_2 from cold reservoir and ejects 100 J of heat to hot reservoir.

$\therefore W + q_2 = q_1$

$10 + q_2 = 100$ or $q_2 = 90$ J

3. An engine operates by taking n moles of an ideal gas through the cycle ABCDA shown in figure. The thermal efficiency of the engine is : (Take C_v =1.5 R, where R is gas constant)



(a) 0.24 (b) 0.15

(c) 0.32 **(d) 0.08**

Solution:

(b) Work-done (W) = P_0V_0

According to principle of calorimetry

Heat given = $Q_{AB} = Q_{BC}$

$= nC_VdT_{AB} + nC_PdT_{BC}$

$$=\frac{3}{2}\left(nRT_B - nRT_A\right)+\frac{5}{2}\left(nRT_C - nRT_B\right)$$

$$=\frac{3}{2}\left(2P_0V_0 - P_0V_0\right)+\frac{5}{2}\left(4P_0V_0 - 2P_0V\right)=\frac{13}{2}P_0V_0$$

Thermal efficiency of engine

$$(\eta)=\frac{W}{Q_{given}}=\frac{2}{13}=0.15$$

4. A Carnot engine absorbs 1000 J of heat energy from a reservoir at 127°C and rejects 600 J of heat energy during each cycle. The efficiency of engine and temperature of sink will be:

(a) 20% and – 43°C (b) 40% and – 33°C

(c) 50% and –20°C (d) 70% and – 10°C

Solution:

(b) Given: $Q_1 = 1000$ J

$Q_2 = 600$ J

$T_1 = 127°C = 400$ K

$T_2 = ?$

$\eta = ?$

Efficiency of carnot engine,

$$\eta=\frac{W}{Q_1}\times 100\%$$

Or, $\eta=\frac{Q_2 - Q_1}{Q_1}\times 100\%$

Or, $\eta=\frac{1000-600}{1000}\times 100\%$

$\eta = 40\%$

Now, for carnot cycle $\frac{Q_2}{Q_1}=\frac{T_2}{T_1}$

$$\frac{600}{1000}=\frac{T_2}{400}$$

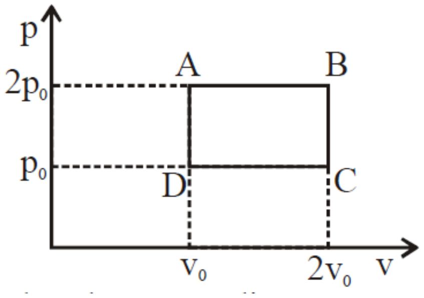
$$T_2=\frac{600\times 400}{1000}$$

= 240 K

= 240 – 273

$\therefore T_2 = -33°C$

5.



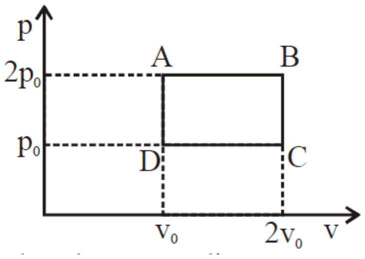
The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monatomic gas. The amount of heat, extracted from the source in a single cycle is

(a) p_0v_0 **(b)** $\left(\frac{13}{2}\right)p_0v_0$

(c) $\left(\frac{11}{2}\right)p_0v_0$ **(d)** $4p_0v_0$

Solution:

Heat is extracted from the source would be in path DA and AB.



Thus heat supplied

$= nC_V(2T_0 - T_0) + nC_P(4T_0 - T_0)$

$$\frac{3nRT_0}{2}+\frac{5nR(2T_0)}{2}$$

$$=\frac{13nRT_0}{2}=\frac{13}{2}P_0V_0$$

6. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work ?

(a) $T=\frac{2T_1T_2}{T_1+T_2}$ **(b)** $T=\frac{T_1+T_2}{2}$

(c) $T=\sqrt{T_1T_2}$ **(d)** $T = 0$

Solution:

(b) Let Q_H = Heat taken by first engine

Q_L = Heat rejected by first engine

Q_2 = Heat rejected by second engine

Work done by 1st engine = work done by 2nd engine

$W = Q_H - Q_L = Q_L - Q_2$

$2Q_L = Q_H + Q_2$

$$2 = \frac{\theta_H}{\theta_L} + \frac{\theta_2}{\theta_L}$$

Let T be the temperature of cold reservoir of first engine.

Then in carnot engine.

$$\frac{Q_H}{Q_L} = \frac{T_1}{T} \text{ and } \frac{Q_L}{Q_2} = \frac{T}{T_2}$$

$$2 = \frac{T_1}{T} + \frac{T_2}{T} \quad \text{using (i)}$$

$2T = T_1 + T_2$

$$T = \frac{T_1 + T_2}{2}$$

7. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are, respectively.

(a) 62°C, 124°C **(b) 99°C, 37°C**

(c) 124°C, 62°C **(d) 37°C, 99°C**

Solution:

(b) Using, $n = 1 - \frac{T_2}{T_1}$

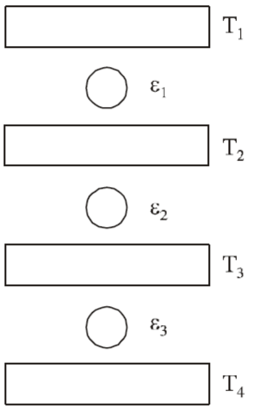
$$n = \frac{1}{6} = 1 - \frac{T_2}{T_1}$$

and $\frac{2}{3} = 1 - \frac{T_2 - 62}{T_1}$

On solving, we get

T_1 = 99°C and T_2 = 37°C

8. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T(4)$ The three engines are equally efficient if :



(a) $T_2 = (T_1T_4)^{1/2}$ **;** $T_3 = (T_1^2T_4)^{1/3}$

(b) $T_2 = (T_1^2T_4)^{1/3}$ **;** $T_3 = (T_1T_4^2)^{1/3}$

(c) $T_2 = (T_1T_4^2)^{1/3}$ **;** $T_3 = (T_1^2T_4)^{1/3}$

(d) $T_2 = (T_1^3T_4)^{1/4}$ **;** $T_3 = (T_1T_4^3)^{1/4}$

Solution:

(b) According to question, $\eta_1 = \eta_2 = \eta_3$

$$\therefore 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

[∵ Three engines are equally efficient]

$$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$T_2 = \sqrt{T_1T_3}$ ... (i)

$T_3 = \sqrt{T_2T_4}$ ... (ii)

From (i) and (ii)

$T_2 = (T_1^2T_4)^{1/3}$ $T_3 = (T_1T_4^2)^{1/3}$

9. Two Carnot engines A and B are operated in series. The first one, A receives heat at T_1 (= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and in turn, rejects to a heat reservoir at T_3 (= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal:

(a) 600K **(b) 400K**

(c) 300K **(d) 500 K**

Solution:

(d) $\eta_A = \frac{T_1 - T_2}{T_1} = \frac{w_A}{Q_1}$

and, $\eta_B = \frac{T_2 - T_3}{T_2} = \frac{w_B}{Q_2}$

According to question,

$W_A = W_B$

$$\therefore \frac{Q_1}{Q_2} = \frac{T_1}{T_2} \times \frac{T_2 - T_3}{T_1 - T_2} = \frac{T_1}{T_2}$$

$$\therefore T_2 = \frac{T_1 + T_3}{2}$$

$$= \frac{600 + 400}{2} = 500 \text{ K}$$

Pro **2nd Law – Heat Engine**

1. A scientist says that the efficiency of his heat engine which operates at source temperature 127°C and sink temperature 27°C is 26%, then

(a) It is impossible

(b) It is possible but less probable

(c) It is quite probable

(d) Data are incomplete

Solution:

Correct answer is A.

$$\eta = 1 - \frac{300}{400} = \frac{100}{400} = 25\%$$

26% efficiency is not possible.

2. Efficiency of a Carnot engine is 50% when temperature of outlet is 500 K. In order to increase efficiency up to 60% keeping temperature of intake the same what is temperature of outlet

(a) 200 K **(b) 400 K**

(c) 600 K **(d) 800 K**

Solution:

Correct answer is B.

$$\frac{50}{100} = 1 - \frac{500}{T}$$

$$\frac{1}{2} = \frac{T - 500}{T}$$

$$T = 2T - 1000$$

$$T = 1000K$$

$$\frac{60}{100} = 1 - \frac{T}{1000}$$

$$\frac{6}{10} = \frac{1000 - T}{1000}$$

$$600 = 1000 - T$$

$$T = 400K$$

3. In which case will the efficiency of a Carnot cycle be higher: when the hot body temperature is increased by ΔT, or when the cold body temperature is decreased by the same magnitude?

Solution:

In carnot cycle is:

$$\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{T_2}{T_1}$$

When hot is made hotter

$$\eta_h = 1 - \frac{T_2}{T_1 + \Delta T}$$

$$= \frac{T_1 - T_2 + \Delta T}{T_1 + \Delta T}$$

$$\therefore \eta_c > \eta_h$$

4. Hydrogen is used in a Carnot cycle as a working substance. Find the efficiency of the cycle, if as a result of an adiabatic expansion

(a) The gas volume increases n = 2.0 times;

(b) The pressure decreases n = 2.0 times.

Solution:

$$\eta = 1 - \frac{T_2}{T_1}$$

(a) $pv^r = const$

$$\Rightarrow TV^{r-1} = const$$

$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{1}{n}\right)^{r-1}$$

$$\therefore \eta = 1 - \frac{1}{n^{\gamma-1}}$$

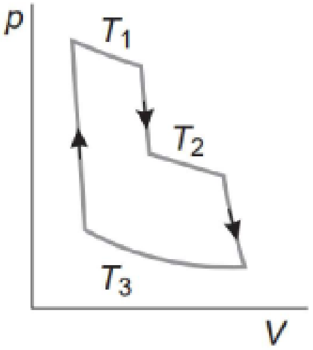
(b) $pv^r = const$

$$\Rightarrow p^{1-r}T^r = const$$

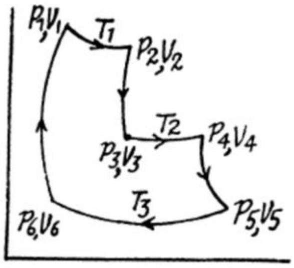
$$\Rightarrow \frac{T_2}{T_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1-r}{r}} = n^{\frac{1-r}{r}}$$

$$\therefore \eta = 1 - n^{\frac{1-\gamma}{\gamma}}$$

5. An ideal gas goes through a cycle consisting of alternate isothermal and adiabatic curves (Fig.). The isothermal processes proceed at the temperatures T_1; T_2, and T_3. Find the efficiency of such a cycle, if in each isothermal expansion the gas volume increases in the same proportion.



Solution:



$$= RT_1 \ln n + R\,T_2 \ln n = R\,(T_1+T_2)\ln n$$

Now $T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$

or $V_3 = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} V_2$

Similarly $V_5 = \left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma-1}} V_4$

, $V_6 = \left(\frac{T_1}{T_3}\right)^{\frac{1}{\gamma-1}} V_1$

Thus Q_2 = heat ejected at the lower temperature

$$= -RT_3 \ln \frac{V_6}{V_5}$$

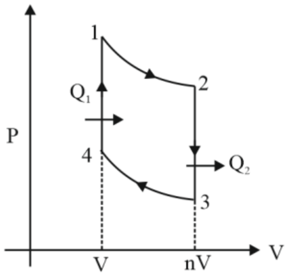
$$= -R\,T_3 \ln \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} \frac{V_1}{V_4} = -RT_3 \ln \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} \frac{V_2}{n^2 V_3}$$

$$= -RT_3 \ln \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} \frac{1}{n^2}\left(\frac{T_1}{T_2}\right)^{-\frac{1}{\gamma-1}} = 2R\,T_3 \ln n$$

$$\eta = 1 - \frac{2T_3}{T_1+T_2}$$

6. Find the efficiency of a cycle consisting of two isochoric and two adiabatic lines, if the volume of the ideal gas changes n = 10 times within the cycle. The working substance is nitrogen.

Solution:



$$nC_V\Delta T = C_V\left(n\Delta T\right)$$

$$= C_v \Delta pV$$

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - |Q_2|}{Q_1}$$

$$= 1 - \frac{|Q_2|}{Q_1}$$

$$= 1 - \frac{C_v\left(p_2 - p_3\right)nV}{C_v\left(p_1 - p_4\right)vV} = 1 - n^{1-\gamma}$$

For the two adiabatic processes:

$\Pr^{r} = const$

$\Rightarrow 1-2: p_1V^r = P_2\left(nV\right)^r \qquad ...(i)$

$3-4: p_4V^r = P_3\left(nV\right)^r \qquad .....(ii)$

Subtracting (i), (ii):

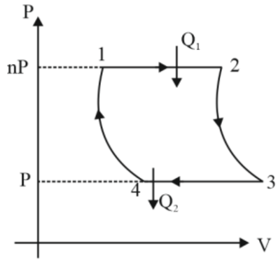
$\left(p_1 - p_4\right)v^r = \left(p_2 - p_3\right)\left(nv\right)^r$

$$\Rightarrow \frac{p_2 - p_3}{p_1 - p_4} = \frac{1}{n^r}$$

7. Find the efficiency of a cycle consisting of two isobaric and two adiabatic lines, if the pressure changes n times within the cycle. The working substance is an ideal gas whose adiabatic exponent is equal to γ.

Solution:

$$\Delta Q = nC_p\Delta T = C_p\left(n\Delta T\right) = C_p p\Delta v$$



$$\eta = \frac{w}{Q_1} = \frac{Q_1 - |Q_2|}{Q_1}$$

$$= 1 - \frac{|Q_2|}{Q_1} = 1 - \frac{C_p p\left(v_3 - v_4\right)}{C_p np\left(v_2 - v_1\right)}$$

$$= 1 - n^{\frac{1-r}{r}}$$

For the two adiabatic processes:

$$\mathrm{P}V^{r}=const$$

$$\Rightarrow 2-3:npV_2^r$$

$$\Rightarrow n^{\frac{1}{r}}v_2=v_3 \qquad ...(i)$$

$$4-1:nPv_1^r=pv_4^r$$

$$\Rightarrow n^{\frac{1}{r}v_1}=v_4 \qquad ....(ii)$$

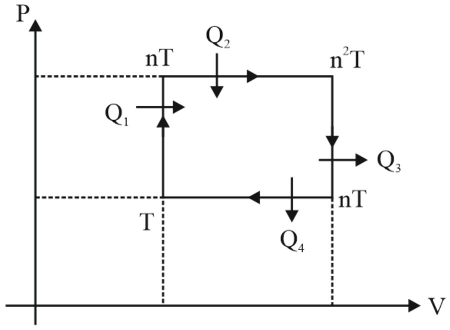
Subtracting (i), (ii):

$$(v_2-v_1)n^{\frac{1}{r}}=v_3-v_4$$

$$\Rightarrow \frac{v_3-v_4}{v_2-v_1}=n^{\frac{1}{r}}$$

8. An ideal gas whose adiabatic exponent equal γ goes through a cycle consisting of two isochoric and two isobaric lines. Find the efficiency of such a cycle, if the absolute temperature of the gas rises n times both in the isochoric heating and in the isobaric expansion.

Solution:



$$\eta=\frac{w}{Q_1+Q_2}$$

$$\frac{Q_1+Q_2-|(Q_3+Q_4)|}{Q_1+Q_2}=1-\frac{|Q_3+Q_4|}{Q_1+Q_2}$$

$$Q_1=n_0Cv(n-1)T$$

$$Q_2=n_0Cp(n^2-n)T$$

$$|Q_3|=n_0Cv(n^2-n)T$$

$$|Q_4|=n_0Cp(n-1)T$$

$$\therefore \eta=1-\frac{Cv(n^2-n)+Cp(n-1)}{Cv(n-1)+Cp(n^2-n)}$$

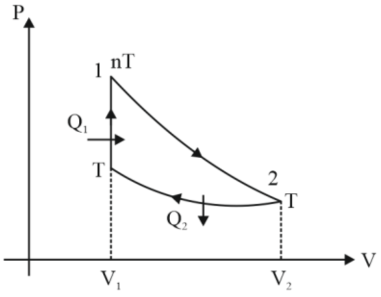
$$=1-\frac{Cv[n-1]}{Cv(n-1)}\frac{\left[n+\frac{Cp}{Cv}\right]}{\left[1+\frac{Cp}{Cv}.n\right]}=1-\frac{n+\gamma}{1+n^{\gamma}}$$

9. An ideal gas goes through a cycle consisting of (a) Isochoric, adiabatic, and isothermal lines, (b) Isobaric, adiabatic, and isothermal lines, With the isothermal process proceeding at the minimum temperature of the whole cycle. Find the efficiency of each cycle if the absolute temperature varies n-fold within the cycle.

Solution:

$$Q_1=n_0Cv(n-1)T$$

$$|Q_2|=n_0RT\ell n\frac{V_2}{V_1}$$



$$\eta=\frac{w}{Q_1}=\frac{Q_1-|Q_2|}{Q_1}=1-\frac{|Q_2|}{Q_1}$$

For adiabatic process 1-2:

$$(nT)^{\frac{1}{r-1}}v_1$$

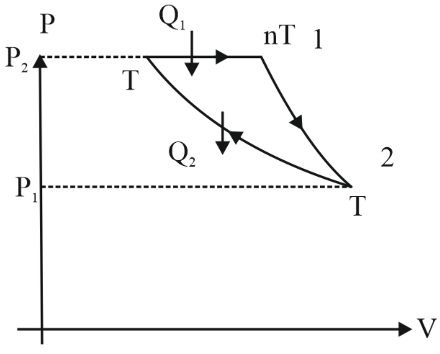
$$=T^{\frac{1}{r-1}}v_2$$

$$\Rightarrow \frac{v_2}{v_1}=n^{\frac{1}{r-1}}$$

$$\therefore \eta=1-\frac{n_0RT\ell n\,n^{\frac{1}{r-1}}}{n_0\frac{R}{r-1}(n-1)T}=1-\frac{\frac{1}{r-1}\ell n\;\eta}{\frac{1}{r-1}(n-1)}$$

$$=1-\frac{\ell n\;n}{n-1}$$

(b)



$$Q_1 = \eta c_p (n-1)T$$

$$|Q_2| = \eta_o RT \ell n \frac{\rho_2}{\rho_1}$$

$$\eta = 1 - \frac{|Q_2|}{Q_1}$$

For adiabatic process 1 – 2

$$\rho_1 (nT)^{\frac{n}{1-n}} = \rho_2 T^{\frac{n}{1-n}}$$

$$\Rightarrow \frac{\rho_2}{\rho_1} = n^{\frac{n}{1-n}}$$

$$\therefore \eta = 1 - \frac{n_o \frac{Rr}{r-1}(n-1)T}{n_o RT \ \ell n\, \eta^{\frac{r}{1-n}}}$$

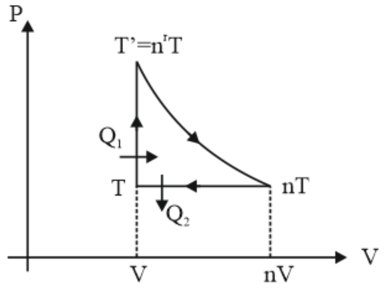
$$= 1 - \frac{\frac{r^1(n-1)}{n-1}}{\frac{r}{r^1-1}\ell n\, \eta} = 1 - \frac{\ell n\, n}{n-1}$$

10. An ideal gas with the adiabatic exponent γ goes through a direct (clockwise) cycle consisting of adiabatic, isobaric and isochoric lines. Find the efficiency of the cycle if in the adiabatic process the volume of the ideal gas

(a) Increases n-fold

(b) Decreases n-fold.

Solution:(a)

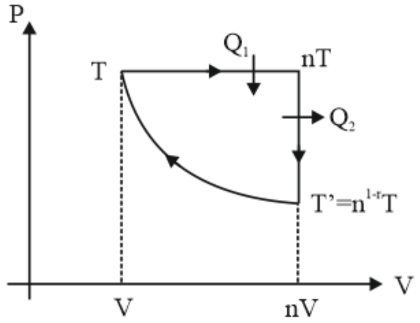


$$\eta = 1 - \frac{|Q_2|}{Q_1} = 1 - \frac{n_0 C_p (n-1)T}{n_o C_v (n^r - 1)T} = 1 - \frac{r(n-1)}{n^x - 1}$$

$$T^1 V^{x-1} = (nT)(nT)^{x-1}$$

$$\Rightarrow T^1 = n^n T$$

(b)



$$T^1 (nv)^{n-1} = TV^{n-1}$$

$$\Rightarrow T^1 = n^{1-n} T$$

$$\eta = 1 - \frac{|Q_2|}{Q_1}$$

$$= \frac{1 - n_o C_v (n - n^{1-n})T}{n_o C_p (n-1)T} = 1 - \frac{n(1-n^{-r})}{n(n-1)}$$

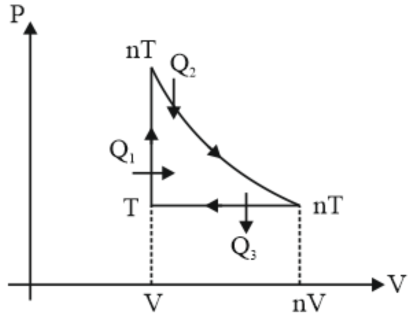
11. Calculate the efficiency of a cycle consisting of isothermal, isobaric, and isochoric lines, if in the isothermal process the volume of the ideal gas with the adiabatic exponent y

(a) Increases n-fold

(b) Decreases n-fold

Solution:

(a)

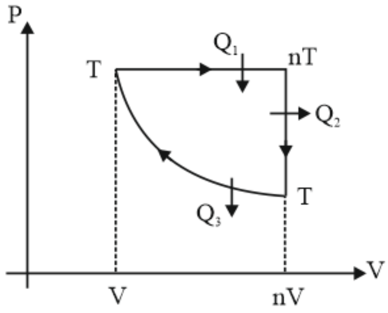


$$\eta = \frac{W}{Q_1 + Q_2} = \frac{Q_1 + Q_2 - |Q_3|}{Q_1 + Q_2} = 1 - \frac{|Q_3|}{Q_1 + Q_2}$$

$$= \frac{1 - n_o C_p (n-1)T}{n_o C_v (n-1)T + n_o R(nT)\ell n \frac{nV}{V}}$$

$$= 1 - \frac{r(n-1)}{n - 1 + (r-1)n\ \ell n\, n}$$

(b)



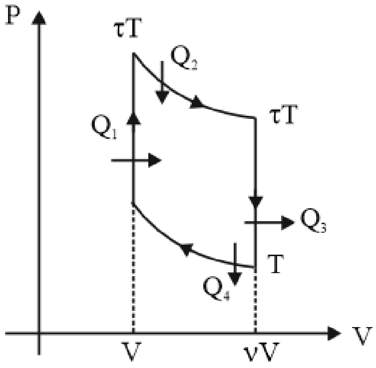
$$\eta=\frac{W}{Q_1}=\frac{Q_1-|Q_2|-|Q_3|}{Q_1}=1-\frac{|Q_2|-|Q_3|}{Q_1}$$

$$=1-\frac{n_oC_v\left(n-1\right)T+n_oRT\ell n\frac{nV}{V}}{n_oC_p\left(n-1\right)T}$$

$$=1-\frac{n-1+(r-1)\ln n}{r(n-1)}$$

12. **Find the efficiency of a cycle consisting of two isochoric and two isothermal lines if the volume varies v-fold and the absolute temperature τ -fold within the cycle. The working substance is an ideal gas with the adiabatic exponent γ .**

Solution:



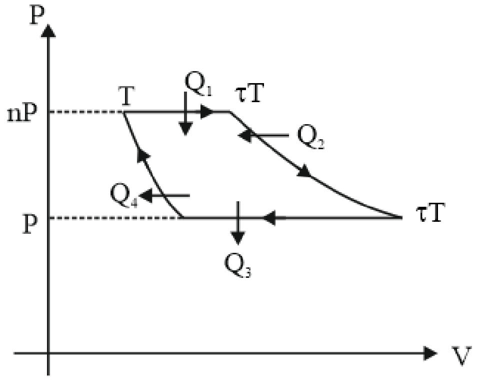
$$\eta=1-\frac{|Q_3|+|Q_4|}{Q_1+Q_2}$$

$$=1-\frac{n_oC_v\left(\tau-1\right)T+n_oRT\ \ell n\,v}{n_oC_v\left(\tau-1\right)T+n_oR\left(\tau T\right)\ell n\,v}$$

$$=1-\frac{\tau-1+\left(r\ \ -1\right)\ell n\,v}{\tau-1+\left(r-1\right)\tau\ \ell nv}$$

13. **Find the efficiency of a cycle consisting of two isobaric and two isothermal lines if the pressure varies n-fold and the absolute temperature τ -fold within the cycle. The working substance is an ideal gas with the adiabatic exponent γ .**

Solution:



$$\eta=1-\frac{|Q_3|+|Q_4|}{Q_1+Q_2}$$

$$=1-\frac{n_oC_p\left(\tau-1\right)T+n_oRT\ \ell n\,n}{n_oC_p\left(\tau-1\right)T+n_oR\left(\tau T\right)\ell n\,n}$$

$$=1-\frac{r\ \left(\tau-1\right)+\left(r-1\right)\ell n\,n}{r\ \left(\tau-1\right)+\tau\left(r\ \ -1\right)\ell n\,n}$$

Beginner **2nd Law – Heat Pump**

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1. **An ideal refrigerator has a freezer at a temperature of –13°C. The coefficient of performance of the engine is 5. The temperature of the air (to which heat is rejected) will be**

(a) 325°C	**(b) 325K**
(c) 39°C	**(d) 320°C**

Solution:

The answer is C.

Given:

$temperature\ of\ freezer=T_2=-13℃$

$=-13+273=260K$

$Coefficient\ performance=\beta=5$

And

$\beta=[T_2/(T_1-T_2)]$

$\therefore 5=[(260)/(T_1-260)]$

$\therefore T_1=260+52=312K=(312-273)℃$

$=39℃$

2. **In a mechanical refrigerator, the low temperature coils are at a temperature of –23°C and the compressed gas in the condenser**

has a temperature of 27°C. The theoretical coefficient of performance is

(a) 5 **(b) 8**

(c) 6 **(d) 6.5**

Solution:

The answer is A.

Coefficient of performance

$$K = \frac{T_2}{T_1 - T_2}$$

$$\frac{(273-23)}{(273+27)-(273-23)} = \frac{250}{300-250} = 5$$

3. The coefficient of performance of a refrigerator is 5. If the temperature inside freezer is -20°C, the temperature of the surroundings to which it rejects heat is

(a) 31°C **(b) 41°C**

(c) 11°C **(d) 21°C**

Solution:

Coefficient of performance (β) of a refrigerator is defined as the ratio of quantity of heat removed per cycle to the work done on the working substance per cycle to remove this heat.

Given, coefficient of performance of a refrigerator $\beta = 5$

Temperature of surface i.e. inside freezer,
$T_2 = -20°C = -20 + 273 = 253K$

Temperature of surrounding i.e. heat rejected outsider $T_1 = ?$

$$\text{So, } \beta = \frac{T_2}{T_1 - T_2}$$

$$5 = \frac{253}{T_1 - 253}$$

$$5T_1 = 1518T_1$$

$$= \frac{1518}{5} = 303.6K$$

$$T_1 = 303.6 - 273 = 31°C$$

4. A refrigerator is to maintain eatables kept inside at 9°C. If room temperature is 36°C, the coefficient of performance is:

(a) 9.3 **(b) 12.4**

(c) 11.2 **(d) 10.4**

Solution:

Hint: The coefficient of performance is $\beta = \frac{T_C}{T_H - T_C}$

Step 1: Calculate the coefficient of performance by putting the values Here,

T_H = Temperature outside or temperature of hot source = 36° C

Tc - Temperature of sink or temperature of the cold source = 9°C.

So,

$$\beta = \frac{T_C}{T_H - T_C} = \frac{273+9}{36-9} \approx 10.4$$

5. A Carnot freezer takes heat from water at 0°C inside it and rejects it to the room at a temperature of 27°C. The latent heat of ice is 336 × 10^3 J kg^{-1}. If 5 kg of water at 0°C is converted into ice at 0°C by the freezer, then the energy consumed by the freezer is close to:

(a) 1.51 × 10^5 J **(b) 1.68 × 10^6 J**

(c) 1.71 × 10^7 J **(d) 1.67 × 10^5 J**

Solution:

(d) $\Delta H = mL = 5 \text{ v } 336 \times 10^3 = Q_{sink}$

$$\frac{Q_{\sin k}}{Q_{source}} = \frac{T_{\sin k}}{T_{source}}$$

$$\therefore\ Q_{source} = \frac{T_{source}}{T_{\sin k}} \times Q_{\sin k}$$

Energy consumed by freezer

$$W_{output} = Q_{source} - Q_{sink} = Q_{sink}\left(\frac{T_{source}}{T_{\sin k}} - 1\right)$$

Given: $T_{source} = 27°C + 273 = 300K$,

$T_{sink} = 0°C + 273 = 273$ k

$$W_{output} = 5 \times 336 \times 10^3 \left(\frac{300}{273} - 1\right) = 1.67 \times 10^5 \text{ J}$$

6. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer?

Solution:

Given,

Heat absorbed, $Q_2 = mL = 80 \times 100 = 8000$ Cal

Temperature of ice, $T_2 = 273$ K

Temperature of surrounding,

$T_1 = 273 + 27 = 300$ K

$$\text{Efficiency} = \frac{w}{Q_2} = \frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2} = \frac{300-273}{273}$$

$$\frac{Q_1 - 8000}{8000} = \frac{27}{273}$$

$Q_1 = 8791$ Cal

7. A refrigerator works between 4°C and 30°C. It is required to remove 600 calories of heat every second in order to keep the temperature of the refrigerated space constant. The power required is (Take, 1 cal = 4.2 Joules)

(a) 23.65 W **(b) 236.5 W**
(c) 2365 W **(d) 2.365 W**

Solution:

(b) Given, temperature of source,
T = 30°C = 30 + 273 ⇒ T_1 =303 K
Temperature of sink, T_2 = 4°C = 4 + 273
T_2 = 277 K
As, we know that

$$\frac{Q_1}{Q_2}=\frac{T_1}{T_2}\Rightarrow\frac{Q_2+W}{Q_2}=\frac{T_1}{T_2}\ \{\because W=Q_1-Q_2\}$$

Where Q_2 is the amount of heat drawn from the sink (at T_2), W is work done on working substance, Q_1 is amount of heat rejected to source (at room temperature T_1).

$$WT_2 + T_2Q_2 = T_1Q_2$$
$$WT_2 = T_1Q_2 - T_2Q_2$$
$$WT_2 = Q_2(T_1 - T_2)$$
$$W=Q_2\left(\frac{T_1}{T_2}-1\right)$$
$$W=600\times4.2\times\left(\frac{303}{277}-1\right)$$
$$W=600\times4.2\times\left(\frac{26}{277}\right)$$

W = 236.5 Joules

$$\text{Power}=\frac{\text{Work done}}{\text{Time}}=\frac{W}{t}=\frac{236.5}{5}=236.5\text{ W}$$

8. The temperature inside a refrigerator is t_2 °C and the room temperature is t_1 °C. The amount of heat delivered to the room for each joule of electrical energy consumed ideally will be

(a) $\frac{t_1}{t_1-t_2}$ **(b)** $\frac{t_1+273}{t_1-t_2}$
(c) $\frac{t_2+273}{t_1-t_2}$ **(d)** $\frac{t_1+t_2}{t_1+273}$

Solution:

(b) For a refrigerator, we know that

$$\frac{Q_1}{W}=\frac{Q_1}{Q_1-Q_2}=\frac{T_1}{T_1-T_2}$$

where,
Q_1 = amount of heat delivered to the room
W = electrical energy consumed
T_1 = room temperature = t_1 + 273
T_2 = temperature of sink = t_2 + 273

$$\therefore\ \frac{Q_1}{1}=\frac{t_1+273}{t_1+273-(t_2-273)}$$
$$Q_1=\frac{t_1+273}{t_1-t_2}$$

Expert **2nd Law – Heat Pump**

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1. Calculate the least amount of work that must be done to freeze 1 gram of water at 0°C by mean of refrigerator. T_s = 27°C. How much heat is passed to surrounding in this process. L_f = 80 cal/g

Solution:

The answer is.

$$L=80\frac{cal}{g}$$
$$m=1g$$
$$T_1=27°=300K$$
$$T_2=0°=273K$$
$$\text{Least work done } W=L\times m\times\frac{T_1}{T_2}$$
$$=80\times1\times\frac{300}{273}$$
$$=87.912cal$$

2. A Carnot's engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is:

(a) 420J **(b) 2100J**
(c) 772J **(d) 2520J**

Solution:

(a) Given: Temperature of cold body,
T_2 = 250 K
Temperature of hot body; T_1 = 300 K
Heat received, Q_2 = 500 cal work done, W = ?

$$\text{Efficiency}=1-\frac{T_2}{T_1}=\frac{W}{Q_2+W}$$
$$1-\frac{250}{300}=\frac{W}{Q_2+W}$$
$$W=\frac{Q_2}{5}=\frac{500\times4.2}{5}J$$

= 420 J

3. A refrigerator works between 4°C and 30°C. It is required to remove 600 calories of heat every second to keep the temperature of the refrigerated space constant. The power required will be: (Take 1 cal = 4.2 Joules)

(a) 23.65 W **(b) 236.5 W**
(c) 2365 W **(d) 2.365 W**

Solution:

The temperature of the source, T_1= 30°C i.e. 303 K
Temperature of the sink, T_2 = 4°C i.e. 277 K
It is known that,

$$\frac{Q_1}{Q2}=\frac{T_1}{T_2}$$

$$\Rightarrow \frac{Q_2+W}{Q_2}=\frac{T_1}{T_2}\{W=Q_1-Q_2\}$$

Here, Q_2 is the amount of heat drawn from the sink at temperature T_2 Q_1 is the amount of heat given to the source at temperature T_1 W is the amount of work done on working substances

$$\Rightarrow WT_2+T_2Q_2=T_1Q_2$$

$$\Rightarrow WT_2=T_1Q_2-T_2Q_2$$

$$\Rightarrow WR_2=Q_2\left(T_1-T_2\right)$$

$$\Rightarrow W=Q_2\left(\frac{T_1}{T_2}-1\right)$$

$$\Rightarrow W=600\times 4.2\times\left(\frac{303}{277}-1\right)$$

$$W=600\times 4.2\times\left(\frac{26}{277}\right)$$

W = 236.5 Joules

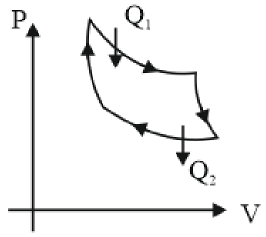
$$\text{Power}=\frac{\text{Work}}{\text{doetime}}$$

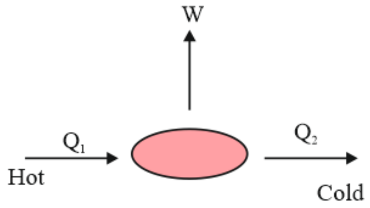
$$=\frac{W}{t}=\frac{236.5}{1}=236.5W$$

4. A heat engine employing a Carnot cycle with an efficiency of η = 10% is used as a refrigerating machine, the thermal reservoirs being the same. Find its refrigerating efficiency ε .

Solution:

Carnot cycle:

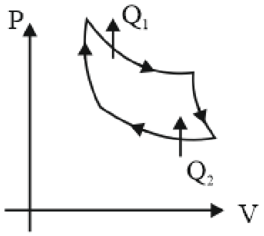


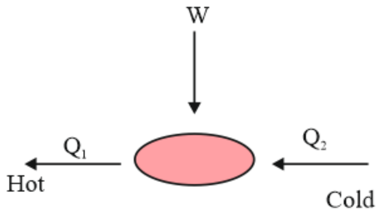


$$\eta=\frac{W}{Q_1}=\frac{Q_1-Q_2}{Q_1}=1-\frac{Q_2}{Q_1}$$

$$\Rightarrow\frac{Q_2}{Q_1}=1-\eta$$

Refrigeration cycle :





$$\in=\frac{Q_2}{W}=\frac{Q_2}{Q_1-Q_2}$$

$$=\frac{\frac{Q_2}{Q_1}}{1-\frac{Q_2}{Q_1}}=\frac{1-\eta}{\eta}$$

Beginner **Miscellaneous Problems**

1. 200g water is heated from 40°C to 60°C. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water = 4184 J/kgK):

(a) 167.4kJ **(b) 8.4kJ**
(c) 4.2 kJ **(d) 16.7 kJ**

Solution:

(d) Volume of water does not change, no work is done on or by the system (W = 0)
According to first law of thermodynamics
Q = dU + W
For Isochoric process Q = dU
ΔU = nCdT = 2 × 4184 × 20 = 16.7 kJ.

2. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently, the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_p}{C_v}=\gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by:

(a) 2 **(b)** $\frac{1}{2}$

(c) $\left(\frac{1}{2}\right)^{\gamma}$ **(d)** $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$

Solution:

We know that Relaxation time,

$$\tau = \frac{V}{\sqrt{T}} \quad \text{.......(i)}$$

Equation of adiabatic process is

$TV^{\gamma-1}$ = constant

$$T \propto \frac{1}{V^{\gamma-1}}$$

$$\tau \propto V^{1+\frac{\gamma-1}{2}} \quad \text{using (i)}$$

$$\tau \propto V^{\frac{\gamma+1}{2}}$$

$$\frac{\tau_f}{\tau_i} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}} = (2)^{\frac{1+\gamma}{2}}$$

3. **Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_p}{C_v}\right)$**

(a) $\frac{\gamma+1}{2}$ **(b)** $\frac{\gamma-1}{2}$

(c) $\frac{3\gamma+5}{6}$ **(d)** $\frac{3\gamma-5}{6}$

Solution:

(a) $$\tau = \frac{1}{\sqrt{2}\pi d^2\left(\frac{N}{V}\right)\sqrt{\frac{3RT}{M}}}$$

$$\tau \propto \frac{V}{\sqrt{T}}$$

As, $TV^{\gamma-1} = K$, So, $\tau \propto V^{\frac{\gamma+1}{2}}$

Therefore, $q = \frac{\gamma+1}{2}$

4. **1g of water, of volume 1 cm^3 at 100°C is converted into steam at same temperature under normal atmospheric pressure = ($\simeq 1 \times 10^5$ Pa). The volume of steam formed equals 1671 cm^3. If the specific latent heat of vaporization of water is 2256 J/g, the change in internal energy is**

(a) 2423 J **(b) 2089 J**

(c) 167 J **(d) 2256 J**

Solution:

(b) Given, mass of water, m = 1g

Volume of 1g of water = 1cm^3 = 10^{-6} m^3

Volume of 1g of steam = 1671 cm^3

= 1671×10^{-6} m^3

Pressure, p = 1×10^5 Pa

Latent heat of vaporization of water,

L = 2256 J/g

Change in volume,

$\Delta V = (1671 - 1) \times 10^{-6}$ m^3

= 1670×10^{-6} m^3 … (i)

Heat supplied,

$\Delta Q = mL = 1 \times 2256 = 2256$ J ... (ii)

As the steam expands, so the work done in expansion is

$\Delta W = p\Delta V$

= $1 \times 10^5 \times 1670 \times 10^{-6}$ [from Eq.(i)]

= 167 J ... (iii)

According to first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$\Delta U = \Delta Q - \Delta W$

= 2256 – 167 [from Eq. (ii) and (iii)]

= 2089 J

5. **A sample of 0.1 g of water at 100°C and normal pressure (1.013×10^5 Nm^{-2}) requires 54 cal of heat energy to convert to steam at 100°C. If the volume of the steam produced is 167.1 cc, the change in internal energy of the sample, is**

(a) 42.2 J **(b) 208.7 J**

(c) 104.3 J **(d) 84.5 J**

Solution:

(b) According to the question,

Heat spent during the conversion of sample of water at 100°C to steam is,

ΔQ = 54 cal = 54×4.18 J = 225.72 J

Normal pressure, p = 1.013×10^5 Nm^{-2}

Net work done during the conversion would be given as

$\Delta W = p\Delta V = p\,[V_{steam} - V_{water}]$

Here, V_{steam} = 167.1 cc = 167.1×10^{-6} m^3

V_{water} = 0.19 = 0.1cc = 0.1×10^{-6} m^3

$\therefore \Delta W = 1.013 \times 10^5\,[(167.1 - 0.1) \times 10^{-6}]$

= $1.013 \times 167 \times 10^{-1}$ = 16.917 J

Now, by the first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

Where, ΔU is the change in internal energy of the sample.

$\Delta U = \Delta Q - \Delta W$

Substituting the values in the above equation, we get

$\Delta U = 225.72 - 16.917 = 208.7$ J

Expert Miscellaneous Problems

1. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

(a) $T \propto \frac{1}{R}$ **(b)** $T \propto \frac{1}{R^3}$

(c) $T \propto e^{-R}$ **(d)** $T \propto e^{-3R}$

Solution:

(a) As, $P = \frac{1}{3}\left(\frac{U}{V}\right)$

But $\frac{U}{V} = KT^4$

So, $P = \frac{1}{3}KT^4$

or $\frac{uRT}{V} = \frac{1}{3}KT^4$ [As PV = uRT]

$\frac{4}{3}\pi R^3 T^3$ = constant

Therefore, $T \propto \frac{1}{R}$

2. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is :

(a) ln2, 2ln2 **(b) 2ln2, 8ln2**

(c) ln2, 4ln2 **(d) ln2, ln2**

Solution:

(d) The entropy change of the body in the two cases is same as entropy is a state function.

Test Yourself

Beginner Test - I

1. Air pressure in a car tyre increases during driving. Explain.
2. If a gas is suddenly compressed, its temperature increases, why?
3. Write conditions for an isothermal process.
4. Apply first law of thermodynamics for isothermal expansion of an ideal gas.
5. When a bottle of cold carbonated drink is opened, a slight fog is formed around the opening, why?
6. A person of mass 60 kg wants to lose 5 kg by going up and down a 10 m high stairs. Assume he burns twice as much fat while going up than coming down. If 1 kg of fat is burnt on expending 7000 kcal calories, how many times must he go up and down to reduce his weight by 5 kg?
7. What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise its temperature by 45°C at constant pressure? (Molecular mass of $N_2 = 28$, $R = 8.3 \text{J mol}^{-1} \text{K}^{-1}$)
8. Two bodies at different temperatures T_1 and T_2 are brought in contact. Under what condition, they settle to mean temperature? (after they attain equilibrium)
9. The initial state of a certain gas is (p_i, V_i, T_i). It undergoes expansion till its volume becomes V_f. Consider the following two cases

   (i) The expansion takes place at constant temperature.

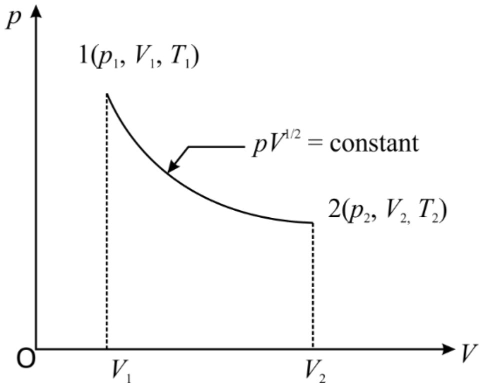
   (ii) The expansion takes place at constant pressure.

   Plot the p-V diagram for each case. In which of the two cases, is the work done by the gas more?
10. Explain, why?

    (i) 500 J of work is done on a gas to reduce its volume by compression adiabatically.
    What is the change in internal energy of the gas?

    (ii) The coolant in a chemical or a nuclear plant, i.e. the liquid used to prevent the different parts of a plant from getting too hot should have high specific heat.

    (iii)The climate of a harbour town is more temperate than that of a town in a desert at the same latitude
11. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case? (Take, 1 cal = 4.19 J). According to first law of thermodynamics, if ΔQ heat energy is given (or taken) to a thermodynamic system which is partially utilised in doing work (ΔW) and remaining part increases (or decreases) the internal energy of the system.
12. Calculate the work done for adiabatic expansion of a gas.
13. Consider a p-V diagram in which the path followed by one mole of perfect gas in a cylindrical container is shown.

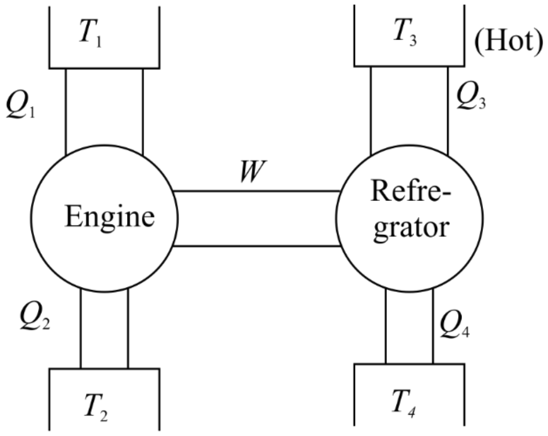


    (i) Find the work done when the gas is taken from state 1 to state 2.

    (ii)What is the ratio of temperatures T_1/T_2, if $V_2 = 2V_1$?

    (iii) Given the internal energy for one mole of gas at temperature T is (3/2) RT, find the heat supplied to the gas when it is taken from states 1 to 2, with $V_2 = 2V_1$.

14. Consider that an ideal gas (n moles) is expanding in a process given by p = f(V), which passes through a point (V_o, p_o). Show that the gas is absorbing heat at (p_o, V_o), if the slope of the curve p = f(V) is larger than the slope of the adiabat passing through (p_o, V_o).
15. What is a cyclic process? What is change in internal energy of the system in a cyclic process? In changing the state of gas adiabatically from an equilibrium states A to B. an amount of 40.5 J of work is done on the system. If the gas is taken from states A to B via a process in which net heat absorbed by the system is 12.6 cal. How much is the net work done by the system in the later case? (1 cal = 4.19 J)
16. If a refrigerator's door is kept open, will the room become cool or hot? Explain.
17. Can a ship be moved with the energy of the sea water?
18. What are the forces which make any process irreversible?
19. The coefficient of performance of a refrigerator depends on which factor?
20. Find the efficiency of the Carnot engine working between boiling point and freezing point of water.
21. Which thermodynamic law put restrictions on the complete conversion of heat into work?
22. Consider a Carnot cycle operating between T_1 = 500 K and T_2 = 300 K producing 1 kJ of mechanical work per cycle. Find the heat transferred to the engine by the reservoirs.
23. (i) The efficiency of a heat engine is more in hilly area than in plain. Explain it.
    (ii) Is the coefficient of performance of a refrigerator, a constant quantity?
    (iii) Why is it theoretically not possible to have a device which create no thermal pollution?
24. A Carnot engine is operating between 600 K and 200 K. Consider that the actual energy produced is 2 kJ per kilocalorie of heat absorbed. Compare the real efficiency with the efficiency of Carnot engine.
25. Explain, what do you understand by the efficiency of a heat engine?
26. Under what condition, an ideal Carnot engine has 100% efficiency?
27. Temperatures of the hot and cold reservoirs of a Carnot engine is raised by equal amounts. How the efficiency of the Carnot engine affected?
28. A refrigerator transfers 250 J heat per second from –23°C to 25°C. Find the power consumed, assuming no loss of energy.
29. Find out whether these phenomena are reversible or not.
    (i) Waterfall and
    (ii) Rusting of iron.
30. An ideal engine works between temperatures T_1 and T_2. It derives an ideal refrigerator that Works between temperatures T_3 and T_4. Find the ratio Q_3/Q_1 in terms of T_1, T_2, T_3 and T_4



31. Explain with the suitable example that a reversible process must be carried slowly and a fast process is necessarily irreversible.
32. Two Carnot engines A and B are operated in series. The first one A receives heat at 800 K and rejects to a reservoir at temperature T K. The second engine B receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 300 K. Calculate the temperature T K for the following cases.

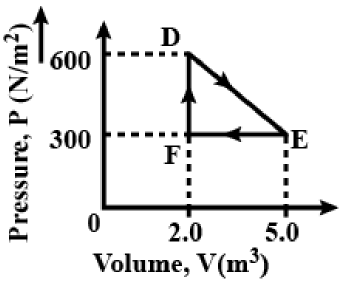
(i) When the outputs of the two engines are equal.
(ii) When the efficiencies of the two engines are equal.

Answer Key

Refer. Solutions

Beginner Test - II

1. A geyser heats water flowing at the rate of 3.0 litres per minute from 27 °C to 77 °C. If the geyser operates on a gas burner, what is the rate of consumption of the fuel if its heat of combustion is 4.0×10^4 J/g?
2. What amount of heat must be supplied to 2.0×10^{-2} kg of nitrogen (at room temperature) to raise its temperature by 45 °C at constant pressure? (Molecular mass of N2 = 28; R = 8.3 J mol^{-1} K^{-1}.)
3. Explain why
   (a) Two bodies at different temperatures T_1 and T_2 if brought in thermal contact do not necessarily settle to the mean temperature $(T_1 + T_2)/2$.
   (b) The coolant in a chemical or a nuclear plant (i.e., the liquid used to prevent the different parts of a plant from getting too hot) should have high specific heat.
   (c) Air pressure in a car tyre increases during driving.
   (d) The climate of a harbour town is more temperate than that of a town in a desert at the same latitude.
4. A cylinder with a movable piston contains 3 moles of hydrogen at standard temperature and pressure. The walls of the cylinder are made of a heat insulator, and the piston is insulated by having a pile of sand on it. By what factor does the pressure of the gas increase if the gas is compressed to half its original volume?
5. In changing the state of a gas adiabatically from an equilibrium state A to another equilibrium state B, an amount of work equal to 22.3 J is done on the system. If the gas is taken from state A to B via a process in which the net heat absorbed by the system is 9.35 cal, how much is the net work done by the system in the latter case? (Take 1 cal = 4.19 J)
6. Two cylinders A and B of equal capacity are connected to each other via a stopcock. A contains a gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stopcock is suddenly opened.
   Answer the following:
   (a) What is the final pressure of the gas in A and B?
   (b) What is the change in internal energy of the gas?
   (c) What is the change in the temperature of the gas?
   (d) Do the intermediate states of the system (before settling to the final equilibrium state) lie on its P-V-T surface?
7. A steam engine delivers 5.4×10^8 J of work per minute and services 3.6×10^9 J of heat per minute from its boiler. What is the efficiency of the engine? How much heat is wasted per minute?
8. An electric heater supplies heat to a system at a rate of 100 W. If system performs work at a rate of 75 Joules per second. At what rate is the internal energy increasing?
9. A thermodynamic system is taken from an original state to an intermediate state by the linear process shown in Fig.



Its volume is then reduced to the original value from E to F by an isobaric process. Calculate the total work done by the gas from D to E to F

10. A refrigerator is to maintain eatables kept inside at 9°C. If room temperature is 36° C, calculate the coefficient of performance.

Answer Key

1. 15.75 g/min
2. 933.38 J
3. *Refer Solution*
4. 2.639
5. 16.88 J
6. (a) 0.5 atm, (b) Zero, (c) Zero, (d) No
7. 3.06×10^9 J
8. 25 W
9. 450 J
10. 10.44

Expert Test - I

1.1g of water, of volume 1 cm^3 at 100°C is converted into steam at same temperature under normal atmospheric pressure = ($\simeq 1 \times 10^5$ Pa). The volume of steam formed equals 1671 cm^3. If the specific latent heat of vaporization of water is 2256 J/g, the change in internal energy is

(a) 2423 J (b) 2089 J

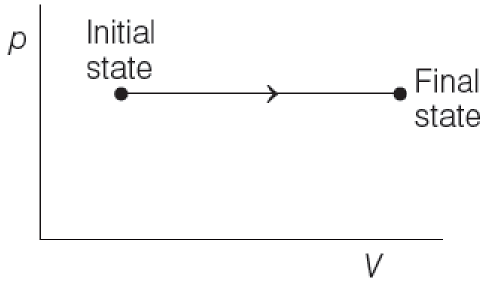
(c) 167 J (d) 2256 J

2. A sample of 0.1 g of water at 100°C and normal pressure (1.013×10^5 Nm^{-2}) requires 54 cal of heat energy to convert to steam at 100°C. If the volume of the steam produced is 167.1 cc, the change in internal energy of the sample, is

(a) 42.2 J (b) 208.7 J

(c) 104.3 J (d) 84.5 J

3. The p-V diagram for an ideal gas in a piston cylinder assembly undergoing a thermodynamic process is shown in the figure. The process is



(a) Adiabatic (b) Isochoric

(c) Isobaric (d) Isothermal

4. Two cylinders A and B of equal capacity are connected to each other via a stop cock. A contains an ideal gas at standard temperature and pressure. B is completely evacuated. The entire system is thermally insulated. The stop cock is suddenly opened. The process is

(a) Adiabatic (b) Isochoric

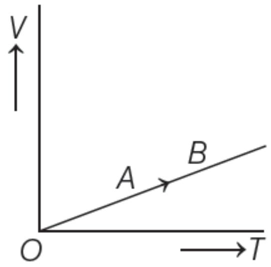
(c) Isobaric (d) Isothermal

5. In which of the following processes, heat is neither absorbed nor released by a system?

(a) Adiabatic (b) Isobaric

(c) Isochoric (d) Isothermal

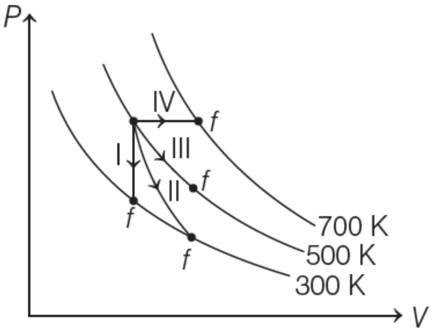
6. The volume (V) of a monoatomic gas varies with its temperature (T), as shown in the graph. The ratio of work done by the gas, to the heat absorbed by it, when it undergoes a change from state A to state B, is



(a) $\frac{1}{3}$ (b) $\frac{2}{3}$

(c) $\frac{2}{5}$ (d) $\frac{2}{7}$

7. Thermodynamic processes are indicated in the following diagram



Match the following:

	Column-I		Column-II
P.	Process I	a.	Adiabatic
Q.	Process II	b.	Isobaric
R.	Process III	c.	Isochoric
S.	Process IV	d.	Isothermal

(a) P → a, Q → c, R → d, S →b (b) P → c, Q → a, R → d, S → b

(c) P → c, Q → d, R → b, S → a (d) P → d, Q → b, R → a, S → c

8. A gas is compressed isothermally to half its initial volume. The same gas is compressed separately through an adiabatic process until its volume is again reduced to half. Then

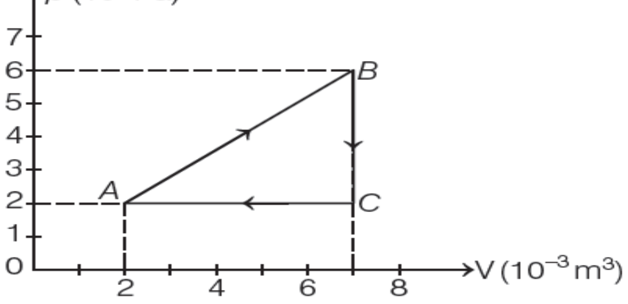
(a) Compressing the gas through adiabatic process will require more work to be done.

(b) Compressing the gas isothermally or adiabatically will require the same amount of work.

(c) Which of the case (whether compression through isothermal or through adiabatic process) requires more work will depend upon the atomicity of the gas.

(d) Compressing the gas isothermally will require more work to be done.

9. A gas is taken through the cycle A →B→C→A, as shown. What is the net work done by the gas?



(a) 2000 J (b) 1000 J

(c) Zero (d) –2000 J

10. The efficiency of a Carnot engine depends upon
    (a) The temperature of the sink only
    (b) The temperatures of the source and sink
    (c) The volume of the cylinder of the engine
    (d) The temperature of the source only

11. The efficiency of an ideal heat engine working between the freezing point and boiling point of water, is
    (a) 6.25%
    (b) 20%
    (c) 26.8%
    (d) 12.5%

12. A Carnot engine having an efficiency of $\frac{1}{10}$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is
    (a) 1 J
    (b) 90 J
    (c) 99 J
    (d) 100 J

13. A refrigerator works between 4°C and 30°C. It is required to remove 600 calories of heat every second in order to keep the temperature of the refrigerated space constant. The power required is (Take, 1 cal = 4.2 Joules)
    (a) 23.65 W
    (b) 236.5 W
    (c) 2365 W
    (d) 2.365 W

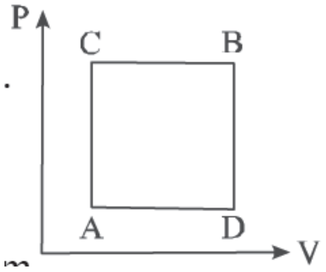
14. The temperature inside a refrigerator is t_2 °C and the room temperature is t_1 °C. The amount of heat delivered to the room for each joule of electrical energy consumed ideally will be
    (a) $\frac{t_1}{t_1 - t_2}$
    (b) $\frac{t_1 + 273}{t_1 - t_2}$
    (c) $\frac{t_2 + 273}{t_1 - t_2}$
    (d) $\frac{t_1 + t_2}{t_1 + 273}$

Answer Key

1. (b)	2. (b)	3. (c)	4. (a)
5. (a)	6. (c)	7. (b)	8. (a)
9. (b)	10. (b)	11. (c)	12. (b)
13. (b)	14. (b)		

Expert Test - II

1. A gas can be taken from A to B via two different processes ACB and ADB. When path ACB is used 60 J of heat flows into the system and 30J of work is done by the system. If path ADB is used work done by the system is 10 J. The heat Flow into the system in path ADB is:



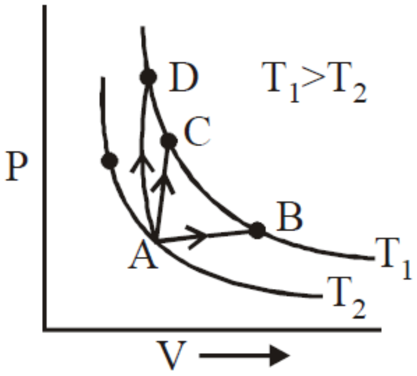
    (a) 40J
    (b) 80J
    (c) 100 J
    (d) 20J

2. 200g water is heated from 40°C to 60°C. Ignoring the slight expansion of water, the change in its internal energy is close to (Given specific heat of water = 4184 J/kgK):
    (a) 167.4kJ
    (b) 8.4kJ
    (c) 4.2 kJ
    (d) 16.7 kJ

3. A gas is compressed from a volume of $2m^3$ to a volume of $1m^3$ at a constant pressure of 100 N/m^2. Then it is heated at constant volume by supplying 150 J of energy. As a result, the internal energy of the gas:

(a) Increases by 250 J
(b) Decreases by 250 J
(c) Increases by 50 J
(d) Decreases by 50 J

4. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as A → B, A → C and A →D. The change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the work done as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameters are:

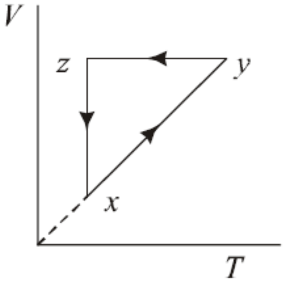


(a) $E_{AB} = E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
(b) $E_{AB} = E_{AC} = E_{AD}$, $W_{AB} > 0$, $W_{AC} = 0$, $W_{AD} < 0$
(c) $E_{AB} < E_{AC} < E_{AD}$, $W_{AB} > 0$, $W_{AC} > W_{AD}$
(d) $E_{AB} > E_{AC} > E_{AD}$, $W_{AB} < W_{AC} < W_{AD}$

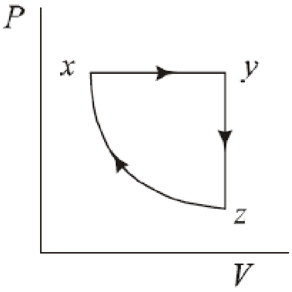
5. In an adiabatic process, the density of a diatomic gas becomes 32 times its initial value. The final pressure of the gas is found to be n times the initial pressure. The value of n is :

(a) 32
(b) 326
(c) 128
(d) $\frac{1}{32}$

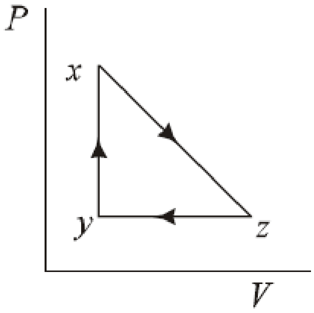
6. A thermodynamic cycle xyzx is shown on a V-T diagram.



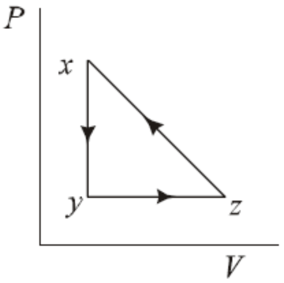
The P-V diagram that best describes this cycle is: (Diagrams are schematic and not to scale)



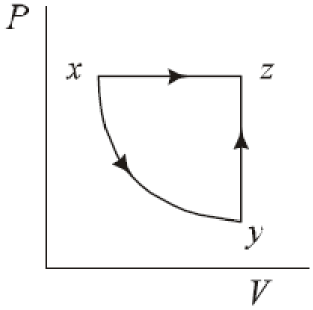
(a)



(b)



(c)

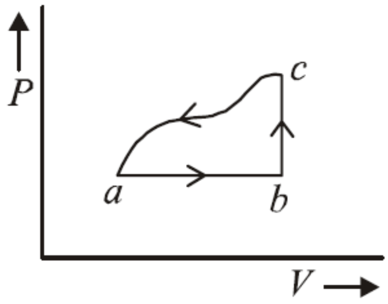


(d)

7. A litre of dry air at STP expands adiabatically to a volume of 3 litres. If $\gamma = 1.40$, the work done by air is: ($3^{1.4} = 4.6555$) [Take air to be an ideal gas]
   (a) 60.7J (b) 90.5J
   (c) 100.8J (d) 48 J

8. Under an adiabatic process, the volume of an ideal gas gets doubled. Consequently the mean collision time between the gas molecule changes from τ_1 to τ_2. If $\frac{C_p}{C_v} = \gamma$ for this gas then a good estimate for $\frac{\tau_2}{\tau_1}$ is given by:
   (a) 2 (b) $\frac{1}{2}$
   (c) $\left(\frac{1}{2}\right)^{\gamma}$ (d) $\left(\frac{1}{2}\right)^{\frac{\gamma+1}{2}}$

9. A sample of an ideal gas is taken through the cyclic process abca as shown in the figure. The change in the internal energy of the gas along the path ca is – 180 J, The gas absorbs 250 J of heat along the path ab and 60 J along the path bc. The work down by the gas along the path abc is:



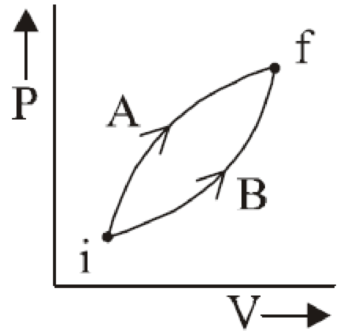
   (a) 120J (b) 130J
   (c) 100J (d) 140J

10. A cylinder with fixed capacity of 67.2 lit contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is : [Given that R = 8.31 J mol^{-1} K^{-1}]
    (a) 350J (b) 374J
    (c) 748J (d) 700J

11. One mole of an ideal gas passes through a process where pressure and volume obey the relation $P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here P_0 and V_0 are constants. Calculate the charge in the temperature of the gas if its volume changes from V_0 to $2V_0$.
    (a) $\frac{1}{2}\frac{P_0V_0}{R}$ (b) $\frac{5}{4}\frac{P_0V_0}{R}$

(c) $\frac{3}{4}\frac{P_0V_0}{R}$ (d) $\frac{1}{4}\frac{P_0V_0}{R}$

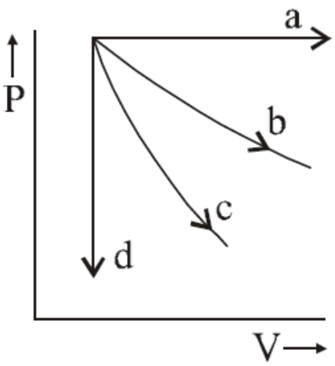
12. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies, respectively, then:



(a) $\Delta Q_A < \Delta Q_B$, $\Delta U_A < \Delta U_B$ (b) $\Delta Q_A > \Delta Q_B$, $\Delta U_A > \Delta U_B$
(c) $\Delta Q_A > \Delta QB$, $\Delta U_A = \Delta U_B$ (d) $\Delta Q_A = \Delta Q_B$; $\Delta U_A = \Delta U_B$

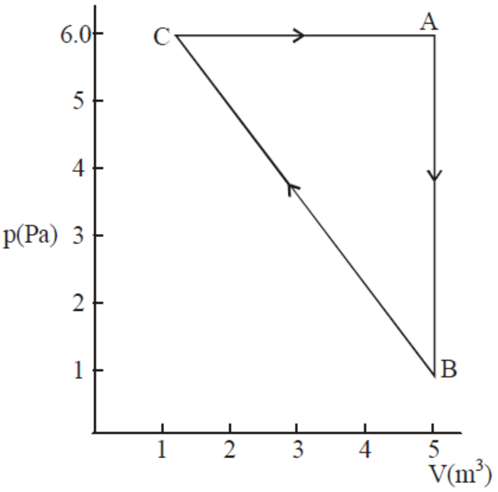
13. A thermally insulted vessel contains 150 g of water at 0°C. Then the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closed to: (Latent heat of vaporization of water = 2.10×10^6Jkg^{-1} and Latent heat of Fusion of water = 3.36×10^5 J kg^{-1})
(a) 150 g (b) 20 g
(c) 130 g (d) 35 g

14. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by:



(a) a d b c (b) d a c b
(c) a d c b (d) d a b c

15. For the given cyclic process CAB as shown for gas, the work done is:



(a) 30J
(b) 10J
(c) 1J
(d) 5J

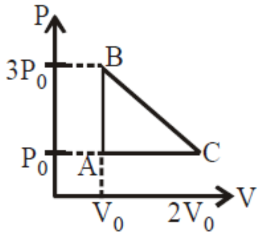
16. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is:

(a) $\frac{3}{5}$
(b) $\frac{2}{5}$
(c) $\frac{2}{3}$
(d) $\frac{5}{3}$

17. Half mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to: (Gas constant R = 8.31 J/mol-K)

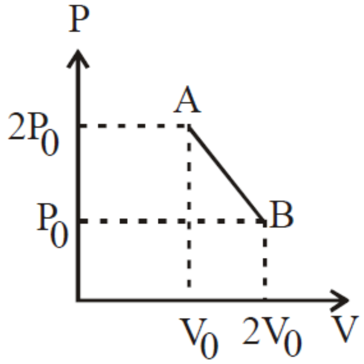
(a) 581 J
(b) 291 J
(c) 146 J
(d) 73 J

18. One mole of an ideal monoatomic gas is taken along the path ABCA as shown in the PV diagram. The maximum temperature attained by the gas along the path BC is given b



(a) $\frac{25}{8}\frac{P_0V_0}{R}$
(b) $\frac{25}{4}\frac{P_0V_0}{R}$
(c) $\frac{25}{16}\frac{P_0V_0}{R}$
(d) $\frac{5}{8}\frac{P_0V_0}{R}$

19. 'n' moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature of the gas during the process will be :



(a) $\frac{9P_0V_0}{2nR}$ (b) $\frac{9P_0V_0}{nR}$

(c) $\frac{9P_0V_0}{4nR}$ (d) $\frac{3P_0V_0}{2nR}$

20. The ratio of work done by an ideal monoatomic gas to the heat supplied to it in an isobaric process is:

(a) $\frac{2}{5}$ (b) $\frac{3}{2}$

(c) $\frac{3}{5}$ (d) $\frac{2}{3}$

21. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_p}{C_v}\right)$

(a) $\frac{\gamma+1}{2}$ (b) $\frac{\gamma-1}{2}$

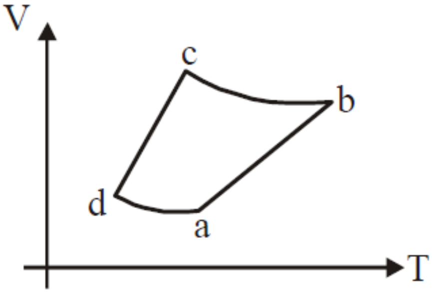
(c) $\frac{3\gamma+5}{6}$ (d) $\frac{3\gamma-5}{6}$

22. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

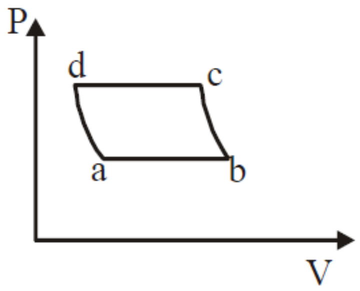
(a) $T \propto \frac{1}{R}$ (b) $T \propto \frac{1}{R^3}$

(c) $T \propto e^{-R}$ (d) $T \propto e^{-3R}$

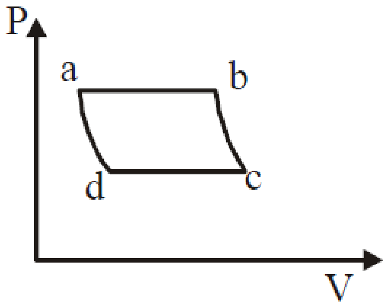
23. An ideal gas goes through a reversible cycle a → b → c → d has the V - T diagram shown below. Process d → a and b → c are adiabatic.



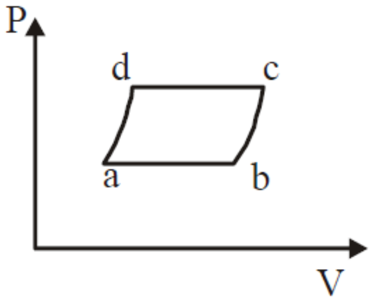
The corresponding P - V diagram for the process is (all figures are schematic and not drawn to scale) :



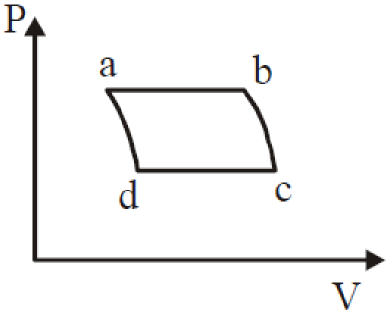
(a)



(b)

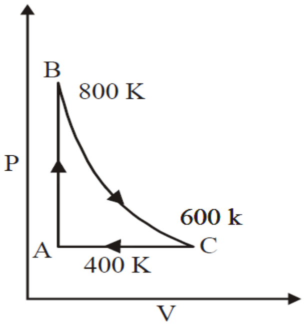


(c)



(d)

24. One mole of a diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K respectively. Choose the correct statement:



(a)The change in internal energy in whole cyclic process is 250 R.
(b)The change in internal energy in the process CA is 700 R.
(c)The change in internal energy in the process AB is –350 R.
(d)The change in internal energy in the process BC is –500 R.

25. During an adiabatic compression, 830 J of work is done on 2 moles of a diatomic ideal gas to reduce its volume by 50%. The change in its temperature is nearly: (R = 8.3 JK^{-1}mol^{-1})

(a) 40K	(b) 33K
(c) 20K	(d) 14K

26. An ideal gas at atmospheric pressure is adiabatically compressed so that its density becomes 32 times of its initial value. If the final pressure of gas is 128 atmospheres, the value of ‘γ’of the gas is :

(a) 1.5	(b) 1.4
(c) 1.3	(d) 1.6

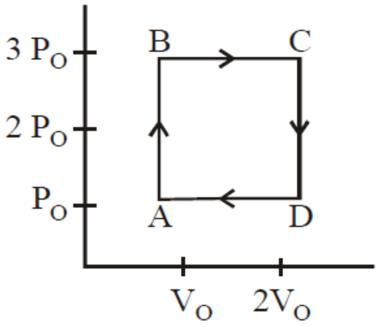
27. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is

(a) $\frac{9}{8}RT_1$	(b) $\frac{3}{2}RT_1$
(c) $\frac{15}{8}RT_1$	(d) $\frac{9}{2}RT_1$

28. An engine takes in 5 mole of air at 20°C and 1 atm, and compresses it adiabaticaly to 1/10th of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be XkJ. The value of X to the nearest integer is _________.

29. Starting at temperature 300 K, one mole of an ideal diatomic gas ($\gamma = 1.4$) is first compressed adiabatically from volume V_1 to $V_2 = \frac{V_1}{16}$. It is then allowed to expand isobarically to volume $2V_2$. If all the processes are the quasi-static then the final temperature of the gas (in °K) is (to the nearest integer) ___________.

30. An engine operates by taking a monatomic ideal gas through the cycle shown in the figure. The percentage efficiency of the engine is close to



(a) 19
(b) 16
(c) 8
(d) 4

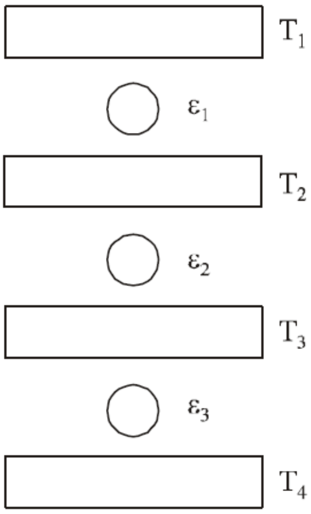
31. A heat engine is involved with exchange of heat of 1915 J, – 40 J, +125 J and – Q J, during one cycle achieving an efficiency of 50.0%. The value of Q is:
(a) 640J
(b) 40J
(c) 980J
(d) 400J

32. A Carnot engine having an efficiency of $\frac{1}{10}$ is being used as a refrigerator. If the work done on the refrigerator is 10 J, the amount of heat absorbed from the reservoir at lower temperature is:
(a) 99J
(b) 100J
(c) 1J
(d) 90J

33. Two ideal Carnot engines operate in cascade (all heat given up by one engine is used by the other engine to produce work) between temperatures, T_1 and T_2. The temperature of the hot reservoir of the first engine is T_1 and the temperature of the cold reservoir of the second engine is T_2. T is temperature of the sink of first engine which is also the source for the second engine. How is T related to T_1 and T_2, if both the engines perform equal amount of work ?
(a) $T = \frac{2T_1T_2}{T_1 + T_2}$
(b) $T = \frac{T_1 + T_2}{2}$
(c) $T = \sqrt{T_1T_2}$
(d) T = 0

34. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are, respectively.
(a) 62°C, 124°C
(b) 99°C, 37°C
(c) 124°C, 62°C
(d) 37°C, 99°C

35. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperature T_2 and T_3, as shown, with $T_1 > T_2 > T_3 > T(4)$ The three engines are equally efficient if :



(a) $T_2 = (T_1T_4)^{1/2}$; $T_3 = (T_1^2T_4)^{1/3}$ (b) $T_2 = (T_1^2T_4)^{1/3}$; $T_3 = (T_1T_4^2)^{1/3}$

(c) $T_2 = (T_1T_4^2)^{1/3}$; $T_3 = (T_1^2T_4)^{1/3}$ (d) $T_2 = (T_1^3T_4)^{1/4}$; $T_3 = (T_1T_4^3)^{1/4}$

36. Two Carnot engines A and B are operated in series. The first one, A receives heat at T_1 (= 600 K) and rejects to a reservoir at temperature T_2. The second engine B receives heat rejected by the first engine and in turn, rejects to a heat reservoir at T_3 (= 400 K). Calculate the temperature T_2 if the work outputs of the two engines are equal:

(a) 600K (b) 400K

(c) 300K (d) 500 K

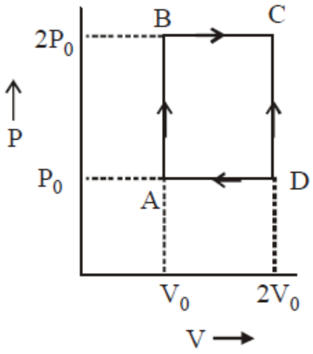
37. A Carnot's engine works as a refrigerator between 250 K and 300 K. It receives 500 cal heat from the reservoir at the lower temperature. The amount of work done in each cycle to operate the refrigerator is:

(a) 420J (b) 2100J

(c) 772J (d) 2520J

38. An engine operates by taking n moles of an ideal gas through the cycle ABCDA shown in figure. The thermal efficiency of the engine is : (Take C_v =1.5 R, where R is gas constant)

[Main Online April 8, 2017]



(a) 0.24 (b) 0.15

(c) 0.32 (d) 0.08

39. A Carnot freezer takes heat from water at 0°C inside it and rejects it to the room at a temperature of 27°C. The latent heat of ice is 336×10^3 J kg^{-1}. If 5 kg of water at 0°C is converted into ice at 0°C by the freezer, then the energy consumed by the freezer is close to:

(a) 1.51×10^5 J (b) 1.68×10^6 J

(c) 1.71×10^7 J (d) 1.67×10^5 J

40 A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two

ways:

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

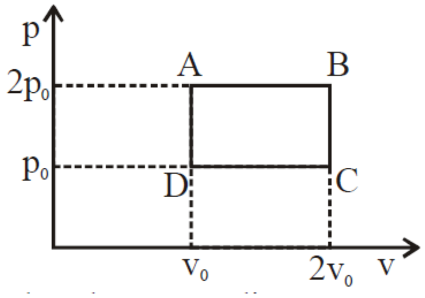
In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is :

(a) ln2, 2ln2
(b) 2ln2, 8ln2
(c) ln2, 4ln2
(d) ln2, ln2

41. A Carnot engine absorbs 1000 J of heat energy from a reservoir at 127°C and rejects 600 J of heat energy during each cycle. The efficiency of engine and temperature of sink will be:

(a) 20% and – 43°C
(b) 40% and – 33°C
(c) 50% and –20°C
(d) 70% and – 10°C

42.



The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monatomic gas. The amount of heat, extracted from the source in a single cycle is

(a) p_0v_0
(b) $\left(\frac{13}{2}\right)p_0v_0$
(c) $\left(\frac{11}{2}\right)p_0v_0$
(d) $4p_0v_0$

43. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0°C to ice, how much heat (in calories) is released to the surroundings at temperature 27°C (Latent heat of ice = 80 Cal/gram) to the nearest integer?

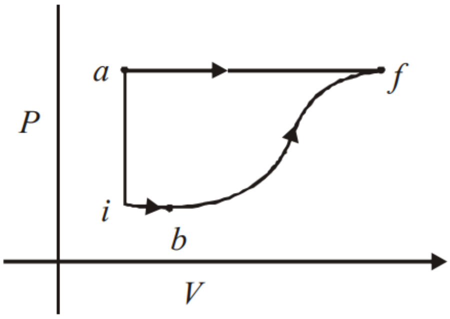
44. A Carnot engine operates between two reservoirs of temperatures 900 K and 300 K. The engine performs 1200 J of work per cycle. The heat energy (in J) delivered by the engine to the low temperature reservoir, in a cycle, is ________________.

Answer Key

1. (a)	2. (d)	3. (a)	4. (b)
5. (c)	6. (a)	7. (b)	8. (Bonus)
9. (b)	10. (c)	11. (b)	12. (c)
13. (b)	14. (d)	15. (b)	16. (b)
17. (b)	18. (a)	19. (c)	20. (a)
21. (a)	22. (a)	23. (b)	24. (d)
25. (c)	26. (b)	27. (a)	28. (46)
29. (1818K)	30. (a)	31. (c)	32. (d)
33. (b)	34. (b)	35. (b)	36. (d)
37. (a)	38. (b)	39. (d)	40. (d)
41. (b)	42. (b)	43. (8791)	44. (600)

Pro Test - I

1. A thermodynamic system is taken from an initial state i with internal energy U_i=100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} =200 j, W_{ib}=50 J and W_{bf}=100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf}, Q_{ib} and Q_{bf} respectively. If the internal energy of the system in the state b is U_b= 200 J and Q_{iaf}=500 J, The ratio $\frac{Q_{bf}}{Q_{ib}}$ is



2. An ideal monatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. The gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statements(s) is (are)

(a) If V_2 = $2V_1$ and T_2 = $3T_1$, then the energy stored in the spring is 1/4 P_1V_1
(b) If V_2 = $2V_1$ and T_2 = $3T_1$, then the change in internal energy is 3 P_1V_1
(c) If V_2 = $3V_1$ and T_2 = $4T_1$, then the work done by the gas is 7/3 P_1V_1
(d) If V_2 = $3V_1$ and T_2 = $4T_1$, then the heat supplied t the gas is 17/6 P_1V_1

3. One mole of a monatomic gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II give the characteristics in the cycle. Match them with each of the processes given in Column I.

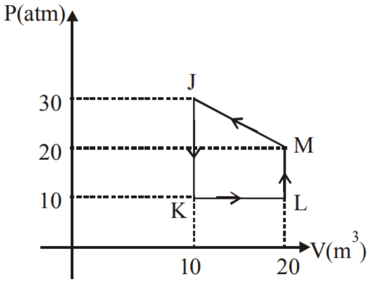
Column I	Column II
(A) Process A B	(p) Internal energy decreases
(B) Process B C	(q) Internal energy increases
(C) Process C D	(r) Heat is lost
(D) Process D A	(s) Heat is gained
	(t)Work is done on the gas

4. Column I contains a list of processes involving expansion of an ideal gas. Match this with Column II describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS.

Column I	**Column II**
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened.	(p) The temperature of the gas decreases.
(B) An ideal monoatomic gas expands to twice its remains original volume such that its pressure P ∞ $1/V^2$ where V is the volume of the gas	(q) The temperature of the gas increases or constant
(C) An ideal monoatomic gas expands to twice its original volume such that its	(r) The gas loses heat where V is its volume

pressure P ∞ $1/V^{4/3}$	
(D) An ideal monoatomic gas expands such that its pressure P and volume V follows the behavior shown in the graph	(s) The gas gains heat

5. Heat given to process is positive, match the following option of Column I with the corresponding option of column II:



Column I	**Column II**
(A) JK	(p) $\Delta W > 0$
(B) KL	(q) $\Delta Q < 0$
(C) LM	(r) $\Delta W < 0$
(D) MJ	(s) $\Delta Q > 0$

6. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure P_i = 10^5 Pa and volume V_i = $10^{-3} m^3$ changes to a final state at P_f = (1/32) ×10^5 Pa and V_f = 8× $10^{-3} m^3$ in an adiabatic quasi-static process, such that $P^3 V^5$ = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps: an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volumeV_f. The amount of heat supplied to the system in the two-step process is approximately

(a) 112 J	(b) 294 J
(c) 588 J	(d) 813 J

7. A diatomic ideal gas is compressed adiabatically to 1/32 of its initial volume. If the initial temperature of the gas is T_i(in Kelvin) an the final temperature is a T_i, the value of a is

8. Consider one mole of helium gas enclosed in a container at initial pressure P_1 and volumeV_1. It expands isothermally to volume becomes $32V_1$. The work done by the gas during isothermal and adiabatic expansion processes are $W_{iso}$and W_{adia} , respectively. If the ratio $\frac{W_{iso}}{W_{adia}}$ = f In 2, then f is______.

9. A spherical bubble inside water has radius R. Take the pressure inside the bubble and the water pressure to be p_0. The bubble now gets compressed radially in an adiabatic manner so that its radius becomes (R-a). For a<<R the magnitude of the work done in the process is given by, where X is a constant and $\gamma = C_p/C_v$ = 41/30. The value of X is ______.

10. A mixture of ideal gas containing 5 moles of monatomic gas and 1 mole of rigid diatomic gas is initially at pressureP_0 , volumeV_0, and temperatureT_0. If the gas mixture is adiabatically compressed to a volume $V_0/4$, then the correct statement(s) is/are,(Given $2^{1.2}$=2.3; $2^{3.2}$ 9.2; R is gas constant)

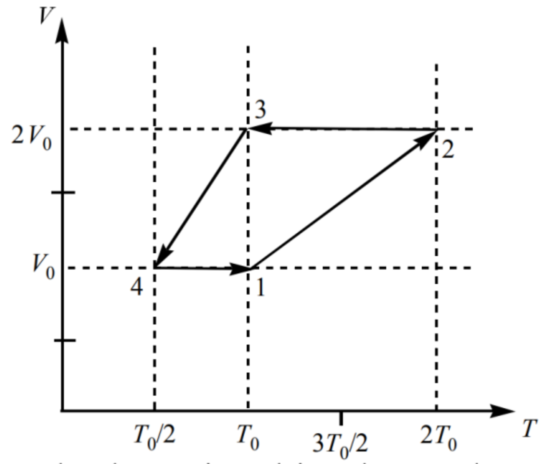
(a)The work |W| done during the process is $13RT_0$

(b)The final pressure of the gas m0069ture after compression is in between $9P_0$ and $10P_0$

(c)The average kinetic energy of the gas mixture aft4ee4r compression is in between $18RT_0$ and $19RT_0$

(d)Adiabatic constant of the gas mixture is 1.6

11. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are: [R is the gas constant



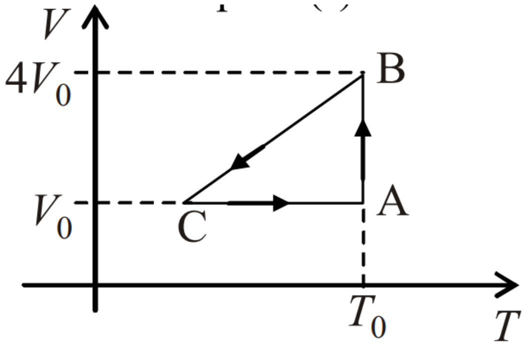
(a) Work done in this thermodynamic cycle
$(1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 1)$is $|W| = \frac{1}{2} RT_0$

(b) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(c) The ratio of heat transfer during processes 1 → 2 and 3 → 4 is $Q_{1\rightarrow2}/Q_{2\rightarrow3} = 5/3$

(d) The ratio of heat transfer during processes 1 → 2 and 3 → 4 is $Q_{1\rightarrow2}/Q_{2\rightarrow3} = 1/2$

12. One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following



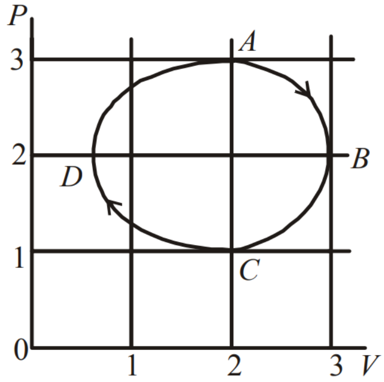
(a)Internal energies at A and B are the same

(b)Work done by the gas in process AB is P_0V_0 In 4

(c)Pressure at C is $\frac{P_0}{4}$

(d) Temperature at C is $\frac{T_0}{4}$

13. The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is an semi circle and CDA is half of an ellipse. Then,



(a) The process during the path $A \rightarrow B$ is isothermal

(b) Heat flows out of the gas during the path $B \rightarrow C \rightarrow D$

(c) Work done during the $A \rightarrow B \rightarrow C$ is zero

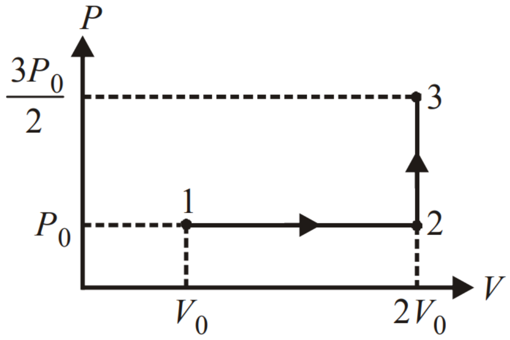
(d) Positive work is done by the gas in the cycle ABCDA

14. In a thermodynamic process on an ideal monatomic gas, the infinitesimal heat absorbed by the gas is given by

$T\Delta X$, where T is temperature of the system and ΔX is the infinitesimal change in a thermodynamic quantity X of the system. For a mole of monatomic ideal gas

X= $\frac{3}{2}$R in ($\frac{T}{T_A}$)+R In ($\frac{V}{V_A}$). Here R is gas constant, V is volume of gas T_A and $V$are constants. The List-I below gives some quantities involved in a process and List-II gives some possible values of these quantities.

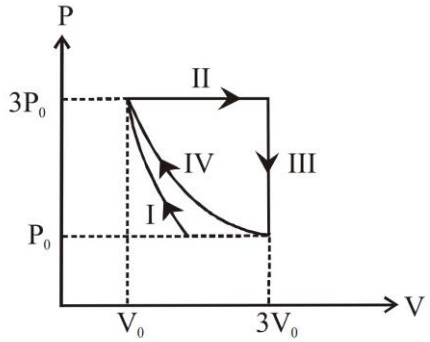
List-I	**List-II**
(1) Work done by the	(P) $\frac{1}{3}RT_0/ln2$
(2) System in process 1 → 2 → 3	
(3) Change in internal Energy in process 1 → 2 → 3	(Q) $\frac{1}{3}RT_0$
(4) Heat absorbed by The system in Process 1 → 2 → 3	(R) RT_0
(5) Heat absorbed By the system	(S) $\frac{4}{3}RT_0$
In process 1 → 2	(T) $\frac{1}{3}RT_0\,(3+ln2)$
	(U) $\frac{5}{6}RT_0$



If the process carried out on one mole of monatomic ideal gas is shown in the figure PV-diagram with $P_0V_0 = \frac{1}{3}RT_0$, the correct match is,

(a) $I \to Q, II \to R, III \to S, IV \to U$
(b) $I \to S, II \to R, III \to Q, IV \to T$
(c) $I \to Q, II \to R, III \to P, IV \to U$
(d) $I \to Q, II \to S, III \to R, IV \to U$

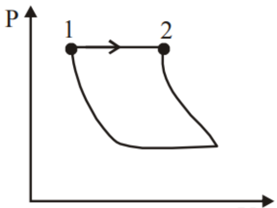
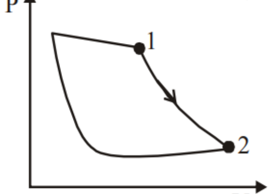
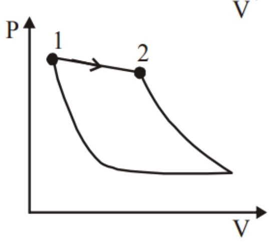
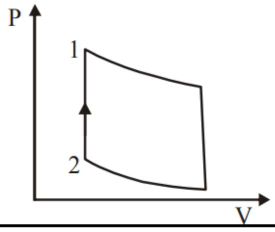
15. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List-II.



List-I	**List-II**
P. In process I	1. Work done by the gas is zero
Q. In process II	2. Temperature of the gas Remain unchanged
R. In process III	3. No heat is exchanged Between the gas and its Surroundings
S. In process IV	4. Work done by the gas is $6P_0V_0$

(a) $P \rightarrow 4;\ Q \rightarrow 3;\ R \rightarrow 1;\ S \rightarrow 2$
(b) $P \rightarrow 1;\ Q \rightarrow 3;\ R \rightarrow 2;\ S \rightarrow 4$
(c) $P \rightarrow 3;\ Q \rightarrow 4;\ R \rightarrow 1;\ S \rightarrow 2$
(d) $P \rightarrow 3;\ Q \rightarrow 4;\ R \rightarrow 2;\ S \rightarrow 1$

DIRECTIONS Q.NO.16, 17, and 18: By appropriately matching the information given in the three columns of the following table. An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P-V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic processes. Here Y is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas in n.

Column 1	**Column 2**	**Column 3**
(I) $W_{1\to 2} = \frac{1}{\gamma - 1}(P_2V_2 - P_1V_1)$	(i) Isothermal	(P) 
(II) $W_{1\to 2} = -PV_2 + PV_1$	(ii) Isochoric	(Q) 
(III) $W_{1\to 2} = 0$	(iii) Isobaric	(R) 
(IV) $W_{1\to 2} = -nRT\ \ln\left(\frac{V_2}{V_1}\right)$	(iv) Adiabatic	(S) 

16. Which one of the following options correctly represents a thermodynamic process that is used as a correction in the determination of the speed of sound in an ideal gas?

(a) (I)(ii)(Q) (b) (IV)(ii)(R)

(c) (III)(iv)(R) (d) (I)(iv)(Q)

17. Which one of the following options is the correct combination?

(a) (IV)(ii)(S) (b) (III)(ii)(S)

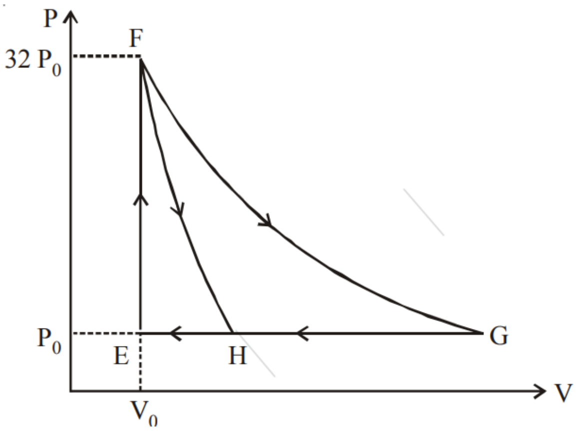
(c) (II)(iv)(P) (d) (II)(iv)(R)

18. Which one of the following options is the only correctly representation of a process in which $\Delta U = \Delta Q$ $P\Delta V$?

(a) (II)(iv)(R) (b) (III)(iii)(P)

(c) (II)(iii)(S) (d) (II)(iii)(P)

19. One mole of a monatomic ideal gas is taken along two cyclic processes $E \to F \to G \to E\ and\ E \to H \to E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic. Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists.



List I	**List II**
P. $G \rightarrow E$	1. $160P_0V_0\ In\ 2$
Q. $G \rightarrow H$	2. $36P_0V_0$
R. $F \rightarrow H$	3. $24P_0V_0$
S. $F \rightarrow G$	4. $31P_0V_0$

Codes:

P Q R S

(a) 4 3 2 1
(b) 4 3 1 2
(c) 3 1 2 4
(d) 1 3 2 4

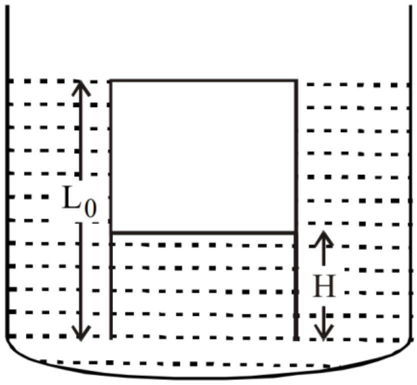
20. The piston is now pulled out slowly and held at a distance 2L from the top. The pressure in the cylinder between its top and the piston will then be

(a) P_0
(b) $\frac{P_0}{2}$
(c) $\frac{P_0}{2} + \frac{M_g}{pR2}$
(d) $\frac{P_0}{2} - \frac{M_g}{pR2}$

21. While the piston is at a distance 2L from the top, the hole at the top is sealed. The piston is then released, to a position where it can stay in equilibrium. In this condition, the distance of the piston from the top is

(a) $\{\frac{2P_0pR^2}{\pi R^2P_0+Mg}\}(2L)$
(b) $\{\frac{P_0pR^2-Mg}{\pi R^2P_0}\}(2L)$
(c) $\{\frac{P_0pR^2+Mg}{\pi R^2P_0}\}(2L)$
(d) $\{\frac{P_0pR^2}{\pi R^2P_0-Mg}\}(2L)$

22. The piston is taken completely out of the cylinder. The hole at the top is sealed. A water tank is brought below the cylinder and put in a position so that the water surface in the tank is at the same level as the top of the cylinder as shown in the figure. The density of the water is p. In equilibrium, the height H of the water column in the cylinder satisfies



(a) $pg(L_0 - H)^2 + P_0(L_0 - H) + L_0P_0 = 0$
(b) $pg(L_0 - H)^2 - P_0(L_0 - H) - L_0P_0 = 0$
(c) $pg(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0 = 0$
(d) $pg(L_0 - H)^2 - P_0(L_0 - H) + L_0P_0 = 0$

Answer Key

1.(2)
2.(a, b, c)
3.A – p,r,t; B – p, r; C – q, s; D – r, t

4.A- q; B-p, r; C- p, s; D- q, s.
5. A- q; B- p, s; C- s; D- r, q.
6.(c)
7.(4)
8.(1.78)
9.(2.05)
10.(a, b, d)
11.(a, c)
12.(a, b)
13.(b, d)
14.(a)
15.(c)
16.(d)
17.(b)
18.(d)
19.(d)
20.(a)
21.(d)
22.(c)

Solution:
Test Papers

Solutions: Beginner Test - I

1.

During driving, temperature of the gas increases while its volume remains constant. So, according to Charles' law, at constant V. p ∝ T. Therefore, pressure of gas increases.

2.

Because sudden compression of a gas is an adiabatic process. According to the first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$\because \Delta Q = 0$

$\therefore \Delta U = -\Delta W$

$\because \Delta W$ = negative because work done on the gas

Thus, ΔU increases and gas temperature rises.

3.

The conditions for an isothermal process are

(i) The walls should be diathermic.

(ii) The process should be quasi-static.

4.

First law of thermodynamics gives,

$\Delta Q = \Delta U + \Delta W$

For an isothermal expansion, $\Delta U = 0$.

So, $\Delta Q = \Delta W$

For an isothermal expansion of the ideal gas work done by the gas is equal to the heat given to the ideal gas.

5.

In opening of bottle, adiabatic expansion of gas causes lowering of temperature.

6.

Here, $m = 60$ kg, $g = 10$ m/s^2, $h = 10$ m

In going up and down once, number of kilocalories burnt $= (mgh + mgh/2) = \frac{3}{2}$ mgh

$$= \frac{3}{2} \times \frac{60 \times 10 \times 10}{4.2 \times 1000} = \frac{15}{7} kcal$$

Total number of kilocalories to be burnt for losing 5 kg of weight = 5 × 7000 = 35000 kcal

∴ Number of times of the person has to go up and down the stairs

$$= \frac{35000}{15/7} = \frac{35 \times 7}{15} \times 10^3 = 16.3 \times 10^3 \, times$$

7.

Here, mass of gas, $m = 2 \times 10^{-2}$ kg = 20 g

Rise in temperature, $\Delta T = 45°C$

Heat required, $\Delta Q = ?$

Molecular mass, $M = 28$

Number of moles, $n = \frac{m}{M} = \frac{20}{28} = 0.714$

As nitrogen is a diatomic gas, molar specific heat at constant pressure is

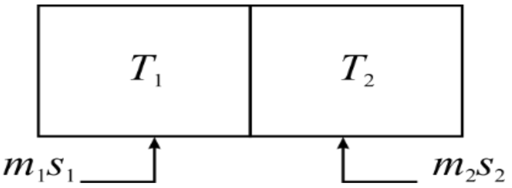
$$C_p = \frac{7}{2} R = \frac{7}{2} \times 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$$

As $\Delta Q = nC_p \Delta T$

$$\therefore \Delta Q = 0.714 \times \frac{7}{2} \times 8.3 \times 45 \text{ J} = 933.4 \text{ J}$$

8.

Let m_1 and m_2 are masses of bodies with specific heats s_1 and s_2, then if their temperature after they are in thermal equilibrium is T.



Then, if $T_1 > T > T_2$ and assuming no heat loss.

Heat lost by hot body = heat gained by cold

$m_1s_1 (T_1 - T) = m_2s_2 (T - T_2)$

$$\frac{m_1s_1T_1 + m_2s_2T_2}{m_1s_1 + m_2s_2} = T \quad \text{[equilibrium temperature]}$$

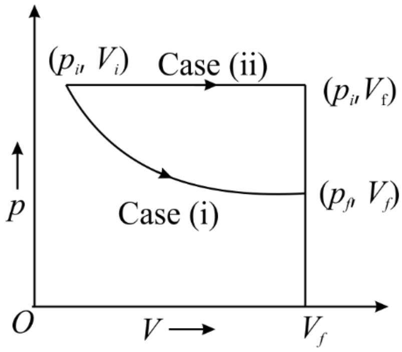
So for, bodies to settle down to mean temperature,

$m_1 = m_2$ and $s_1 = s_2$

means bodies have same specific heat and have equal

Then, $T = \frac{T_1 + T_2}{2}$ [mean temperature]

9.



The p-V diagram for each case is shown in the figure. In case (i) $p_iV_i = p_fV_f$, therefore process is isothermal. In case (ii), pressure is constant, work done = area under the p-V curve, so work done is more when the gas expands at constant pressure.

10.

(i) ∵ process is adiabatic

$\therefore \Delta Q = 0$

Work done on the gas, $\Delta W = -500J$

According to the first law of thermodynamics.

$\Delta Q = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W = 500J$

(ii) This is because heat absorbed by a substance (coolant) is directly proportional to the specific heat of the substance.

(iii) This is because in a harbour town, the relative humidity is more than in a desert town. Hence, the climate of a harbour town is without extremes of hot and cold.

11.

Given, work done (W) = – 22.3 J

Work done is taken negative as work is done on the system.

In an adiabatic change, $\Delta Q = 0$

Using first law of thermodynamics,

$\Delta U = \Delta Q - W = 0 - (-22.3) = 22.3$ J

For another process between states A and B,

Heat absorbed $(\Delta Q) = +9.35$ cal

$= +(9.35 \times 4.19)$ J $= +39.18$ J

Change in internal energy between two states via different paths are equal.

$\therefore \Delta U = 22.3$ J

∴ From first law of thermodynamics,

$\Delta U = \Delta Q - W$

or $W = \Delta Q - \Delta U$

$= 39.18 - 22.3 = 16.88$ J ≈ 16.9J

12.

Consider (say μ mole) an ideal gas, which is undergoing an adiabatic expansion. Let the gas expands by an infinitesimally small volume dV, at pressure p, then the infinitesimally small work done given by dW = pdV

The net work done from an initial volume V_1 to final volume V_2 is given by

$$W = \int_{V_1}^{V_2} pdV$$

For an adiabatic process, pV^γ = constant = K

$$p = \frac{K}{V^\gamma} = KV^{-\gamma}$$

$$\therefore W = \int_{V_1}^{V_2} \left(KV^{-\gamma}\right) dV = K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$= \frac{KV_2^{-\gamma+1} - KV_1^{-\gamma+1}}{(1-\gamma)}$$

For an adiabatic process,

$$K = p_1V_1^\gamma = p_2V_2^\gamma$$

$$W = \frac{p_2V_2^\gamma \cdot V_2^{-\gamma+1} - p_1V_1^\gamma \cdot V_2^{-\gamma+1}}{(1-\gamma)}$$

$$= \frac{1}{(1-\gamma)}\left(p_2V_2 - p_1V_1\right)$$

For an ideal gas, $p_1V_1 = \mu RT_1$ and $p_2V_2 = \mu RT_2$. So, we have

$$W = \frac{1}{(1-\gamma)}\left[\mu RT_2 - \mu RT_1\right]$$

$$= \frac{\mu R}{(\gamma-1)}\left[T_1 - T_2\right]$$

13.

(i) Work done by the gas (let $pV^{1/2} = A$)

$$\Delta W = \int_{V_1}^{V_2} pdV = A\int_{V_1}^{V_2} \frac{dV}{\sqrt{V}} = A\left[\frac{\sqrt{V}}{1/2}\right]_{V_1}^{V_2}$$

$$= 2A\left(\sqrt{V_2} - \sqrt{V_1}\right)$$

$$= 2p_1V_1^{1/2}\left[V_2^{1/2} - V_1^{1/2}\right]$$

(ii) Since, $T = pV/nR = \frac{A}{nR} \cdot \sqrt{V}$ T

Thus, $\frac{T_2}{T_1} = \sqrt{\frac{V_2}{V_1}} = \sqrt{2}$

(iii) Then, the change in internal energy

$\Delta U = U_2 - U_1 = \frac{3}{2}R(T_2 - T_1) = \frac{3}{2}RT_1(\sqrt{2} - 1)$

$\Delta W = 2A\sqrt{V_1}(\sqrt{2} - 1) = 2RT_1(\sqrt{2} - 1)$

According to first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$\Delta Q = (7/2)RT_1(\sqrt{2} - 1)$

14.

Slope of p = f(V), curve at $(V_0\ p_0) = f(V_0)$

Slope of adiabat at (V_0, p_0)

$$= K(-\gamma)V_0^{-1-\gamma} = -\gamma p_0/V_0$$

Now, heat absorbed in the process p = f(V)

$dQ = dU + DW = nC_V dt + pdV$

Since, $T = (1/nR)\, pV = (1/nR)\, V f(V)$

$\therefore dT = (1/nR)\, [f(V) + V f'(V)]\, dV$

Thus, $\left.\frac{dQ}{dV}\right|_{V=V_0} = \frac{C_V}{R}\left[f(V_0)+V_0 f'(V_0)\right]+f(V_0)$

$$=\left[\frac{1}{\gamma-1}+1\right]f(V_0)+\frac{V_0 f'(V_0)}{\gamma-1}$$

$$=\frac{\gamma}{\gamma-1}p_0+\frac{V_0}{\gamma-1}f'(V_0)$$

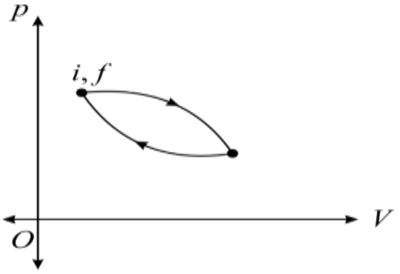
Heat is absorbed when dQ/dV > 0 when gas expands, that is when

$$\gamma p_0 + V_0 f'(V_0) > 0$$

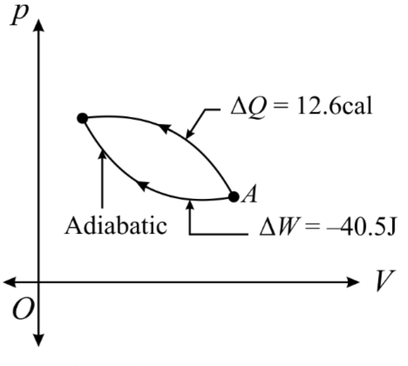
$$f'(V_0) > -\gamma p_0 / V_0$$

15.

A cyclic process restores the system back to its initial state after completion of the cycle.



As internal energy is a state function, so its value at initial point is same as that at final point (initial state is same as that of final state).



For adiabatic process A → B,

ΔQ = 0, so by first law of thermodynamics,

$\Delta Q = \Delta W + \Delta U$

or $\Delta U = -\Delta W = -(-40.5) = 40.5$ J

For another process, A → B,

$\Delta Q = +12.6$ cal

$= 12.6 \times 4.19$ J $= 52.8$ J

and by first law of thermodynamics,

$\Delta Q = \Delta W + \Delta U$

$\Delta W = \Delta Q - \Delta U$

$\Delta W = 52.8 - 40.5 = 12.3$ J

[as ΔU is a state function, ΔU_{AB} is same for this process also]

16.

Here, heat removed is less than the heat supplied and hence the room become hotter.

17.

No, a ship cannot be moved in a sea by the use of the energy of sea water because refrigerator is against the second law of thermodynamics.

18.

All sorts of dissipative forces, e.g. force of friction, viscous drag, electrical resistance, non-elasticity, thermal radiation, convection etc., make a real process irreversible.

19.

The coefficient of performance of a refrigerator depends on the temperatures of source and sink.

20.

Efficiency of Carnot engine, $\eta = 1-\frac{T_2}{T_1}$

$$=1-\frac{273K}{373K}=\frac{100}{373}$$

= 0.268 = 26.8%

21.

According to second law of thermodynamics, heat energy cannot converted into work completely.

22.

As we know, $\frac{Q_2}{Q_1}=\frac{T_2}{T_1}=\frac{3}{5}$

$$\because 1-\frac{Q_2}{Q_1}=1-\frac{T_2}{T_1}$$

$$\frac{Q_1-Q_2}{Q_1}=\frac{500-300}{500} \qquad [\because Q_1 - Q_2 = W]$$

$$\frac{W}{Q_1}=\frac{2}{5}$$

$$\therefore Q_1 = 10^3 \times \frac{5}{2} = 2500 \text{ J}$$

23.

(i) Because in the hilly area, temp of surrounding is lower than that of plains.

As $\eta = 1 - \frac{T_2}{T_1}$

(ii) No, it is not constant quantity, as inside, temperature of the refrigerator decreases, it is coefficient of performance also decreases.

(iii) According to the second law of thermodynamics whole of the heat cannot be converted completely into work. Some part of the heat that is not converted into work is released by the engine to the atmosphere (as sink).

Thus, thermal pollution will always occur.

24.

Given, $T_1 = 600$ K, $T_2 = 200$ K

Efficiency of Carnot engine,

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{600 - 200}{600}$$

$$= \frac{400}{600} = \frac{2}{3} = 66\%$$

$$\text{Real efficiency} = \frac{\text{Energy output}}{\text{Energy input}}$$

$$= \frac{2}{1 \times 4.2} = 0.47 = 47\%$$

$$\frac{\text{Real efficiency}}{\text{Carnot engine efficiency}}$$

$$= \frac{47}{66} = 0.71$$

25.

The efficiency of a heat engine is stated as the ratio of the net work done by the heat engine and heat absorbed by the working substance.

Suppose a heat engine absorbs Q_1 heat from the hot reservoir and gives Q_2 heat rejected to the colder reservoir. So, the work done by the working substance is

$W = Q_1 - Q_2$

So, efficiency of heat engine

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

26.

Efficiency of a Carnot engine is given by

$$\eta = \left(1 - \frac{T_2}{T_1}\right)$$

where, T_2 = temperature of sink

and T_1 = temperature of sink source.

So, for η = 1 or 100%, $T_2 = 0$ K or heat is rejected into a sink at 0 K temperature.

27.

Let the initial temperatures of hot and cold reservoirs were T_1 and T_2. The efficiency of the Carnot engine is given by

So, initially $\eta = \frac{T_1 - T_2}{T_1}$ ... (i)

As given the temperature of both the reservoirs is raised by equal amount t, so $T_1' = T_1 + t$ and $T_2' = T_2 + t$. The final efficiency of the Carnot engine will be

$$\eta' = \frac{T_1' - T_2'}{T_1'} = \frac{(T_1 + t) - (T_2 + t)}{(T_1 + t)}$$

$$= \frac{T_1 - T_2}{(T_1 + t)} \quad \text{... (ii)}$$

Dividing Eq. (ii) by Eq. (i), we have

$$\frac{\eta'}{\eta} = \frac{\left(\frac{T_1 - T_2}{T_1 + t}\right)}{\left(\frac{T_1 - T_2}{T_1}\right)} = \frac{T_1}{T_1 + t} \quad \text{... (iii)}$$

As $\eta' = \eta$, i.e. the efficiency of Carnot engine decreases.

28.

Here, $Q_2 = 250$ Js^{-1}

$T_2 = -23°C = -23 + 273 = 250$ K

$T_1 = 25°C = 25 + 273 = 298$ K

We know, $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\therefore \quad W = \frac{Q_2 (T_1 - T_2)}{T_2}$$

$$W = \frac{250(298 - 250)}{250} = \frac{250 \times 48}{250}$$

W = 48 Js^{-1}

29.

(i) Waterfall- The falling of water cannot be reversible process. During the water fall, its potential energy convert into kinetic energy of the water.

On striking the ground, some part of potential energy converts into heat and (sound not possible that heat the sound). In nature, it automatically convert the kinetic energy and potential energy so that the water will rise back so waterfall is not a reversible process.

(ii) Rusting of iron- In rusting of iron, the iron become oxidised with the oxygen of the air as it is a chemical reaction, it cannot be reversed.

30.

W = work done by engine = $Q_1 - Q_2$

and W = work done supplied to refrigerator

$= Q_3 - Q_4$

$Q_1 - Q_2 = Q_3 - Q_4$

Dividing by Q_1, $1 - \frac{Q_2}{Q_1} = \frac{Q_3}{Q_1} - \frac{Q_4}{Q_1}$

$$\frac{Q_3}{Q_1} = 1 - \frac{Q_2}{Q_1} + \frac{Q_4}{Q_1}$$

$$= 1 - \frac{T_2}{T_1} + \frac{T_4}{T_1}$$

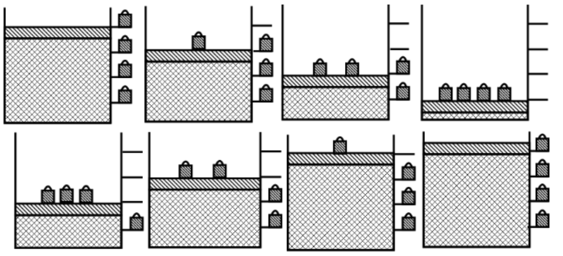
$$= \frac{T_1 - T_2 + T_4}{T_1}$$

31.

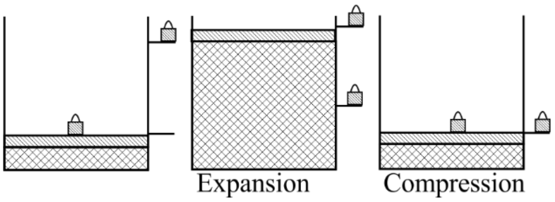
A reversible process must pass through equilibrium states which are very close to each other so that when process is reversed, it passes back through these equilibrium states.

Then, it is again decompressed or it passes through same equilibrium states, system can be restored to its initial state without any change in surroundings.

e.g. If a gas is compressed as shown



But a reversible process can proceeds without reaching equilibrium in intermediate states.



32.

For engine A, $T_1 = 800$ K, $T_2 = T$ K

Efficiency, $\eta_A = 1 - \frac{T_2}{T_1} = 1 - \frac{T}{800}$

Also, $\frac{Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{T}{800}$

Work output, $W_A = Q_1 - Q_2 = \eta_A \times Q_1$

$$\left[\because \eta_A = 1 - \frac{Q_2}{Q_1}\right]$$

Or $W_A = \left(1 - \frac{T}{800}\right)Q_1$

For engine B, $T_1' = TK, \; T_2' = 300K$

Efficiency, $\eta_B = 1 - \frac{T_2'}{T_1'} = 1 - \frac{300}{T}$

Work output,

$$W_B = Q_1' - Q_2' = \eta_B \times Q_1' = \left(1 - \frac{300}{T}\right)Q_1'$$

Since, the engine B absorbs the heat rejected by the engine A, so

$$Q_1' = Q_2 \qquad \therefore W_B = \left(1 - \frac{300}{T}\right)Q_2$$

(i) When outputs of the two engines are equal,

$W_A = W_B$

Or $\left(1 - \frac{T}{800}\right)Q_1 = \left(1 - \frac{300}{T}\right)Q_2$

Or $\left(1 - \frac{T}{800}\right) = \left(1 - \frac{300}{T}\right)\frac{Q_2}{Q_1}$

$$= \left(1 - \frac{300}{T}\right)\frac{T}{800}$$

On solving, we get T = 550 K

(ii) When the efficiencies are equal, $\eta_A = \eta_B$

Or $1 - \frac{T}{800} = 1 - \frac{300}{T}$

Or $T^2 = 24 \times 10^4$

$\therefore$ T = 489.9 K

Solutions: Beginner Test - II

1.

Water is flowing at a rate of 3.0 litre/min.

The geyser heats the water, raising the temperature from 27°C to 77°C.

Initial temperature, T_1 = 27°C

Final temperature, T_2 = 77°C

$\therefore$Rise in temperature, $\Delta T = T_2 - T_1$

= 77 – 27= 50°C

Heat of combustion = 4×10^4 J/g

Specific heat of water, c = 4.2 J g^{-1} °c^{-1}

Mass of flowing water, m = 3.0 litre/min = 3000 g/min

Total heat used, ΔQ = mc ΔT

= 3000 × 4.2 × 50

= 6.3 × 10^5 J/min

∴Rate of consumption =

$\frac{6.3\times10^5}{4\times10^4} = \frac{15.75g}{min}$

2.

Mass of nitrogen, m = 2.0 × 10^{-2}kg = 20 g

Rise in temperature, ΔT = 45°C

Molecular mass of N_2, M = 28

Universal gas constant, R = 8.3 J mol^{-1} K^{-1}

Number of moles, n=$\frac{m}{M} = \frac{2.0\times10^{-2}\times10^{3}}{28} = 0.714$

Molar specific heat at constant pressure for nitrogen

$$C_p = \frac{7}{2}R = \frac{7}{2}\times 8.3 = 29.05 Jmol^{-1}K^{-1}$$

The total amount of heat to be supplied is given by the relation:

ΔQ = nC_P ΔT

= 0.714 × 29.05 × 45

= 933.38 J

Therefore, the amount of heat to be supplied is 933.38 J.

3.

a) When two bodies at different temperaturesT_1 and T_2 are brought in thermal contact, heat flows from the body at the higher temperature to the body at the lower temperature till equilibrium is achieved, i.e., the temperatures of both the bodies become equal. The equilibrium temperature is equal to the mean temperature (T_1 + T_2)/2.

only when the thermal capacities of both the bodies are equal.

b) The coolant in a chemical or nuclear plant should have a high specific heat. This is because higher the specific heat of the coolant, higher is its heat-absorbing capacity and vice versa. Hence, a liquid having a high specific heat is the best coolant to be used in a nuclear or chemical plant. This would prevent different parts of the plant from getting too hot.

c) When a car is in motion, the air temperature inside the car increases because of the motion of the air molecules. According to Charles' law, temperature is directly proportional to pressure. Hence, if the temperature inside a tyre increases, then the air pressure in it will also increase.

d) A harbour town has a more temperate climate (i.e., without the extremes of heat or cold) than a town located in a desert at the same latitude. This is because the relative humidity in a harbour town is more than it is in a desert town.

4.

The cylinder is completely insulated from its surroundings. As a result, no heat is exchanged between the system (cylinder) and its surroundings. Thus, the process is adiabatic.

Initial pressure inside the cylinder = P_1

Final pressure inside the cylinder = P_2

Initial volume inside the cylinder = V_1

Final volume inside the cylinder = V_2

Ratio of specific heats, γ = 1.4

For an adiabatic process, we have:

$$P_1V_1^y = P_2V_2^y$$

The final volume is compressed to half of its initial volume.

$$\therefore V_2 = \frac{V_1}{2}$$

$$P_1(V_1)^y = P_2\left(\frac{V_1}{2}\right)^y$$

$$\frac{P_2}{P_2} = \frac{(V_1)^y}{\left(\frac{V_1}{2}\right)^y} = (2)^y = (2)^{1.4} = 2.639$$

Hence, the pressure increases by a factor of 2.639.

5.

The work done (W) on the system while the gas changes from state A to state B is 22.3 J. This is an adiabatic process. Hence, change in heat is zero.

∴ΔQ = 0

ΔW = –22.3 J (Since the work is done on the system)

From the first law of thermodynamics, we have:

ΔQ = ΔU + ΔW

Where,

ΔU = Change in the internal energy of the gas

∴ΔU = ΔQ – ΔW = – (– 22.3 J)

ΔU = + 22.3 J

When the gas goes from state A to state B via a process, the net heat absorbed by the system is:

$\Delta Q = 9.35$ cal $= 9.35 \times 4.19 = 39.1765$ J

Heat absorbed, $\Delta Q = \Delta U + \Delta Q$

$\therefore \Delta W = \Delta Q - \Delta U$

$= 39.1765 - 22.3$

$= 16.8765$ J

Therefore, 16.88 J of work is done by the system.

6.

a) 0.5 atm

b) Zero

c) Zero

d) No

Explanation:

a) The volume available to the gas is doubled as soon as the stopcock between cylinders A and B is opened. Since volume is inversely proportional to pressure, the pressure will decrease to one-half of the original value. Since the initial pressure of the gas is 1 atm, the pressure in each cylinder will be 0.5 atm.

b) The internal energy of the gas can change only when work is done by or on the gas. Since in this case no work is done by or on the gas, the internal energy of the gas will not change.

c) Since no work is being done by the gas during the expansion of the gas, the temperature of the gas will not change at all.

d) The given process is a case of free expansion. It is rapid and cannot be controlled. The intermediate states do not satisfy the gas equation and since they are in nonequilibrium states, they do not lie on the P-V-T surface of the system.

7.

Work done by the steam engine per minute, $W = 5.4 \times 10^8$ J

Heat supplied from the boiler, $H = 3.6 \times 10^9$ J

Efficiency of the engine $= \frac{Output\ energy}{Input\ energy}$

$$\therefore \eta = \frac{W}{H} = \frac{5.4 \times 10^8}{3.6 \times 10^9} = 0.15$$

Hence, the percentage efficiency of the engine is 15 %.

Amount of heat wasted $= 3.6 \times 10^9 - 5.4 \times 10^8$

$= 30.6 \times 10^8 = 3.06 \times 10^9$ J

Therefore, the amount of heat wasted per minute is 3.06×10^9 J.

8.

Heat is supplied to the system at a rate of 100 W.

$\therefore$Heat supplied, Q = 100 J/s

The system performs at a rate of 75 J/s

$\therefore$Work done, W = 75 J/s

From the first law of thermodynamics, we have:

$Q = U + W$

Where, U = Internal energy

$\therefore U = Q - W$

$= 100 - 75$

$= 25$ J/s

$= 25$ W

Therefore, the internal energy of the given electric heater increases at a rate of 25 W.

9.

Total work done by the gas from D to E to F = Area of ΔDEF

Area of ΔDEF $= \frac{1}{2} DE \times EF$

Where,

DF = Change in pressure

$= 600\ N/m^2 - 300\ N/m^2$

$= 300\ N/m^2$

FE = Change in volume

$= 5.0\ m^3 - 2.0\ m^3$

$= 3.0\ m^3$

Area of ΔDEF $= \frac{1}{2} 300 \times 3 = 450$J

Therefore, the total work done by the gas from D to E to F is 450 J.

10.

Temperature inside the refrigerator,

$T_1 = 9^oC = 282K$

Room temperature, $T_2 = 36^oC = 309K$

Coefficient of performance $= T_1 / (T_2 - T_1)$

$= 282/(309-282) = 10.44$

Therefore, the coefficient of performance of the given refrigerator is 10.44.

Solutions: Expert Test - I

1.

(b) Given, mass of water, m = 1g

Volume of 1g of water $= 1cm^3 = 10^{-6}\ m^3$

Volume of 1g of steam $= 1671\ cm^3$

$= 1671 \times 10^{-6}\ m^3$

Pressure, $p = 1 \times 10^5$ Pa

Latent heat of vaporization of water,

L = 2256 J/g

Change in volume,

$\Delta V = (1671 - 1) \times 10^{-6}\ m^3$

$= 1670 \times 10^{-6}\ m^3$ … (i)

Heat supplied,

$\Delta Q = mL = 1 \times 2256 = 2256\ J$ ... (ii)

As the steam expands, so the work done in expansion is

$\Delta W = p\Delta V$

$= 1 \times 10^5 \times 1670 \times 10^{-6}$ [from Eq.(i)]

$= 167\ J$ ... (iii)

According to first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

$\Delta U = \Delta Q - \Delta W$

$= 2256 - 167$ [from Eq. (ii) and (iii)]

$= 2089\ J$

2.

(b) According to the question,

Heat spent during the conversion of sample of water at 100°C to steam is,

$\Delta Q = 54\ cal = 54 \times 4.18\ J$

$= 225.72\ J$

Normal pressure, $p = 1.013 \times 10^5\ Nm^{-2}$

Net work done during the conversion would be given as

$\Delta W = p\Delta V$

$= p\,[V_{steam} - V_{water}]$

Here, $V_{steam} = 167.1\ cc = 167.1 \times 10^{-6}\ m^3$

$V_{water} = 0.19 = 0.1cc = 0.1 \times 10^{-6}\ m^3$

$\therefore \Delta W = 1.013 \times 10^5\,[(167.1 - 0.1) \times 10^{-6}]$

$= 1.013 \times 167 \times 10^{-1}$

$= 16.917\ J$

Now, by the first law of thermodynamics,

$\Delta Q = \Delta U + \Delta W$

Where, ΔU is the change in internal energy of the sample.

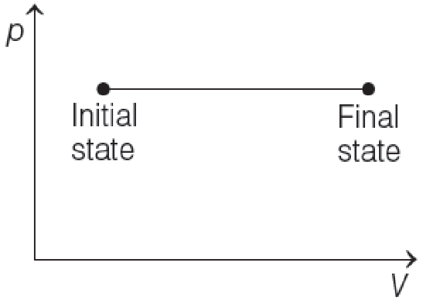
$\Delta U = \Delta Q - \Delta W$

Substituting the values in the above equation, we get

$\Delta U = 225.72 - 16.917 = 208.7\ J$

3.

(c) pV-diagram for an ideal gas is given as



From the pV diagram, it is clear that pressure of ideal gas in the piston cylinder is constant during thermodynamic process. Hence, this process is isobaric.

4.

(a) Since, the entire system is thermally insulated. Thus, their will be no transfer of heat between the system and the surrounding. Hence, when the stop cock is suddenly opened, a sudden process of expansion will takes place with no heat transfer. Thus, the given process will be adiabatic, as it is a process that occurs without transferring heat or mass between a thermodynamic system and its surroundings.

Hence, correct option is (a).

5.

(a) In an adiabatic process, the system is completely insulated from the surroundings. Thus, heat is neither absorbed nor released by the system to the surroundings. So, $\Delta Q = 0$.

Sudden processes are adiabatic like bursting of cycle tyre, etc.

If the pressure of gas is kept constant, then the process is called isobaric, i.e. $\Delta p = 0$.

If the temperature of the system remains constant, then it is called isothermal process, i.e. $\Delta T = 0$.

If the volume of gas is constant in a system, then it is called isochoric process, i.e. $\Delta V = 0$.

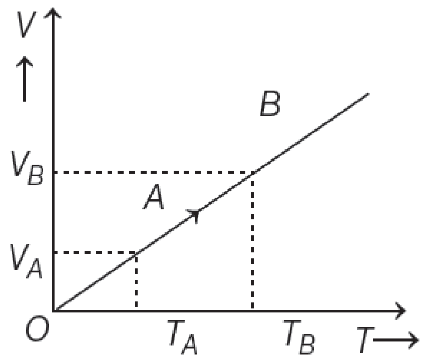
6.

(c) According to the given graph,

Volume (V) ∝ Temperature (T)

Or $\frac{V}{T}$ = constant

Thus, the process is isobaric.



∴ Work done, $\Delta W = p\Delta V$

$= nR\Delta T = nR(T_B - T_A)$ ... (i)

Heat absorbed $\Delta Q = nC_p \, \Delta T$

$= nC_p\,(T_B - T_A)$ ...(ii)

As, $C_p = \dfrac{\gamma R}{\gamma - 1}$, $\left[where,\ \gamma = 1 + \dfrac{2}{f} \right]$

For a monoatomic gas, f = 3

$$C_p = \left(R + \frac{3}{2}R \right) = \frac{5}{2}R$$

Substituting the value of C_p in Eq. (ii), we get

$$\Delta Q = n\left(\frac{5}{2}R\right)(T_B - T_A)$$

Hence, $$\frac{\Delta W}{\Delta Q} = \frac{nR(T_B - T_A)}{n\left(\frac{5}{2}R\right)(T_B - T_A)} = \frac{2}{5}$$

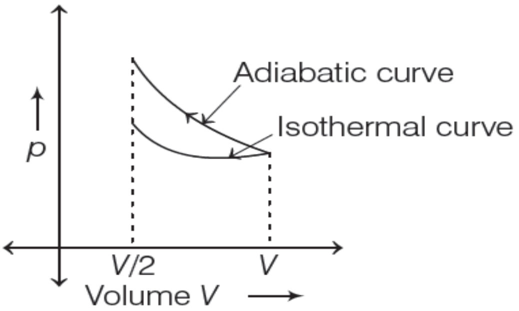
7.

(b) In isochoric process, the curve is parallel to y-axis because volume is constant. Isobaric is parallel to x-axis because pressure is constant. Along the curve, it will be isothermal because temperature is constant.

So, P → C ⇒ Q → a ⇒ R → d ⇒ S → b

8.

(a) The solution of this question can be understood by plotting a p-V graph for the compression of a gas isothermally and adiabatically simultaneously to half of its initial volume. i.e.



Since, the isothermal curve is less steeper than the adiabatic curve. So, area under the p-V curve for adiabatic process has more magnitude than isothermal curve. Hence, work done in adiabatic process will be more than in isothermal process.

9.

(b) According to question,

Net work done = Area enclosed in pv curve i.e. ΔABC

Area of ΔABC

$= \frac{1}{2} \times 5 \times 10^{-3} \times 4 \times 10^{5}$ J

$= 10^3$ J = 1000 J

10.

(b) Efficiency of Carnot engine is given as

$$\eta = 1 - \frac{T_2}{T_1}$$

where, T_2 = temperature of sink

and T_1 = temperature of source.

Hence, η depends upon the temperature of source and sink both.

11.

(c) Efficiency of an ideal heat engine is given as

$$\eta = 1 - \frac{T_2}{T_1}$$

Where, T_1 is the temperature of the source and T_2 is the temperature of the sink.

Here, $T_2 = 100 + 273 = 373$K

$T_2 = 0 + 273 = 273$ K

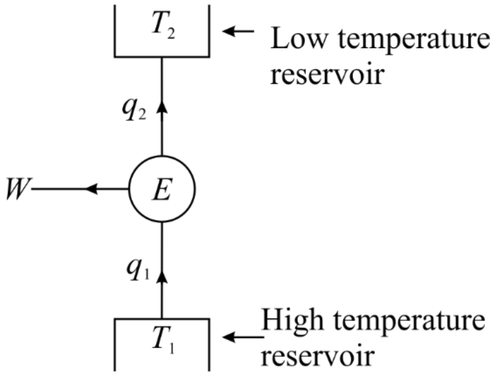
$$\eta = 1 - \frac{273}{373}$$

$$= \frac{373 - 273}{373} = \frac{100}{373} = 0.268$$

∴ η% = 0.268 × 100 = 26.8%

12.

Consider schematic diagram for a Carnot engine as shown below.



In case of engine,

Engine efficiency = $\frac{work}{heat\,absorbed} = \frac{W}{q_1}$

$\therefore \frac{W}{q_1} = \frac{1}{10}$

$\frac{10J}{q_1} = \frac{1}{10}$ or q_1 = 100 J

When this engine is reversed, it takes in work W and heat q_2 from cold reservoir and ejects 100 J of heat to hot reservoir.

$\therefore$ W + q_2 = q_1

10 + q_2 = 100 or q_2 = 90 J

13.

(b) Given, temperature of source,

T = 30°C = 30 + 273 ⇒ T_1 =303 K

Temperature of sink, T_2 = 4°C = 4 + 273

T_2 = 277 K

As, we know that

$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \Rightarrow \frac{Q_2 + W}{Q_2} = \frac{T_1}{T_2}$ {∵ W =$Q_1 - Q_2$}

Where Q_2 is the amount of heat drawn from the sink (at T_2), W is work done on working substance, Q_1 is amount of heat rejected to source (at room temperature T_1).

$WT_2 + T_2Q_2 = T_1Q_2$

$WT_2 = T_1Q_2 - T_2Q_2$

$WT_2 = Q_2 (T_1 - T_2)$

$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

$$W = 600 \times 4.2 \times \left(\frac{303}{277} - 1\right)$$

$$W = 600 \times 4.2 \times \left(\frac{26}{277}\right)$$

W = 236.5 Joules

Power = $\frac{Work\,done}{Time} = \frac{W}{t} = \frac{236.5}{5}$

= 236.5 W

14.

(b) For a refrigerator, we know that

$$\frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$$

where,

Q_1 = amount of heat delivered to the room

W = electrical energy consumed

T_1 = room temperature = t_1 + 273

T_2 = temperature of sink = t_2 + 273

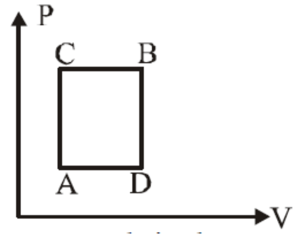
$$\therefore \frac{Q_1}{1} = \frac{t_1 + 273}{t_1 + 273 - (t_2 - 273)}$$

$$Q_1 = \frac{t_1 + 273}{t_1 - t_2}$$

Solutions: Expert Test - II

1.

(a)ΔU remains same for both paths ACB and ADB



$\Delta Q_{ACB} = \Delta W_{ACB} + \Delta U_{ACB}$

60 J = 30 J + ΔU_{ACB}

U_{ACB} = 30 J

$\therefore \Delta U_{ADB} = \Delta U_{ACB}$ = 30 J

$\Delta Q_{ADB} = \Delta U_{ADB} + \Delta W_{ADB}$

= 10 J + 30 J = 40 J

2.

(d) Volume of water does not change, no work is done on or by the system (W = 0)

According to first law of thermodynamics

Q = dU + W

For Isochoric process Q = dU

ΔU = μcdT = 2 × 4184 × 20 = 16.7 kJ.

3.

(a) As we know,

ΔQ = Δu + Δw (Ist law of thermodynamics)

ΔQ = Δu + PΔv

or 150 = Δu + 100(1 – 2)

= Δu – 100

$\therefore \Delta u = 150 + 100 = 250$ J

Thus the internal energy of the gas increases by 250 J

4.

(b)Temperature change ΔT is same for all three processes $A \rightarrow B$; $A \rightarrow C$ and $A \rightarrow D$

$\Delta U = nC_v\Delta T$ = same

$E_{AB} = E_{AC} = E_{AD}$

Work done, $W = P \times \Delta V$

$AB \rightarrow$ volume is increasing $\Rightarrow W_{AB} > 0$

$AD \rightarrow$ volume is decreasing $\Rightarrow W_{AD} < 0$

$AC \rightarrow$ volume is constant $\Rightarrow W_{AC} = 0$

5.

(c) In adiabatic process

PV^{γ} = constant

$$\therefore P\left(\frac{m}{\rho}\right)^{\gamma} = \text{constant} \quad \left(\because V = \frac{m}{\rho}\right)$$

As mass is constant

$$\therefore P \propto \rho^{\gamma}$$

If P_i and P_f be the initial and final pressure of the gas and ρ_i and ρ_f be the initial and final density of the gas. Then

$$\frac{P_f}{P_i} = \left(\frac{\rho_f}{\rho_i}\right)^{\gamma} = (32)^{7/5}$$

$$\frac{nP_i}{P_i} = \left(2^5\right)^{7/5} = 2^7$$

$n = 2^7 = 128$.

6.

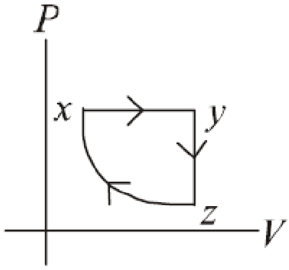
(a) From the corresponding V-T graph given in question,

Process $xy \rightarrow$ Isobaric expansion,

Process $yz \rightarrow$ Isochoric (Pressure decreases)

Process $zx \rightarrow$ Isothermal compression

Therefore, corresponding PV graph is as shown in figure



7.

(b) Given, $V_1 = 1$ litre, $P_1 = 1$ atm

$V_2 = 3$ litre, $\gamma = 1.40$,

Using, PV^{r} = constant

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$P_2 = P_1 \times \left(\frac{1}{3}\right)^{1.4} = \frac{1}{4.6555}\,atm$$

$$\therefore \text{Work done, } W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$= \frac{\left(1\times1 - \frac{1}{4.6555}\times 3\right)1.01325\times10^{5}\times10^{-3}}{0.4}$$

= 90.1 J

Closest value of W = 90.5 J

8.

We know that Relaxation time,

$$\tau = \frac{V}{\sqrt{T}} \qquad \text{.......(i)}$$

Equation of adiabatic process is

$TV^{\gamma-1}$ = constant

$$T \propto \frac{1}{V^{\gamma-1}}$$

$\tau \propto V^{1+\frac{\gamma-1}{2}}$ using (i)

$$\tau \propto V^{\frac{\gamma+1}{2}}$$

$$\frac{\tau_f}{\tau_i} = \left(\frac{2V}{V}\right)^{\frac{1+\gamma}{2}} = (2)^{\frac{1+\gamma}{2}}$$

9.

(b) $\Delta U_{ac} = -(\Delta U_{ca}) = -(-180) = 180$ J

$Q = 250 + 60 = 310$ J

Now $Q = \Delta U + W$

Or $310 = 180 + W$

Or $W = 130$ J

10.

(c) As the process is isochoric so,

$$Q = nc_v\Delta T = \frac{67.2}{22.4}\times\frac{3R}{2}\times 20$$

$= 90R = 90 \times 8.31$

$\simeq 748$ J.

11.

(b) We have given,

$$P = P_0\left[1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$$

When $V_1 = V_0$

$$P_1 = P_0\left[1-\frac{1}{2}\right] = \frac{P_0}{2}$$

When $V_2 = 2V_0$

$$P_2 = P_0\left[1-\frac{1}{2}\left(\frac{1}{4}\right)\right] = \left(\frac{7P_0}{8}\right)$$

$$\Delta T = T_2 - T_1 = \left|\frac{P_1V_1}{nR} - \frac{P_2V_2}{nR}\right| \quad \left[\because T = \frac{PV}{nR}\right]$$

$$\Delta T = \left|\left(\frac{1}{nR}\right)(P_1V_1 - P_2V_2)\right|$$

$$= \left(\frac{1}{nR}\right)\left|\left(\frac{P_0V_0}{2} - \frac{7P_0V_0}{4}\right)\right|$$

$$= \frac{5P_0V_0}{4nR} = \frac{5P_0V_0}{4R} \qquad (\because n = 1)$$

12.

(c) Internal energy depends only on initial and final state

So, $\Delta U_A = \Delta U_B$

Also $\Delta Q = \Delta U + W$

As $W_A > W_B$

$\Delta Q_A > \Delta Q_B$

13.

(b) Suppose amount of water evaporated be M gram.

Then (150 – M) gram water converted into ice.

So, heat consumed in evaporation = Heat released in fusion

$M \times L_v = (150 - M) \times L_s$

$M \times 2.1 \times 10^6 = (150 - M) \times 3.36 \times 10^5$

M = 20 g

14.

(d) a → Isobaric, b → Isothermal, c → Adiabatic, d → Isochoric

15.

(b) Total work done by the gas during the cycle is equal to area of triangle ABC.

$$\therefore \Delta W = \frac{1}{2}\times 4 \times 5$$

= 10 J

16.

(b) Equation of adiabatic change is

$TV^{\gamma-1}$ = constant

Put $\gamma = \frac{7}{5}$, we get: $\gamma - 1 = \frac{7}{5} - 1$

$$\therefore x = \frac{2}{5}$$

17.

(b) Work done,

$W = P\Delta V = nR\Delta T$

$$= \frac{1}{2}\times 8.31 \times 70$$

≃ 291 J

18.

(a) Equation of the BC

$$P = P_0 - \frac{2P_0}{V_0}(V - 2V_0)$$

Using PV = nRT

$$\text{Temperature, } T = \frac{P_0V - \dfrac{2P_0V^2}{V_0} + 4P_0V}{1\times R}$$

(∵ n = 1 mole given)

$$T = \frac{P_0}{R}\left[5V - \frac{2V^2}{V_0}\right]$$

$$\frac{dT}{dV} = 0 \Rightarrow 5 - \frac{4V}{V_0} = 0$$

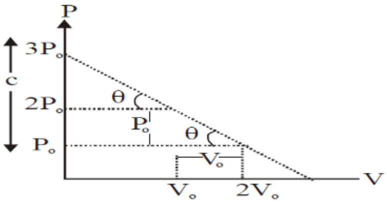
$$V = \frac{5}{4}V_0$$

$$T = \frac{P_0}{R}\left[5\times\frac{5V_0}{4} - \frac{2}{V_0}\times\frac{25}{16}V_0^2\right]$$

$$= \frac{25}{8}\frac{P_0V_0}{R}$$

19.

(c) The equation for the line is



$$P = \frac{-P_0}{V_0}V + 3P \qquad \left[\text{slope} = \frac{-P_0}{V_0}, \ c = 3P_0\right]$$

$PV_0 + P_0V = 3P_0V_0$ ... (i)

But $pV = nRT$

$$\therefore P = \frac{nRT}{V} \qquad \text{... (ii)}$$

From (i) & (ii)

$$\frac{nRT}{V}V_0 + P_0V = 3P_0V_0$$

∴ $nRTV_0 + P_0V^2 = 3P_0V_0V$ ... (iii)

For temperature to be maximum $\frac{dT}{dV} = 0$

Differentiating e.q. (iii) by 'V' we get

$$nRV_0\frac{dT}{dV} + P_0\left(2V\right) = 3P_0V_0$$

$$\therefore\ nRV_0\frac{dT}{dV} = 3P_0V_0 - 2P_0V$$

$$\frac{dT}{dV} = \frac{3P_0V_0 - 2P_0V}{nRV_0} = 0$$

$$V = \frac{3V_0}{2}$$

$\therefore\ P = \frac{3P_0}{2}$ [From (i)]

$\therefore\ T_{\max} = \frac{9P_0V_0}{4nR}$ [From (iii)]

20.

(a) Efficiency of heat engine is given by

$$h = \frac{w}{Q} = 1 - \frac{C_V}{C_P} = \frac{R}{C_p} = \frac{R}{\frac{5R}{2}} = \frac{2}{5}$$

(∵ $C_p - C_v = R$)

For monatomic gas $C_P = \frac{5}{2}R$.

21.

(a) $\tau = \frac{1}{\sqrt{2}\pi d^2\left(\frac{N}{V}\right)\sqrt{\frac{3RT}{M}}}$

$$\tau \propto \frac{V}{\sqrt{T}}$$

As, $TV^{\gamma-1} = K$

So, $\tau \propto V^{\frac{\gamma+1}{2}}$

Therefore, $q = \frac{\gamma+1}{2}$

22.

(a) As, $P = \frac{1}{3}\left(\frac{U}{V}\right)$

But $\frac{U}{V} = KT^4$

So, $P = \frac{1}{3}KT^4$

or $\frac{uRT}{V} = \frac{1}{3}KT^4$ [As PV = uRT]

$\frac{4}{3}\pi R^3T^3$ = constant

Therefore, $T \propto \frac{1}{R}$

23.

(b) In VT graph

ab-process : Isobaric, temperature increases.

bc process : Adiabatic, pressure decreases.

cd process : Isobaric, volume decreases.

da process : Adiabatic, pressure increases.

The above processes correctly represented in P-V diagram (b).

24.

(d) In cyclic process, change in total internal energy is zero.

$\Delta U_{cyclic} = 0$

$$\Delta U_{BC} = nC_v\Delta T = 1\times\frac{5R}{2}\Delta T$$

Where, C_v = molar specific heat at constant volume.

For BC, $\Delta T = -200$ K

∴ $\Delta U_{BC} = -500R$

25.

(c) Given : work done, W = 830 J

No. of moles of gas, $\mu = 2$

For diatomic gas $\gamma = 1.4$

Work done during an adiabatic change

$$W = \frac{\mu R\left(T_1 - T_2\right)}{\gamma - 1}$$

$$830 = \frac{2\times 8.3\left(\Delta T\right)}{1.4-1} = \frac{2\times 8.3\left(\Delta T\right)}{0.4}$$

$$\Delta T = \frac{830\times 0.4}{2\times 8.3}$$

= 20 K

26.

(b) Volume of the gas

$v = \frac{m}{d}$ and

Using PV^{γ} = constant

$$\frac{P'}{P} = \frac{V}{V'} = \left(\frac{d'}{d}\right)^{\gamma}$$

Or $128 = (32)^{\gamma}$

$$\therefore\ \gamma = \frac{7}{5} = 1.4$$

27.

(a) Initially, Volume $V_1 = 5.6l$, temperature = T_1

and $\gamma = \frac{5}{3}$ (for monoatomic gas)

The number of moles of gas, $n = \frac{5.6l}{22.4l} = \frac{1}{4}$

Finally (after adiabatic compression)

$V_2 = 0.7l$

For adiabatic process

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\therefore\ T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T_1\left(\frac{5.6}{0.7}\right)^{\frac{5}{3}-1}$$

$= T_1(8)^{2/3} = 4T_1$

Work done in adiabatic process

$$W = \frac{nR\Delta T}{\gamma-1} = \frac{\frac{1}{4}R\left[T_1 - 4T_1\right]}{\left[\frac{5}{3}-1\right]}$$

$$= \frac{9}{8}RT_1$$

28.

For adiabatic process,

$TV^{\gamma-1}$ = constant

or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_1 = 20°C + 273 = 293$ K,

$V_2 = \frac{V_1}{10}$ and $\gamma = \frac{7}{5}$

$$T_1(V_1)^{\gamma-1} = T_2\left(\frac{V_1}{10}\right)^{\gamma-1}$$

$$293 = T_2\left(\frac{1}{10}\right)^{2/5}$$

$T_2 = 293(10)^{2/5} \simeq 736$ K

$\Delta T = 736 - 293 = 443$ K

During the process, change in internal energy

$$\Delta U = NC_V\Delta T = 5\times\frac{5}{2}\times 8.3\times 443$$

$\simeq 46 \times 10^3$ J = X kJ

$\therefore$ X = 46.

29.

For an adiabatic process,

$TV^{\gamma-1}$ = constant

$$\therefore\ T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$T_2 = (300)\times\left(\frac{V_1}{\frac{V_1}{16}}\right)^{1.4-1}$$

$T_2 = 300 \times (16)^{0.4}$

Ideal gas equation, PV = nRT

$$\therefore\ V = \frac{nRT}{P}$$

V = kT (since pressure is constant for isobaric process)

So, during isobaric process

$V_2 = kT_2$ ... (i)

$2V_2 = kT_f$ ... (ii)

Dividing (i) and (ii)

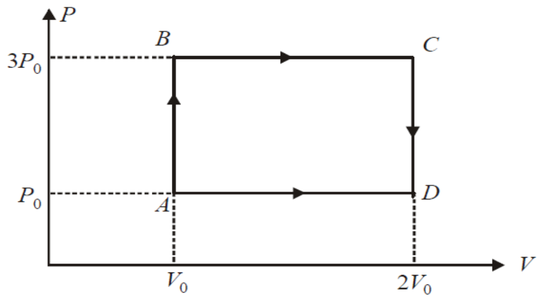
$$\frac{1}{2} = \frac{T_2}{T_f}$$

$T_f = 2T_2 = 300 \times 2 \times (16)^{0.4}$

= 1818 K

30.

(a)



From the figure,

Work, $W = 2P_0V_0$

Heat given, $Q_{in} = W_{AB} + W_{BC}$

$$= n\cdot C_V\Delta T_{AB} + nC_P\Delta T_{BC}$$

$$= n\frac{3R}{2}\left(T_B - T_A\right) + \frac{n5R}{2}\left(T_C - T_B\right)$$

$$\left(\because C_v = \frac{3R}{2}\, and\, C_P = \frac{5R}{2}\right)$$

$$= \frac{3}{2}\left(P_BV_B - P_AV_A\right) + \frac{5}{2}\left(P_CV_C - P_BV_B\right)$$

$$= \frac{3}{2}\times\left[3P_0V_0 - P_0V_0\right] + \frac{5}{2}\left[6P_0V_0 - 3P_0V_0\right]$$

$$= 3P_0V_0 + \frac{15}{2}P_0V_0 = \frac{21}{2}P_0V_0$$

Efficiency, $\eta = \frac{W}{Q_{in}} = \frac{2P_0V_0}{\frac{21}{2}P_0V_0} = \frac{4}{21}$

$$\eta\% = \frac{400}{21} \approx 19.$$

31.

(c) Efficiency, $\eta = \frac{Work\, done}{Heat\, absorbed} = \frac{W}{\sum Q}$

$$= \frac{Q_1 + Q_2 + Q_3 + Q_4}{Q_1 + Q_3} = 0.5$$

Here, Q_1 = 1915 J, Q_2 = – 40 J and Q_3 = 125 J

$$\therefore\ \frac{1915 - 40 + 125 + Q_4}{1915 + 125} = 0.5$$

1915 – 40 + 125 + Q_4 = 1020

Q_4 = 1020 – 2000

Q_4 = – Q = – 980 J

Q = 980 J

32.

(d) For carnot refrigerator

Efficiency = $\frac{Q_1 - Q_2}{Q_1}$

Where,

Q_1 = heat lost from surrounding

Q_2 = heat absorbed from reservoir at low temperature.

Also, $\frac{Q_1 - Q_2}{Q_1} = \frac{w}{Q_1}$

$$\frac{1}{10} = \frac{w}{Q_1}$$

Q_1 = w × 10 = 100 J

So, $Q_1 - Q_2 = w$

$Q_2 = Q_1 - w$

100 – 10 = Q_2 = 90 J

33.

(b) Let Q_H = Heat taken by first engine

Q_L = Heat rejected by first engine

Q_2 = Heat rejected by second engine

Work done by 1st engine = work done by 2nd engine

$W = Q_H - Q_L = Q_L - Q_2$

$2Q_L = Q_H + Q_2$

$$2 = \frac{\theta_H}{\theta_L} + \frac{\theta_2}{\theta_L}$$

Let T be the temperature of cold reservoir of first engine.

Then in carnot engine.

$\frac{Q_H}{Q_L} = \frac{T_1}{T}$ and $\frac{Q_L}{Q_2} = \frac{T}{T_2}$

$2 = \frac{T_1}{T} + \frac{T_2}{T}$ using (i)

$2T = T_1 + T_2$

$$T = \frac{T_1 + T_2}{2}$$

34.

(b) Using, $n = 1 - \frac{T_2}{T_1}$

$$n = \frac{1}{6} = 1 - \frac{T_2}{T_1}$$

and $\frac{2}{3} = 1 - \frac{T_2 - 62}{T_1}$

On solving, we get

T_1 = 99°C and T_2 = 37°C

35.

(b) According to question, $\eta_1 = \eta_2 = \eta_3$

$$\therefore\ 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

[$\because$ Three engines are equally efficient]

$$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$T_2 = \sqrt{T_1T_3}$ ... (i)

$T_3 = \sqrt{T_2T_4}$ ... (ii)

From (i) and (ii)

$T_2 = \left(T_1^2T_4\right)^{1/3}$ $T_3 = \left(T_1T_4^2\right)^{1/3}$

36.

(d) $\eta_A = \dfrac{T_1 - T_2}{T_1} = \dfrac{w_A}{Q_1}$

and, $\eta_B = \dfrac{T_2 - T_3}{T_2} = \dfrac{w_B}{Q_2}$

According to question,

$W_A = W_B$

$\therefore \dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2} \times \dfrac{T_2 - T_3}{T_1 - T_2} = \dfrac{T_1}{T_2}$

$\therefore T_2 = \dfrac{T_1 + T_3}{2}$

$= \dfrac{600+400}{2}$

= 500 K

37.

(a) Given: Temperature of cold body,

T_2 = 250 K

Temperature of hot body; T_1 = 300 K

Heat received, Q_2 = 500 cal work done, W = ?

Efficiency = $1 - \dfrac{T_2}{T_1} = \dfrac{W}{Q_2 + W}$

$1 - \dfrac{250}{300} = \dfrac{W}{Q_2 + W}$

$W = \dfrac{Q_2}{5} = \dfrac{500 \times 4.2}{5} J$

= 420 J

38.

(b) Work-done (W) = P_0V_0

According to principle of calorimetry

Heat given = $Q_{AB} = Q_{BC}$

$= nC_VdT_{AB} + nC_PdT_{BC}$

$= \dfrac{3}{2}\left(nRT_B - nRT_A\right) + \dfrac{5}{2}\left(nRT_C - nRT_B\right)$

$= \dfrac{3}{2}\left(2P_0V_0 - P_0V_0\right) + \dfrac{5}{2}\left(4P_0V_0 - 2P_0V\right)$

$= \dfrac{13}{2}P_0V_0$

Thermal efficiency of engine

$(\eta) = \dfrac{W}{Q_{given}} = \dfrac{2}{13}$

= 0.15

39.

(d) ΔH = mL = 5 v 336 × 10^3 = Q_{sink}

$\dfrac{Q_{\sin k}}{Q_{source}} = \dfrac{T_{\sin k}}{T_{source}}$

$\therefore Q_{source} = \dfrac{T_{source}}{T_{\sin k}} \times Q_{\sin k}$

Energy consumed by freezer

$W_{output} = Q_{source} - Q_{sink} = Q_{sink}\left(\dfrac{T_{source}}{T_{\sin k}} - 1\right)$

Given: T_{source} = 27°C + 273 = 300K,

T_{sink} = 0°C + 273 = 273 k

$W_{output} = 5 \times 336 \times 10^3 \left(\dfrac{300}{273} - 1\right)$

= 1.67 × 10^5 J

40.

(d) The entropy change of the body in the two cases is same as entropy is a state function.

41.

(b) Given: Q_1 = 1000 J

Q_2 = 600 J

T_1 = 127°C = 400 K

T_2 = ?

η = ?

Efficiency of carnot engine,

$\eta = \dfrac{W}{Q_1} \times 100\%$

Or, $\eta = \dfrac{Q_2 - Q_1}{Q_1} \times 100\%$

Or, $\eta = \dfrac{1000 - 600}{1000} \times 100\%$

η = 40%

Now, for carnot cycle $\dfrac{Q_2}{Q_1} = \dfrac{T_2}{T_1}$

$\dfrac{600}{1000} = \dfrac{T_2}{400}$

$T_2 = \dfrac{600 \times 400}{1000}$

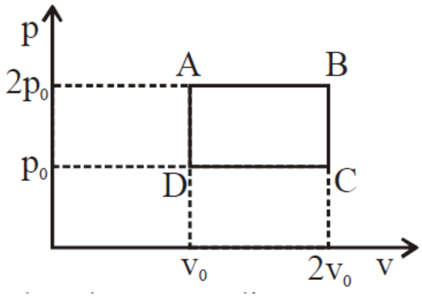
= 240 K

= 240 – 273

∴ T_2 = – 33°C

42.

Heat is extracted from the source would be in path DA and AB.



Thus heat supplied

= $nC_V(2T_0\text{-}T_0)+nC_P(4T_0\text{-}T_0)$

$$\frac{3nRT_0}{2}+\frac{5nR(2T_0)}{2}$$

$$=\frac{13nRT_0}{2}=\frac{13}{2}P_0V_0$$

43.

Given,

Heat absorbed, $Q_2 = mL = 80 \times 100 = 8000$ Cal

Temperature of ice, $T_2 = 273$ K

Temperature of surrounding,

$T_1 = 273 + 27 = 300$ K

Efficiency =

$$\frac{w}{Q_2}=\frac{Q_1-Q_2}{Q_2}=\frac{T_1-T_2}{T_2}=\frac{300-273}{273}$$

$$\frac{Q_1-8000}{8000}=\frac{27}{273}$$

$Q_1 = 8791$ Cal

44.

Given;

$T_1 = 900$ K, $T_2 = 300$K, W = 1200 J

Using, $1-\frac{T_2}{T_1}=\frac{W}{Q_1}$

$$1-\frac{300}{900}=\frac{1200}{Q_1}$$

$$\frac{2}{3}=\frac{1200}{Q_1}$$

$Q_1 = 1800$

Therefore heat energy delivered by the engine to the low temperature reservoir, $Q_2 = Q_1 - W$ = 1800 – 1200 = 600. 00 J

Solutions: Pro Test - I

1.

Applying first law of thermodynamics to path iaf

$Q_{iaf} = \Delta U_{iaf} + W_{iaf}$

$500 = \Delta U_{iaf} + 200$

$\Delta U_{iaf} = 300$ J

Now,

$U_{iaf} = \Delta U_{iaf} + W_{ib} + W_{bf} = 300 + 50 + 100$

$Q_{ib} + Q_{bf} = 450$ J

Also $Q_{ib} = \Delta U_{ib} + W_{ib}$

$Q_{ib} = 100 + 5\ 0 + 150$ J ...(ii)

From eq. (i) & (ii) $\frac{Q_{bf}}{Q_{ib}}=\frac{Q_{ibr-}Q_{ib}}{Q_{ib}}=\frac{300}{150}=2$

2.

(a, b, c)

Let spring is compressed by x on heating the gas.

(a) As gas is ideal monatomic,

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_1}$$

Force on spring by gas = kx

$P_2 = P_1 + \frac{kx}{\text{A}}$ (A = area of cross – section of piston) …(ii)

When $V_2 = 2V_1$, $T_2 = 3T_1$

$$\frac{P_1V_1}{T_1}=\frac{P_2\times(2V_1)}{3T_1}\Longrightarrow \quad P_2=\frac{3}{2}P_1$$

Putting this value of P_2 in eqn. (ii) we get

$$\frac{3}{2}P_1= P_1+\frac{kx}{\text{A}}\Longrightarrow \quad kx=\frac{P_1A}{2}$$

$$x=\frac{V_2-V_1}{\text{A}}=\frac{2V_1-V_1}{\text{A}}=\frac{V_1}{\text{A}}$$

Energy stored in the spring

$$=\frac{1}{2}kx^2=\frac{1}{2}(kx)(x)=\frac{P_1V_1}{4}$$

(b) Change in internal energy,

$$\text{U}=\frac{f}{2}(P_2V_2 - P_1V_1)=\frac{3}{2}(\frac{3}{2}P_1\times 2V_1-P_1V_1)$$

(c) Again, when $V_2 = 3V_1$ and $T_2 = 4T_1$ then

From eqn. (i),

$$\frac{P_1V_1}{T_1}=\frac{P_2\,(3V_1)}{4T_1}\Longrightarrow \quad P_2=\frac{4}{3}P_1$$

$$x=\frac{V_2-V_1}{\text{A}}=\frac{2V_1}{\text{A}}$$

Form eqn. (ii),

$$\frac{4}{3}P_1 = P_1+\frac{kx}{\text{A}}\Longrightarrow \quad kx=\frac{P_1A}{2}$$

Work done by gas = Work done by gas on at

Energy stored I spring.

$W_8 = P_1 \mathrm{A}x + \frac{1}{2}kx^2 = P_1(2V_1) + \frac{1}{2}(\frac{P_1A}{3})(\frac{2V_1}{A})$

$= 2P_1V_1 + \frac{1}{3}P_1V_1 = \frac{7}{3}P_1V_1$

$(d)\ \Delta Q = W_8 + \Delta U$

$= \frac{7}{3}P_1V_1 + \frac{3}{2}(P_2V_2 - P_1V_1)$

$= \frac{7}{3}P_1V_1 + \frac{3}{2}(\frac{4}{3}P_1 \times 3V_1 - P_1V_1)$

3.

A-p, r, t; B-p, r’ C-q, s; D-r, t

(A) Process A→ B

This is an isobaric process, P = constant and volume (V) of the gas decreases. Therefore, work is done on the gas.

W = (3V-V) = 2PV

Also V decreases so temperature at B decreases

∴ Internal energy U decreases.

From, Q = U + W as U and W decreases so Q decreases that means heat is lost.

(B) Process B → C

This is an isochoric process V = constant pressure decreases

P ∝ T is temperature also decreases.

W=0; ΔU = negative so ΔQ negative

Hence heat is lost.

(C) Process C → D

This is isobaric, Pressure P = constant V increases and V $\propto T$ so T increases. Hence $\Delta W, \Delta U$ and $\Delta Q + ve$ so heat gained by the gas.

(D) Process D → A

Applying PV= nRT

for D P (9V) = 1R $T_D \therefore T_D = \frac{9PV}{R}$

for A 3P (3V) = 1R $T_A \therefore T_A = \frac{9PV}{R}$

i.e., the process is isothermal

$\therefore \Delta U = 0$

Now, $\Delta Q = \Delta U + W$

$\therefore \Delta Q = W.$

As volume decrease in this process so W negative i.e., wow done on the gas and ΔQ negative hence heat is lost.

4.

(A) (q): As the ideal gas expands in vacuum, and the container is insulated therefore W = 0 & Q = 0 and according to first law of thermodynamics

ΔU = Q + W $\rightarrow \Delta U = 0$

Hence there is no change in the temperature of the gas of T is constant.

(B)- (p, r): Given P $\propto \frac{1}{V^2} or, PV^2 = constant$

Or, nRTV = constant

∴ V × T = constant

As the gas expands its volume increases so temperature decreases

We know that Q = n C Δ T …(i)

For a polytrophic process

C = C + R/1-n and PV^n = constant

Here PV^2 = Constant. ∴ n = 2

∴ C = $C_V + \frac{R}{1-2} = C_V - R = \frac{3}{2}R - R = \frac{R}{2}$

Q = nC$\Delta T = n \times \frac{R}{2} \times \Delta T.$

i.e. ΔT is negative, Q is negative so heat is lost.

As volume increases, so temperature decreases given

(C)-(p, s) P $\propto \frac{1}{V^{\frac{4}{3}}} \rightarrow$ P$V^{\frac{4}{3}}$ = constant.

∴ n = 4/3.

∴ C = $C_V + \frac{R}{1-4/3} = \frac{3}{2}R + \frac{3R}{-1} = \frac{3}{2}R - 3R = \frac{-3R}{2}$

∴ Q = n$C_V 9\Delta T = n(-\frac{3R}{2})\Delta t$

As ΔT is negative, Q is positive. So gas gains heat.

(D)- (q, s): From PV = nRT → $T = \frac{PV}{nR}\ PV\ increase\ So$ T increases volume increases so W increases.

From Q = $\Delta U + W, Q$ increases.

Hence the gas gains heat.

5.

From the given P-V graph, In process J → K volume,

V = constant p is decreasing and P ∝ T Therefore, T should also decrease.

∴ W = Pdv = 0, ΔV = ΔQ < 0 (negative)

From $\Delta Q = \Delta U + \Delta W$

In process K → L, p = constant = n$C_V \Delta T\ V$ is increasing So temperature should also increase.

∴ ΔW = Pdv > 0, $nC_V\ \Delta T$ >0 and

$Q = mC\Delta T > 0$

In process L → $M\ V = constant\ P\ in\ crease\ to\ T\ increases$

∴ W = 0, ΔU > 0 and Q > 0

In process M → J

V is decreasing $\therefore\ \Delta W < 0$

T is also decreasing $\therefore\ \Delta U < 0\ and\ \Delta Q < 0$

6.

(c) $\therefore$ PV^r = constant

$P^3V^5 = constant \rightarrow PV^{5/3}$ = constant $\therefore \gamma = 5/3$

For the two step process

$W = P_i\ (V_f - V_i) = 10^5(7 \times 10^{-3}) = 7 \times 10^2 J$

$\Delta U = \frac{f}{2}(P_f V_f - P_i V_i) = \frac{1}{\gamma - 1}\left(\frac{1}{4} \times 10^2 - 10^2\right)$

$\Delta U = \frac{3}{2}.\frac{3}{4} \times 10^2 = -\frac{9}{8}10^2$ J

From first law of threnody mimics

$Q - W = \Delta U$

$$\therefore\ Q = 7 \times 10^2 - \frac{9}{8} \times 10^2 = \frac{47}{8} \times 10^2 \text{ J} = 588 \text{ J}$$

7.

(4) For an adiabatic process, applying $TV^{\gamma-1}$ = constant

$T_1 v_1^{\gamma-1} = T_2 1 v_1^{\gamma-1} \Rightarrow T_1 = T_2\ (\frac{V2}{V1})^{\gamma-1}$

γ = 1.4 (for diatomic gas) $V_2 = \frac{V1}{32}$, $T_1 = T_i$, $T_2 = aT_i$

$\therefore T_i = aT_i\ [\frac{1}{32}]^{0.4} = \frac{1}{4}aT_i\ \therefore a = 4$

8.

(1.77 to 1.78)

For monatomic gas , $\gamma\ \frac{5}{3}$

In adiabatic process

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$\Rightarrow \frac{P1}{4}(4V_1)^{5/3} = P_2(32V_1)^{5/3}$

$\Rightarrow P_2 = \frac{P1}{4}(1/8)^{5/3} = \frac{P1}{128}$

And $W_{adi} = P_1V_1 - P_2V_2/\gamma - 1$

9.

$W = (\Delta P)_{avg}\ x\ 4\pi R^2 a$

$= \left|\frac{dP}{2}.4\pi R^2 a\right|$

for small change $(\Delta P)_{avg} <$ P $>$ arthemetic mean

$PV^{\gamma} = c$

$\Rightarrow dP = -\gamma\frac{P}{V}dV = -\frac{\gamma P_0}{V}4\pi R^2 a$

$= \frac{\gamma P_0}{2V}\ x\ 4\pi R^2 a\ x\ 4\pi R^2 a$

$= \frac{\gamma P_0}{2x4\pi R^3}\ x\ 4\pi R^2 a\ x\ 4\pi R^2 a$

$= (4pRP\ x\ a^2)\frac{3\gamma}{2}$

$\therefore x = 2.05$

10.

(a, b, d)

Adiabatic constant of the gas mixture,

$$\gamma_m = \frac{n_1C_{p1} + n_2C_{p2}}{n_1C_{p1} + n_2C_{p2}} = \frac{5\times\frac{5R}{2}+1\times\frac{7R}{2}}{5\times\frac{3R}{2}+1\times\frac{5R}{2}} = 1.6$$

For an adiabatic process, $P_v\gamma$ =Constant

$$\therefore P = P_0\left(\frac{V_0}{V}\right)^{1.6} = P_0(4)^{1.6}$$

$= P_0(2^2)^{1.6} = P_0 2^{3.2} = 9.2P_0$

Work done during the process,

$$W = \frac{P_2V_2 - P_1V_1}{1 - \gamma} = \frac{9.2P_0 \times \left(\frac{V_0}{4}\right) - P_0V_0}{1 - 1.6}$$

$$= -\frac{13P_0V_0}{6}$$

But $P_0V_0 = 6RT_0(as\ n = 5 + 1 = 6)$

$$\therefore W = -\frac{13(6RT_0)}{6} = -13RT_0 \therefore |W| = 13RT_0$$

The average K.E. of the gas mixture,

$K.E = nCV_{mi} \times T_2$

From $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

Or, $T_2 = T_1(2)^{\frac{6}{5}} = 23T_0$

$\therefore K.E = nCV_{mi} \times T_2 = 23RT_0$

11.

The P-V graph of the given V-T graph is given below.

(a) Work done during cyclic process (1→2→3→4 →1) W

= area enclosed in the loop $= \frac{P^0}{2}V^0$

$\therefore P_0V_0 = nRT_0$ $\therefore \frac{P_0V_0}{2} = \frac{nRT_0}{2}$

$\therefore$ Work done $W = \frac{nRT_0}{1} = \frac{RT_0}{2}$ [as n = 1]

(b) Process 1→2 is isobaric

Process 2→ 3 is isobaric

Process 3→ 4 is isobaric

Process 4 → 1 is isobaric

Hence no adiabatic process is involved.

(c) $\left|\Delta Q_{1\to2}\right| = \left|nC_p\Delta T\right| = \left|nC_p(2T_0 - T_0)\right| = \left|nC_pT_0\right|$

$\left|\Delta Q_{2\to3}\right| = \left|\Delta U\right| = \left|nC_v\Delta T\right| = \left|nC_vT_0\right|$

$\therefore \left|\Delta\frac{Q_{1\to2}}{\Delta Q_{2-3}}\right| = \frac{C_p}{C_v} = \frac{5}{3}$

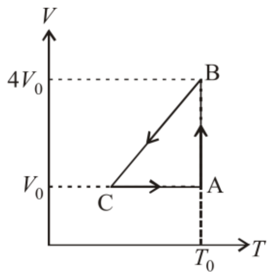
(d) $\left|\Delta Q_{3\to4}\right| = nC_p\frac{T_0}{2}$

$\left|\Delta\frac{Q_{1\to2}}{\Delta Q_{3-4}}\right| = \frac{nC_pT_0}{nC_p\frac{T_0}{2}} = \frac{2}{1}$

12.

(a, b)

Process A to B



Here, $T_A = T_B \therefore U_A = U_B$

Also $P_0V_0 = P_B \times 4V_0$

$\to P_B = \frac{P_0}{4}$

Work done

$$W_{AB} = nRT_0 \log_e \frac{4V_0}{V_0}$$

$= P_0V_0 \log_e 4 \qquad [\because P_0V_0 = nRT_0]$

13.

(b, d)

(a) Process A-B is not isothermal. In case of an isothermal process we get a rectangular hyperbola in a P-V diagram.

(b) In process B→C→D, ΔU is negative. PV decreases and volume also decreases; therefore, W is negative. From first law of thermodynamic, Q is negative i.e., there is a heat loss.

(c) $W_{AB} > W_{BC}. Therefore$ work done during path A→B→C is positive.

(d) Work done is clockwise cycle in a PV diagram is positive.

14.

(a) I. $W_{1\to2\to3} = W_{1\to2} + W_{2\to3}[W_{2\to3} = 0, \Delta V = 0]$

$=P_0 \times V_0 = O$

$= P_0V_0 = \frac{RT_0}{3} \qquad \left[\because P_0V_0 = \frac{1}{3}RT_0\ given\right]$

∴I→Q

II. $\Delta U_{1\to2\to3} = \Delta U_{1\to2} + \Delta U_{2\to3}$

$= nC_v\Delta T_{1\to2} + nC_v\Delta T_{2\to3}$

$$= 1 \times \frac{3R}{2}\left(T_f - T_i\right)_{1\to2} + 1 \times \frac{3R}{2}\left[T_f = T_i\right]_{2\to3}$$

$$= \frac{3}{2}[2P_0V_0 - P_0V_0] + \frac{3}{2}\left[\frac{3P_0}{2} \times 2V_0 - P_0 \times 2V_0\right]$$

$$3P_0V_0 = 3 \times \frac{1}{3}RT_0 = RT_0$$

II→R

III. $Q_{1\to2\to3} = Q_{1\to2} + Q_{2\to3}$

$= nC_p\Delta T_{1\to2} + nC_v\Delta T_{2\to3}$

$$= \frac{5}{2}P_0V_0 + 1 \times \frac{3}{2}\left[\frac{3P_0}{2} \times 2V_0 - P_0 \times 2V_0\right]$$

$$\frac{8}{2}P_0V_0 = \frac{8}{2} \times \frac{RT_0}{3} = \frac{4}{3}RT_0$$

III-S

IV. $Q_{1\to2} = nC_p\Delta T_{1\to2} = nC_p\left(T_f - T_i\right)$

$1 \times \frac{5}{2}R\left[\frac{P_0(2V_0)}{R} - \frac{P_0V_0}{R}\right] \qquad [\because PV = nRT]$

$$= \frac{5}{2}P_0V_0 = \frac{5}{2}\left(\frac{RT_0}{3}\right) = \frac{5RT_0}{6}$$

IV- U

15.

(c) Process I is adiabatic therefore ΔQ=0

Process II is isobaric P = constant therefore $W = P(V_2 - V_1)$

$= 3P_0(3V_0 - V_0)6P_0V_0$

Process III is isochoric V = constant therefore

$W = P(V_2 - V_1) = 0$

Process IV is isothermal, temperature T = constant, Δu=0

16.

(d) Laplace's correction of the speed of sound in ideal gas is related to adiabatic process.

P-V curve in adiabatic process is steeper than isothermal

17.

(b) Work done in isochoric process is zero for which we get a vertical line in P-V graph, graph (s).

18.

(d) ΔU = ΔQ – PΔV

W = -PΔV = $-P(V_2 - V_1) = -PV_2 + PV_1$

which is the formula for isobaric process, so graph (P)

19.

(a) $W_{GE} = P_0(V_0 - 32V_0) = -31P_0V_0$

$W_{GH} = P_0(8V_0 - 32V_0) = -24P_0V_0$

$$(W_{FH})_{asiabatic} = P_fV_f - \frac{P_iV_i}{1-\Upsilon}$$

$$= P_0(8V_0) - \frac{32P_0(V_0)}{1-\frac{5}{3}} = 36P_0V_0$$

$$(W_{FG})_{isothermal} = nRT\ In\left(\frac{V_f}{V_i}\right) = P_0V_0\left(\frac{V_f}{V_i}\right)$$

$$= 1(32P_0V_0)\log_e\frac{32V_0}{V_0}$$

$$= 32P_0V_0\log_e 2^5 = 160P_0V_0\log_e 2$$

(a) is the correct option

20.

(a) AS the cylinder is open at its bottom and has a small hole at its tope, hence the pressure inside the cylinder is atmospheric pressure, P_0 throughout the slow pulling process.

21.

(d) Let x be the distance of the piston from the top.

At equilibrium $Mg = (P_0 - p)\pi R^2$

$\rightarrow\ p = -\frac{Mg}{\pi R^2} + P_0$

Since the cylinder isothermally conducting

temperature, T = constant

Applying $P_1V_1 = P_2V_2$

$$P_0 \times (2L \times \pi R^2) = px \times \pi R^2 \rightarrow x = \frac{P_0}{p} \times 2L$$

$$= \frac{P_0}{\left[P_0 - \frac{Mg}{\pi R^2}\right]} \times 2L$$

$$= \left[P_0 \times \frac{\pi R^2}{P_0\pi R^2 - Mg}\right] \times 2L$$

22.

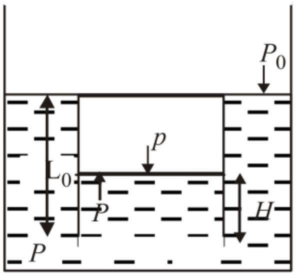
(c) At equilibrium, p = P

$\rightarrow p = P_0 + (L_0 - H)pg\ \ ...(i)$

Also $P_0 \times (\pi R^2 L_0) = p[\pi R^2(L_0 - H)]$

$(P_1V_1 = P_2V_2)$

$\rightarrow p = \frac{L_0P_0}{L_0 - H}$



Substituting this value of E from (i) and (ii)

$$\frac{L_0P_0}{L_0 - H} = P_0 + (L_0 - H)pg$$

$$\rightarrow L_0P_0 = P_0(L_0 - H) + (L_0 - H)^2pg$$

$$\rightarrow pg(L_0 - H)^2 + P_0(L_0 - H) - L_0P_0$$

Kinetic Theory

Chapter Summary

Assumption of Kinetic Theory

1. Molecules of gases are moving randomly in all directions.
2. Size of molecule is very less in comparison to average separation between molecules.
3. Molecules do not exert any force on each other or on wall except during collision.
4. All collisions are elastic.
5. Molecules obey Newton's laws.
6. Gas is in steady state i.e density distribution of molecules is same throughout.

Some Basic Definitions

Avogadro's Number (N_A) :

(a) The number of molecules present in 1 mol of a gas is defined as Avogadro's number.

$N_A = 6.022 \times 10^{23}$ numbers

Molecular weight (M) :

(a) The quantity in 1 mol of matter is its molecular weight.

(b) $M = mN_A$ (m = mass of a single molecule)

Meaning of NTP : NTP means normal temperature and pressure.

(a) Temperature at NTP = 0° C = 273 K

(b) Pressure at NTP = 76 Cm of Hg - column
= 1.013×10^5 Newton/meter^2 or Pascal
= 1 atmosphere.

(c) Volume of 1 mol of gas at NTP = 22.4 litre

Some common Symbols and Constants

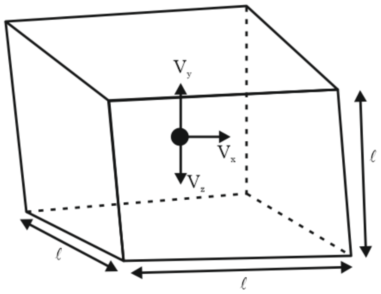
R = Universal gas constant = 8.314 J/mol-K
M_0 = Molecular Mass
m = Mass of each molecule in Kg = M/N_0
N_0 = Avogadro's constant = 6.023×10^{23}
k = Boltzman's constant $= R/N_0 = 1.38\times10^{-23}$J/K
P = Pressure of the gas in N/m^2
T = Temperature in Kelvin
N = Total number of molecules
d = Density of gas = nM/v
n = No. of moles

Pressure of an ideal gas

Derivation:

Lets calculate the change in momentum of each particle in all directions

$$\Delta p_x = 2mV_x$$
$$\Delta p_y = 2mV_y$$
$$\Delta p_z = 2mV_z$$



'm' is the mass of one molecule

n = no of molecules per unit volume

Pressure on one wall, due to collisions

$$P_{inst.} = \frac{F}{A} = \left(\frac{2mV_x}{\Delta t}\right).\frac{1}{A}.n.\left(AV_x\Delta t\right)$$

$$P_{avg} = mnV_x^{\,2}$$

$$V_x^{\,2} + V_y^{\,2} + V_z^{\,2} = \overline{V}^2$$

$$V_x^{\,2} = \frac{1}{3}\overline{V}^2$$

$$P_{ang} = \frac{1}{3}mn\overline{V}^2$$

$$P = \frac{1}{3}\frac{M}{V}\overline{V}^2$$

$$n = \frac{N}{V}$$

Pressure exerted by the gas is

$$P = \frac{1}{3}\rho v_{rms}^2$$

Root Mean Square Speed

$$v_{rms} = \sqrt{\frac{3P}{\rho}}$$

$$v_{rms} = \sqrt{\frac{3PV}{M}}$$ M: total mass of gas

$$v_{rms} = \sqrt{\frac{3RT}{M_0}}$$ M_0 : molar mass of gas

n – no of moles

It is defined as $v_{rms} = \sqrt{\frac{v_1^2 + v_2^2 + ... + v_n^2}{n}}$

Also as $v_{rms} = \left[\frac{\int v^2 dN}{\int dN}\right]^{1/2}$

Here if

N_1 molecules have speed v_1,
N_2 molecules have speed v_2, then

$$v_{rms} = \left[\frac{N_1v_1^2 + N_2v_2^2 + ...}{N_1 + N_2 + ...}\right]^{1/2}$$

Kinetic Interpretation of Temperature

$$P = \frac{1}{3}nmv^2$$

$$PV = \frac{1}{3}Nmv^2 = \frac{1}{3}M_{gas}V^2$$

$$\frac{1}{2}PV = \frac{1}{3}\frac{1}{2}M_{gas}V^2$$

$$KE = \frac{3}{2}PV = \frac{3}{2}nRT$$

$$KE = \frac{3}{2}\left(\frac{N}{N_A}.R\right)T$$

$$= \frac{3}{2}\left(\frac{R}{N_A}\right)NT$$

Kinetic Energy per unit molecule

$$\frac{KE}{N} = \frac{3}{2}k_BT$$

k_B = Boltzmann's constant $= \frac{R}{N_A} = 1.381 \times 10^{-23}$

R = 8.314
$N_A = 6.022 \times 10^{23}$

Different Velocities of Molecules of a Gas

(1) Instantaneous velocity ($\vec{v}$): Velocity of molecule at any instant of time.

(2) Instantaneous speed ($|\vec{v}|$): Speed of molecule at any instant of time

(3) Average velocity ($\bar{v}$):

$$<\vec{v}> = \bar{v} = \frac{\sum\vec{v}}{N} = \frac{\vec{v}_1 + \vec{v}_2 + ...\vec{v}_N}{N} = 0$$

because molecules are in random motion

(4) Average speed ($|\bar{v}|$) :

$$<|\vec{v}|> = |\bar{v}| = \frac{\sum|\vec{v}|}{N} = \frac{|\vec{v}_1| + |\vec{v}_2| + ...|\vec{v}_N|}{N}$$

$$= \sqrt{\frac{8RT}{\pi M}} = 1.59\sqrt{\frac{RT}{M}} = 1.59\sqrt{\frac{kT}{m}}$$

(5) Most probable speed (v_{mp}) : This is defined as the speed with which maximum number of molecules travel.

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$

(6) Mean square velocity ($\bar{v}^2$): Average of the squares of instantaneous velocities of all the molecules.

$$< v^2 > = \bar{v}^2 = \frac{\sum v^2}{N} = \frac{v_1^2 + v_2^2 + ...v_N^2}{N}$$

(7) Root mean square speed (RMS speed) (v_{rms}) **:**

$$v_{rms} = \sqrt{\frac{\sum v^2}{N}} = \sqrt{\frac{v_1^2 + v_2^2 + ...v_N^2}{N}}$$

$$= \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3PV}{nM}} = \sqrt{\frac{3P}{d}}$$

Ideal gas Equation

PV = nRT

$$P = \frac{\rho M}{RT}$$

Where $n = \frac{N_1}{N_A}$ no. of moles

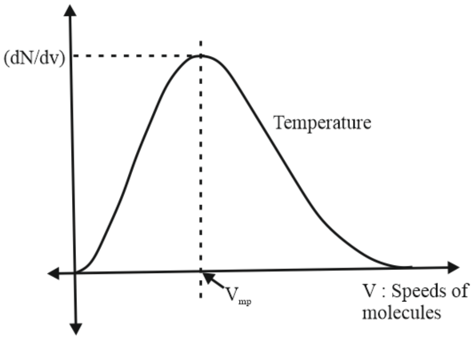
Distribution of Molecular Speed

Maxwell derived an equation giving the distribution of molecules in different speed.

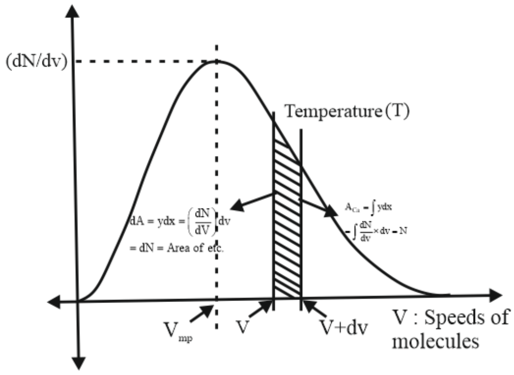
If dN represents the number of molecules with speeds between v and v + dv, then

$$dN = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} v^2e^{-mv^2/2kT}dv$$

$$\frac{dN}{dv} \propto Nv^2T^{-3/2}e^{-mv^2/2kT}$$



The speed v_{mps} at which dN/dv is maximum is called the **most probable speed**



The important features of speed distribution curve:

1. The speed at which dN/dv is maximum, is called most probable speed v_{mp}. Its value is given by

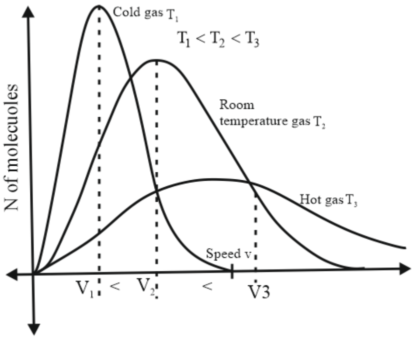
$$v_{mp} = \sqrt{\frac{2kT}{m}}$$

2. The number of molecules with speeds v to v + dv is equal to the area of the strip.

Area = N = constant= total no. of molecules

3. At higher temperatures, molecules have more energy, curve is broader and flatter.

4. At lower temperatures, molecules have lesser energies, curve is narrower and sharper.



Average speed

Defined as the arithmetic mean of the speeds of the molecules of a gas at a given temperature.

$$v_{av} = \frac{v_1 + v_2 + ... + v_n}{n}$$

Most probable speed

Speed with which the largest number of molecules in a gas are moving. At any temperature it is given by

$$v_{mp} = \sqrt{\frac{2RT}{M}} = \sqrt{\frac{2kT}{m}}$$

Degree of Freedom *(f)*

The number of independent ways in which a molecule or an atom can exhibit motion or have energy is called its degrees of freedom.

(1) Translational (2) Rotational (3) Vibrational

For Monoatomic

f = 3 always *(no affect of temperature)*

For Dia-atomic

f = 3 + 2 = 5 *(At normal temperature)*

'2' Additional VKE + VPE will come for diatomic, triatomic and polyatomic linear and non-linear molecules

f = 3 + 2 + 2 = 7 *(At high temperature)*

Law of Equipartition of Energy

"The total energy of an ideal gas molecule is distributed equally among all of its degrees of freedom"

$E = \frac{1}{2}k_BT$ per molecule per degree of freedom

$E = \frac{1}{2}RT$ per mole per degree of freedom

Where k_B is Boltzmann's constant and

T is absolute temperature

$K_B = \frac{R}{N_A} = 1.38 \times 10^{-23}$ J/K

Energy density (E_v)

E = Total energy/Volume = E/V

$$E_v = \frac{1}{2}\left[\frac{M}{V}\right]v_{rms}^2 = \frac{1}{2}\rho v_{rms}^2$$

$$p = \frac{2}{3}\left[\frac{1}{2}\rho v_{rms}^2\right]$$

$$E_v = \frac{3}{2}P$$

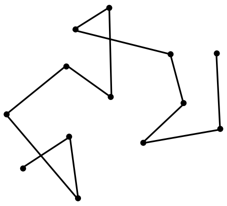
Internal Energy for Different Types of Gases

Total internal energy of n moles of a monoatomic gas at a temperature T is given as

U = f/2 nRT

Mean Free Path (λ)

Average distance travelled by the molecule between two successive collisions.



Mean free path = $\frac{total\,distance\ travelled}{total\ number\ of\ collisions}$

$$\lambda_{mean} = \left(\frac{\lambda_1 + \lambda_2 + ... + \lambda_N}{N}\right)$$

Statistical study of heat gives the mean free path as:

$$\lambda = \frac{1}{\sqrt{2}\pi n d^2} \qquad \lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$$

where d is the diameter of molecule,

n is the number molecules per unit volume

P is the pressure of the gas

k_B is Boltzmann's constant

Dalton's Law of partial pressure:

" The pressure exerted by a mixture of several gases equals the sum of pressures exerted by each gas occupying the same volume as that of the mixture" for n gases

i.e. $P = P_1 + P_2 + P_3 + \ldots\ldots P_n$.

Where P = pressure exerted by mixture

P_1 ,P_2 , P_3, ... P_n = partial pressure of the component gases

Avogadro's Law:

At the same temperature and pressure, equal volumes of all gases contain equal number of molecules or moles

Imp. Facts:

(a) At = T = 273 K and P = 1atm, (STP), 22.4 liter of each gas has $N_0 = 6.023 \times 10^{23}$ molecules or in other words. "One mole of gas at STP occupy 22.4 liter of volume"

(b) This means, that at STP mass of 22.4 litre of a gas is equal to it's mol. wt.

(c) This implies that 32 gm oxygen (O_2), 28 gm Nitrogen (N_2), 2 gm Hydrogen (H_2) etc. occupy the same volume = 22.4 liters at STP.

Beginner **Assumptions of KTG**

---

1. According to kinetic theory of gases, which of following statement will be true.

(a) Ideal gases cannot be liquified

(b) The molecules of ideal gas do not obey newtons laws of motion.

(c) Pressure of gas is always inversely proportional to its volume

(d) molecules of gas never move in straight line.

Solution: Correct option is A.

According to kinetic energy of gases:

The ideal gas cannot be liquefied because the intermolecular forces in ideal gas are negligible. In case of negligible intermolecular forces, the molecules will have free random motion. This is possible in gaseous state. In liquid state, there is restricted motion of molecule. The molecule are held by intermolecular force in liquid state.

2. According to kinetic theory of gases,

(a) The velocity of molecules decreases for each collision

(b) The pressure exerted by a diatomic gas is proportional to the mean velocity of the molecule.

(c) The K.E. of the gas decreases on expansion at constant temperature.

(d) The mean translational K.E. of a diatomic gas increases with increase in absolute temperature.

Solution:

Correct option is D.

Options:

(A): One cannot say velocity of molecule decreases in each collision.

(B): The pressure exerted by a diatomic gas is proportional to rms speed, not mean speed.

(C): Since temp is constant, KE will be constant.

(D): Mean translational KE, is proportional to temp, hence will increase with increase in absolute temp.

3. Which of the statement is correct with reference to kinetic theory of gases?

(a) There is no loss of total kinetic energy of the molecules of the gas due to elastic collision.

(b) There is no potential energy associated with any molecule of the gas

(c) The molecules are moving randomly and obeys the Newton's laws of motion.

(d) All of the above

Solution:(d)

4. Which of the following statement is not according to the postulates of kinetic theory of gases?

(a) Gas molecules are of small size

(b) Gas molecules are always in motion with all possible velocities

(c) There is no force between the molecules

(d) None of these

Solution:(d)

5. In kinetic theory of gases, it is assumed that molecules –

(a) Have same mass but can have different volume

(b) Have same volume but masses can be different.

(c) Have both mass and volume different

(d) Have same mass but negligible volume.

Solution:(d)

6. Which of the following quantities is zero on an average for the molecules of an ideal gas in equilibrium?

(a) Kinetic energy (b) Momentum

(c) Density (d) Speed

Solution:(b)

Beginner Pressure & Kinetic Energy

1. The pressure exerted by the gas on the walls of the container because

(a) It loses kinetic energy

(b) It sticks with the walls

(c) On collision with the walls there is a change in momentum

(d) It is accelerated towards the walls

Solution:

The answer is C.

At microscopic level, pressure is the force exerted by the gas molecules on the wall by virtue of momentum change during their elastic collision with the wall atoms.

2. Gas at a pressure P_0 is contained in a vessel. If the masses of all the molecules are halved and their speeds are doubled, the resulting pressure P will be equal to

(a) $4P_0$ (b) $2\ P_0$

(c) P_0 (d) $\frac{P_0}{2}$

Solution:

The answer is B.

$$P=\frac{1}{3}\frac{mN}{V}{v_{rms}}^2$$

$$\therefore p\propto m{v^2}_{rms}$$

So,

$$\frac{P_2}{P_1}=\frac{m_2}{m_1}\times\left(\frac{v_2}{v_1}\right)^2$$

$$=\frac{m_1/2}{m_1}\left(\frac{2v_1}{v_1}\right)$$

$$=2$$

$$P_2=2P_1$$

$$=2P_0$$

3. A box contains n molecules of a gas. How will the pressure of the gas be affected, if the number of molecules is made 2n?

(a) Pressure will decrease

(b) Pressure will remain unchanged

(c) Pressure will be doubled

(d) Pressure will become three times

Solution:

The answer is C.

We know that from ideal gas equation, PV= nRT now, n=number of moles if gas$\frac{N}{N_A}$, where N = number of molecules of the gas and N_A is the Avogadro's number.

So, putting the value of n in ideal gas equation, we get $PV=\frac{N}{N_A}RT$

This means that pressure of the gas is proportional to the number of molecules of the gas at constant temperature and volume.So, if we double the number of molecules of the gas (2x) , then pressure will also double.

4. The relation between the gas pressure P and average kinetic energy per unit volume E is

(a) $P=\frac{1}{2}E$ **(b) P = E**

(c) $P=\frac{3}{2}E$ **(d)** $P=\frac{2}{3}E$

Solution:

The correct answer is D.

$$Kinetic\ energy=\frac{3RT}{2}$$

$$\Rightarrow K.E=\frac{3PV}{2}\ ,$$

$$Since, PV=nRT\ , here\ n=1mole$$

Therefore,

$$P=\frac{2E}{3V}\ ,(E=kinetic\ energy)$$

As per question,

$$\frac{E}{V}=E\quad(As\ volume=1)$$

So,

$$P=\frac{2E}{3}$$

5. A cylinder of capacity 20 litres is filled with H_2 gas. The total average kinetic energy of translatory motion of its molecules is 1.5×10^5 J . The pressure of hydrogen in the cylinder is

(a) 2 × 10^6 N/m^2 **(b) 3 × 10^6 N/m^2**
(c) 4 × 10^6 N/m^2 **(d) 5 × 10^6 N/m^2**

Solution:

The correct answer is D.

$Kinetic\ energy\ E = 1.5 \times 10^5 J, volume$

$V = 20L = 20 \times 10^{-3} m^3$

$$Pressure = \frac{2}{3}\frac{E}{V} = \frac{2}{3}\left(\frac{1.5 \times 10^5}{20 \times 10^{-3}}\right)$$
$$= 5 \times 10^6 N/m^2$$

6. The root mean square speed of hydrogen molecules of an ideal hydrogen gas kept in a gas chamber at 0°C is 3180 metres/second. The pressure on the hydrogen gas is (Density of hydrogen gas is 8.99 × 10^{-2} kg/m^3 , 1 atmosphere = 1.01 × 10^5 N.m^2)

(a) 1.0 atm **(b) 1.5 atm**
(c) 2.0 atm **(d) 3.0 atm**

Solution:

The correct answer is D.

R.M.S speed of H_2 = 3180 m/s

$$Ph_2 = 8.99 \times \frac{10^{-2} kg}{m^3},$$

$1\ atm = 1.01 \times 10^5 N/m^2$

$$Pressure, P = \frac{1}{3}pc^2$$

$$= \frac{1}{3} \times \frac{8.99 \times 10^{-2} \times 3180 \times 3180}{1.01 \times 10^5}\ atm$$

$= 3\ atm$

7. If the mean free path of atoms is doubled then the pressure of gas will become

(a) P/4 **(b) P/2**
(c) P/8 **(d) P**

Solution:

The correct answer is B.

$\lambda \propto 1/P$

When mean free path is doubled. Pressure becomes half. So, the answer would be simply p/2.

8. Relationship between P, V and E for a gas is

(a) $P = \frac{3}{2}EV$ **(b)** $V = \frac{2}{3}EP$

(c) $PV = \frac{3}{2}E$ **(d)** $PV = \frac{2}{3}E$

Solution:

The correct answer is D.

Pv=$\frac{1}{3}Nmc^2 = \frac{2}{3}\left(\frac{1}{2}Nm\right)c^2 = \frac{2}{3}E$

9. The energy of a given sample of an ideal gas depend only on its –

(a) Volume **(b) Pressure**
(c) Density **(d) Temperature**

Solution: (d)

10. When a gas is forced in a smaller volume without change in temperature, its pressure increases because its molecules –

(a) Strike the unit area of the container walls more often.

(b) Strike the unit area of the container walls at higher speed.

(c) Strike the unit area of the container wall with greater force.

(d) Have more energy

Solution: (a)

11. The pressure of gas increases on raising the temperature of a given gas in a container because –

(a) The average velocity of molecules increase so that per second the number of collisions on the wall increases.

(b) The mass of molecules increases

(c) The molecules get smaller time to remain in contact with the wall

(d) There is a loss of energy in each collisions of the molecules

Solution:(a)

12. Average kinetic energy per molecule of an ideal monatomic gas at 0°C (k = 1.38 × 10^{-23} JK^{-1}) is –

(a) 4.8 × 10^{-18} J **(b) 9.5 × 10^{-23} J**
(c) 5.6 × 10^{-21} J **(d) 11.3 × 10^{-22} J**

Solution:(c)

$$E = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 273 = 5.6 \times 10^{-21}\ J$$

13. The value of universal gas constant is 8.3 J/mole/K, the mean kinetic energy of 32 gm of oxygen at – 73°C will be

(a) 480 J **(b) 4980 J**
(c) 2490 **(d) The information is incomplete**

Solution:

The correct answer is C.

$$KE = \frac{3}{2}nRT$$

$= \frac{3}{2} \times 1 \times 8.3 \times 200$

$= 2490\,J$

14. If number of molecules of H_2 are double than that of O_2, then ratio of kinetic energy of hydrogen and that of oxygen at 300 K is

(a) 1:1 **(b) 1:2**

(c) 2:1 **(d) 1:16**

Solution:

The correct answer is A.

We know that,

$E \propto T$

Therefore, the kinetic energy does not depend upon the molecules of the gas. Hence, the kinetic energies of both the gases at a given temperature are same.

$or\ EH_2 : EO_2 = 1 : 1$

15. The ratio of mean kinetic energy of hydrogen and oxygen at a given temperature is

(a) 1:16 **(b) 1:8**

(c) 1:4 **(d) 1:1**

Solution:

The correct answer is D. Kinetic energy is function of temperature as KE=3/2 KBT. Hence at same temperature rates is 1 : 1.

16. The ratio of mean kinetic energy of hydrogen and nitrogen at temperature 300 K and 450 K respectively is

(a) 3:2 **(b) 2:3**

(c) 2:21 **(d) 4:9**

Solution:

The correct answer is B.

$KE = (3/2)KBT\ i.e\ KE \propto T$

$[\{(KE)_{H2}\}/\{(KE)_{N2}\}] = (T_{H2}/T_{N2})$

Hence,

$[\{(KE)_{H2}\}/\{(KE)_{N2}\}] = (300/450) = (2/3)$

17. Mean kinetic energy (or average energy) per gm molecule of a monoatomic gas is given by

(a) $\frac{3}{2}RT$ **(b)** $\frac{1}{2}KT$

(c) $\frac{1}{2}RT$ **(d)** $\frac{3}{2}KT$

Solution:

The correct answer is A.

One gram molecule is the no. Of molecules present in one mole i.e, 6.022×10^{23} molecules.

Therefore mean kinetic energy per gram molecule of a monoatomic gas:

$= \frac{3}{2}Nk_BT$

$= \frac{3}{2}RT\ since\ R = Nk_B$

18. The translatory kinetic energy of a gas per gm is

(a) $\frac{3}{2}\frac{RT}{N}$ **(b)** $\frac{3}{2}\frac{RT}{M}$

(c) $\frac{3}{2}RT$ **(d)** $\frac{3}{2}NKT$

Solution:

The correct answer is B.

$E = \frac{3}{2}rT = \frac{3}{2}\frac{RT}{M}\left(\because r = \frac{R}{M}\right)$

19. The mean kinetic energy of a gas at 300 K is 100 J. The mean energy of the gas at 450 K is equal to

(a) 100 J **(b) 3000 J**

(c) 450 J **(d) 150 J**

Solution:

The correct answer is D

$Average\ kinetic\ energy\ \propto Temperature$

$\therefore E \propto T$

$(E_2/E_1) = (T_2/T_1)$

$\therefore E_2 = E_1 \times (T_2/T_1)$

$= 100 \times (450/300)$

$= 150\,J$

20. At what temperature is the kinetic energy of a gas molecule double that of its value of 27°C

(a) 54°C **(b) 300 K**

(c) 327°C **(d) 108°C**

Solution:

The correct answer is C.

$Kinetic\ energy\ of\ gas\ (E) = \frac{3}{2} \times k \times T$

Then,

$\frac{E}{T} = k$, Where k is Boltzmann constant

$\frac{E_1}{T_1} = \frac{E_2}{T_2}$

Given,

$E_1 = E; E_2 = 2 \times E; T_1 = 27°\text{C} = 300K; T_2 = ?$

Then, $T_2 = E_2 \times \frac{T_1}{E_1} = 2 \times E \times \frac{300}{E} = 600K$

$T_2 = 327°\text{C}$

21. The average kinetic energy of a gas molecule at 27°C is 6.21×10^{-21} J.

Its average kinetic energy at 227°C will be

(a) 52.2×10^{-21} J (b) 5.22×10^{-21} J

(c) 10.35×10^{-21} J (d) 11.35×10^{-21} J

Solution:

The correct answer is C.

$$Mean\ KE = \frac{3kT}{2}$$

So, $k = 1.38 \times 10^{-23}$

$$KE\ at\ 500\ K, KE = 3 \times 1.38 \times 10^{-23} \times \frac{500}{2}$$

$$= 10.35 \times 10^{-21}\ Joule$$

22. A polyatomic gas with n degrees of freedom has a mean energy per molecule given by

(a) $\frac{nkT}{N}$ **(b)** $\frac{nkT}{2N}$

(c) $\frac{nkT}{2}$ **(d)** $\frac{3kT}{2}$

(N is Avogadro's number)

Solution:

The correct answer is C.

According to law of equipartition of energy, the energy per degree of freedom is 1/2kT. For a polyatomic gas with n degree of freedom, the mean energy per molecule = ½ nkT.

23. Energy of all molecules of a monoatomic gas having a volume V and pressure P is $\frac{3}{2}PV$. The total translational kinetic energy of all molecules of a diatomic gas as the same volume and pressure is

(a) $\frac{1}{2}PV$ **(b)** $\frac{3}{2}PV$

(c) $\frac{5}{2}PV$ **(d) 3 PV**

Solution:

The correct answer is B.

Translational kinetic energy depends upon translational degree of freedom. These degrees are three for monoatomic as well as diatomic gases. Hence, under same pressure, equal volumes of monoatomic and diatomic molecules have same translational kinetic energy i.e. (3/2) PV.

24. Two gases are at absolute temperatures 300 K and 350 K respectively. Ratio of average kinetic energy of their molecules is

(a) 7 : 6 (b) 6 : 7

(c) 36 : 49 (d) 49 : 36

Solution:

The correct answer is B.

$$\frac{KE_1}{KE_1} = \frac{T_1}{T_2} = \frac{300}{350} = \frac{6}{7}$$

25. Find the average translational kinetic energy per molecule if one mole of the gas is contained in a volume 1.23×10^{-3} m^3 at a pressure 2×10^5 N/m^2. Avogadro's number is 6.02×10^{23} molecules/mole.

Solution:

Kinetic energy for translation per molecule is

$$K = \frac{3}{2}K_BT = \frac{3}{2} \times \frac{R}{N_A} \times T \quad but\ PV = net$$

$$= \frac{3}{2} \times \frac{1}{N_A} \times \left(\frac{P \times V}{n}\right)$$

$$= \frac{3}{2} \times \frac{1}{6.02 \times 10^{23}} \times \frac{(2 \times 10^5 \times 1.23 \times 10^{-3})}{1}$$

$$= \frac{3{,}69 \times 10^2}{6.02 \times 10^{23}} = 6.12 \times 10^{-22} Joule$$

26. Which of the following quantities is the same for all ideal gases at the same temperature?

(a) the translational kinetic energy of 1 mole

(b) the translational kinetic energy of 1 g

(c) the number of molecules in 1 mole

(d) the number of molecules in 1 gm

Solution:

Correct options are A & C.

(a) the translational kinetic energy of 1 mole

(c) the number of molecules in 1 mole

Kinetic energy per mole of an ideal gas is directly proportional to T. So, it will be the same for all ideal gases.

Number of molecules in 1 mole of an ideal is the same for all ideal gases because ideal gases obey Avogadro's law.

27. Which of the following quantities is zero on an average for the molecules of an ideal gas in equilibrium?

(a) kinetic energy (b) momentum

(c) density (d) speed

Solution:

Correct option is B.

The molecules move in all possible directions in an ideal gas of equilibrium. Since momentum is a vector quantity for every direction of motion of the molecules, there exists an opposite direction of motion of the other. Hence, the average momentum is zero for an ideal gas at equilibrium.

28. Consider the collision between an oxygen molecule and a hydrogen molecule in a mixture of oxygen and hydrogen kept at a room temperature. Which of the following are possible?

(a) the kinetic energies of both the molecules increases

(b) the kinetic energies of both the molecules decrease

(c) kinetic energy of the oxygen molecule increases and that of the hydrogen molecule decreases

(d) The kinetic energy of the hydrogen molecule increases and that of the oxygen molecule decreases.

Solution:

Correct options are C and D.

Assuming elastic collision between the oxygen and hydrogen molecules, kinetic energy will be conserved.

Increase in KE of oxygen and decrease in KE of hydrogen, keeping the total energy conserved is possible.

Increase in KE of hydrogen and decrease in KE of oxygen, keeping the total energy conserved is possible. Since KE is conserved, both cannot increase or decrease.

29. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is 4 × 10^{-14} erg, g = 980 cm/s^2, density of mercury = 13.6 g/cm^3)

(a) 4.0 × 10^{18} **(b) 4.0 × 10^{16}**

(c) 5.8 × 10^{16} **(d) 5.8 × 10^{18}**

Solution:

(a) Given : $K.E._{mean} = \frac{3}{2}kT = 4\times10^{-14}$

P = 2 cm of Hg, V = 4 cm^3

$$N = \frac{PV}{KT} = \frac{P\rho gV}{KT} \frac{2\times13.6\times980\times4}{\frac{8}{3}\times10^{-14}} \simeq 4\times10^{18}$$

30. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of:

(a) 10^4 N/m^2 **(b) 10^8 N/m^2**

(c) 10^3 N/m^2 **(d) 2 N/m^2**

Solution:

Rate of change of momentum during collision

$$= \frac{mv-(-mv)}{\Delta t} = \frac{2mv}{\Delta t}N$$

So, pressure $P = \frac{N\times(2mv)}{\Delta t\times A}$

$$= \frac{10^{22}\times2\times10^{-26}\times10^{4}}{1\times1} = 2\text{ N/m}^2$$

31. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is:

(a) $\frac{1}{2}$ **(b) 2**

(c) 1 **(d) $\frac{1}{4}$**

Solution:

(c) Pressure, $P = \frac{1}{3}\frac{mN}{V}V_{rms}^2$

or, $P = \frac{(mN)T}{V}$

If the gas mass and temperature are constant then

$P \propto (V_{rms})^2 \propto T$

So, force $\propto (V_{rms})^2 \propto T$

i.e., Value of q = 1

32. A given sample of an ideal gas occupies a volume V at a pressure p and absolute temperature T. The mass of each molecule of the gas is m. Which of the following gives the density of the gas?

(a) p/(kT) **(b) pm/(kT)**

(c) p/(kTV) **(d) mkT**

Solution:

(b) As we know that

Pressure, $p = \frac{1}{3}\cdot\frac{m}{V}v_{rms}^2$

∵ m = mass of the gas, V = volume of the gas

∴ $\frac{mn}{V}$ = density of the gas. Thus,

$$p = \frac{1}{2}pv_{rms}^2 = \frac{1}{3}\rho\frac{3RT}{M_0} = \frac{\rho RT}{M_0}$$

$$\left(\because v_{rms} = \sqrt{\frac{3RT}{M_0}}\right)$$

$$\rho = \frac{pM_0}{RT} = \frac{pmN_A}{kN_AT}$$

[∵ R = N_Ak and M_0 = mN_A]

$$\rho = \frac{pm}{kT}$$

Expert Pressure & Kinetic Energy

1. Consider a gas with density ρ and $\bar{c}$ as the root mean square velocity of its molecules contained in a volume. If the system moves as whole with velocity v, then the pressure exerted by the gas is

(a) $\frac{1}{3}\rho\bar{c}^{2}$ **(b)** $\frac{1}{3}\rho(c+v)^2$

(c) $\frac{1}{3}\rho(\bar{c}-v)^2$ **(d)** $\frac{1}{3}\rho(c^{-2}-v)^2$

Solution:

The answer is A.

Root-mean-square speed is the measure of the speed of particles in a gas which is most convenient for problem solving within the kinetic theory of gases

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$C = \sqrt{\frac{3RT}{M}}$$

$$C = \sqrt{\frac{3PV}{M}}$$

$$C = \sqrt{\frac{3P}{\rho}}$$

$$P = \frac{\rho}{3}C^2 = \frac{1}{3}\rho C^2$$

2. The average kinetic energy of a gas at –23°C and 75 cm pressure is 5 × 10^{-14} erg for H_2. The mean kinetic energy of the O_2 at 227°C and 150 cm pressure will be

(a) 80 × 10^{-14} erg (b) 20 × 10^{-14} erg

(c) 40 × 10^{-14} erg (d) 10 × 10^{-14} erg

Solution:

The answer is D.

$E = avg.KE = (3/2)K_BT$

$\boldsymbol{i.e.} E \propto T$

$(E_1/E_2) = (T_1/T_2)$

$E_1 = 5\times10^{-14} erg\ at\ T_1 = -23°C$

$= (273-23)°K = 250K$

$T_2 = 227°C = 227+273 = 500K$

$\therefore [(5\times10^{-14})/E_2]$

$= [250/500]$

$\therefore E_2 = 10\times10^{-14}\ erg$

3. At 27°C temperature, the kinetic energy of an ideal gas is E_1. If the temperature is increased to 327°C, then kinetic energy would be

(a) 2E_1 **(b)** $\frac{1}{2}E_1$

(c) $\sqrt{2}E_1$ **(d)** $\frac{1}{\sqrt{2}}E_1$

Solution:

The answer is A.

$$E = \frac{3}{2}kT \Rightarrow E \propto T$$

$$\therefore \frac{E_2}{E_1} = \frac{T_2}{T_1} = \frac{600}{300} = 2$$

$$E_2 = 2E_1$$

4. The average translational kinetic energy of O_2 (molar mass 32) molecules at a particular temperature is 0.048 eV. The translational kinetic energy of N_2 (molar mass 28) molecules in eV at the same temperature is

(a) 0.0015 (b) 0.003

(c) 0.048 (d) 0.768

Solution:

Average translational kinetic energy does not depends upon the molar mass of the gas. Different gases will possess same average translational kinetic energy at same temperature Hence c is correct.

5. The average translational energy and the r.m.s. speed of molecules in a sample of oxygen gas at 300 K are 6.21 × 10^{-21} J and 484 m/s respectively. The corresponding values at 600 K are nearly (assuming ideal gas behaviour)

(a) 12.42 × 10^{21}J, 968 m/s
(b) 8.78 × 10^{21}J, 684 m/s
(c) 6.21 × 10^{21}J, 968 m/s
(d) 12.42 × 10 $^{-21}$J, 684 m/s

Solution:

The correct answer is D.

$E = average\ energy = 6.21 \times 10^{-21} J$

$V_{rms} = rms\ speed = 484 m/s$

$T_1 = 300K$

$As\ ET, if\ T = 600K, temperature\ is\ doubled\ so\ energy\ (E) will\ also\ be\ doubled$

$\therefore E = 2 \times 6.21 \times 10^{-21} J = 12.42 \times 10^{-21} J$

6. Vessel A is filled with hydrogen while vessel B, whose volume is twice that of A, is filled with the same mass of oxygen at the same temperature. The ratio of the mean kinetic energies of hydrogen and oxygen is

(a) 16 : 1 **(b) 1 : 8**
(c) 8 : 1 **(d) 1 : 1**

Solution:

The correct answer is D.

Mean Kinetic Energy,

$$KE_m = \frac{P}{2} K_B T$$

So,

$$\frac{KE_m H_2}{KE_m O_2} = \frac{\frac{5}{2} K_B T}{\frac{5}{2} K_B T} = \frac{1}{1}$$

$= 1:1$

7. The kinetic energy of one mole gas at 300K temperature, is E. At 400K temperature kinetic energy is E' The value of E'/E is

(a) 1.33 **(b)** $\sqrt{\left(\frac{4}{3}\right)}$

(c) $\frac{16}{9}$ **(d) 2**

Solution:

The correct answer is A.

$n = 1$

$T_1 = 300K$

$E_1 = E$

$T_2 = 400K$

$E_2 = E_1$

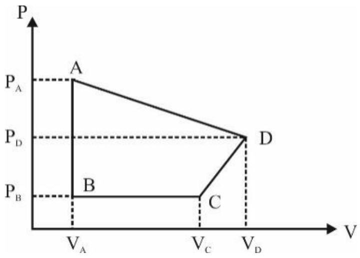
$as\ KE \propto T - as\ KE = (3/2) K_B T$

$(E_2/E_1) = (T_2/T_1)$

$\therefore (E_1/E) = (400/300)$

$\therefore (E_1/E) = (4/3) = 1.33$

8. A parallel beam of particles of mass m moving with velocities v impinges on a wall at an angle θ to its normal. The number of particles per unit volume in the beam is n. If the collision of particles with the wall is elastic, then find the pressure exerted by this beam on the wall.



Solution:

The speed of particles in the direction perpendicular to the wall $= v \cos\theta$,

Since the collision is elastic, the change in velocity occurs in the direction perpendicular to the direction and equal to $\Delta v = v\cos\theta - (-v\cos\theta) = 2v\cos\theta$

Thus the change in momentum for each particle = $2mv\cos\theta$

Thus pressure exerted on the wall = Force per unit area

Total change in momentum per second per unit area = $\frac{\Delta P}{A\Delta t}$

$= nm\Delta v \times 2v\cos\theta = 2nmv^2 cos^2\theta$

9. Pick the correct statement

(a) The r.m.s translational speed for all ideal gas molecules at the same temperature is not the same but it depends on the mass.

(b) Each particle in a gas has average translational kinetic energy and the equation $\frac{1}{2} mv_{rms}^2 = \frac{3}{2} kT$ **establishes the relationship between the average translational kinetic energy per particle and temperature of an ideal gas. It can be concluded that every single particle has a temperature T.**

(c) If the temperature of an ideal gas is doubled from 100°C to 200°C, the average kinetic energy of each particle is also doubled

(d) It is possible for both pressure and volume of a monoatomic ideal gas to change simultaneously without causing the internal energy of the gas to change.

Solution:

Correct options are A & d.

We know that

$$V_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{mN_A}}$$

$$V_{rms} = \sqrt{\frac{3KT}{m}}$$

Here,

T = temperature

m = mass

rms translational speed is depend on mass and temperature. Therefore, option A is correct.

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Since PV = Nrt, therefore P and V both can change simultaneously keeping the temperature constant.

10. At what temperature will the rms speed of oxygen molecules become just sufficient for escaping from the Earth's atmosphere? (Given: mass of oxygen molecule, m = 2.76 × 10^{-26} kg, Boltzmann's constant k_B = 1.38 × 10^{-23} JK^{-1})

(a) 5.016 × 10^4 K (b) 8.362 × 10^4 K

(c) 2.508 × 10^4 K (d) 1.254 × 10^4 K

Solution:

(b) Key Concept The minimum velocity with which the body must be projected vertically upwards, so that it could escape from the Earth's atmosphere, is its escape velocity (v_e).

As, $v_e = \sqrt{2gR}$

Substituting the value of g (9.8 ms^{-2}) and radius of Earth (R = 6.4 × 10^6 m), we get

$$v_e = \sqrt{2\times 9.8\times 6.4\times 10^6}$$

$\cong$ 11.2 km s^{-1} = 11200 m s^{-1}

Let the temperature of molecule be T when it attains v_e.

According to the question,

$$V_{rms} = v_e$$

where, V_{rms} is the rms speed of the oxygen molecule.

$$\sqrt{\frac{3k_B T}{m_{0_2}}} = 11.2\times 10^3$$

or $$T = \frac{\left(11.2\times 10^3\right)^2 \left(m_{0_2}\right)}{\left(3k_B\right)}$$

Substituting the given values, i.e.,

R_B = 1.38 × 10^{-23} JK^{-1} and

m_{0_2} = m = 2.76 × 10^{-26} kg

We get,

$$T = \frac{\left(11.2\times 10^3\right)^2 \left(2.76\times 10^{-26}\right)}{\left(3\times 1.38\times 10^{-23}\right)} = 8.3626 \times 10^4$$ K

Beginner **Various Speeds**

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1. If pressure remains constant, find the temperature at which V_{rms} is half of its value at 27°C.

Solution:

At constant pressure, rms speed depends only on the temperature

$$v_{rms} \propto \sqrt{T}$$

$$\frac{v_{rms}}{v'_{rms}} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{v_{rms}}{\frac{v_{rms}}{2}} = \sqrt{\frac{300}{273+t}}$$

$$2 = \sqrt{\frac{300}{273+t}}$$

Squaring both sides, we get

$$4 = \frac{300}{273+t}$$

$$1092 + 4t = 300$$

$$4t = -1092 + 300 = -792$$

$$t = -\frac{792}{4} = -198°\text{C}$$

2. To what temperature should the hydrogen at

327°C be cooled at constant pressure, so that the root mean square velocity of its molecules become half of its previous value

(a) – 123°C **(b) 123°C**

(c) – 100°C **(d) 0°C**

Solution:

The answer is A.

$$v_{rms} \propto \sqrt{\frac{3RT}{M}}$$

$$T \propto v^2{}_{rms}$$

$$\frac{T_2}{T_1} = \left[\frac{v_2}{v_1}\right]^2 = \frac{1}{4}$$

$$T_2 = \frac{T_1}{4} = \frac{(273+327)}{4}$$

$$= 150K = -123℃$$

3. Determine the average and r.m.s. speed of system of 5 particles of same mass having speeds (each in m/s): 1, 2, 3, 4, 5

Solution:

$$v_{rms} \geq v_{avg}$$

$$\bar{v} = v_{avg} = \frac{1+2+3+4+5}{5} = 3$$

$$v_{rms} = \sqrt{\frac{1^2+2^2+3^2+4^2+5^2}{5}} = \sqrt{11} = 3.31$$

4. The absolute temperature of a gas is increased 3 times. The root mean square velocity of the molecule will become –

(a) 3 times **(b) 9 times**

(c) (1/3) times **(d) √3 times**

Solution:(d)

T → 3T

$$V_{rms} = \sqrt{\frac{3RT}{M_0}} = \sqrt{\frac{3rRT}{M}} = \sqrt{\frac{3PV}{M}} = \sqrt{\frac{3P}{\delta}}$$

5. The temperature of an ideal gas is increased from 27°C to 927°C. The rms speed of its molecules becomes –

(a) Twice **(b) Half**

(c) Four times **(d) One fourth**

Solution:(a)

6. If the r.m.s. velocity of a gas at a given temperature (Kelvin scale) is 300 m/sec. What will be the r.m.s. velocity of a gas having twice the molecular weight and half the temperature on Kelvin scale?

(a) 300 m/sec **(b) 600 m/sec**

(c) 75 m/sec **(d) 150 m/sec**

Solution:(d)

$$V_{rms} = \sqrt{\frac{3RT}{M_0}} = V = 300$$

$$= \sqrt{\frac{3R\left(T/2\right)}{\left(2M_0\right)}} = \sqrt{\frac{3RT}{M_0}} \times \frac{1}{2} = 150 \text{ m/sec}$$

7. The r.m.s speed of oxygen molecule in a gas is v. If the temperature is doubled and the O_2 molecule dissociate into oxygen atoms, the r.m.s speed will become

(a) V **(b)** $v\sqrt{2}$

(c) 2v **(d) 4v**

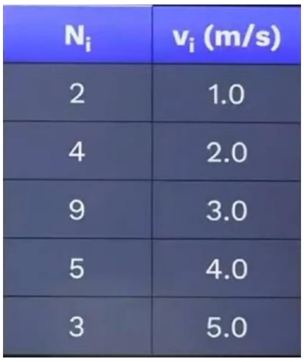
Solution:(c)

$$v = \sqrt{\frac{3R(T)}{32}}$$

$$v' = \sqrt{\frac{3R(2T)}{16}} = 2v$$

8. Given the following group of paritcles, N_i represents the number of molecules with speed v_i Calculate average speed.

Ans till two places after decimal



N_i	v_i (m/s)
2	1.0
4	2.0
9	3.0
5	4.0
3	5.0

Solution:

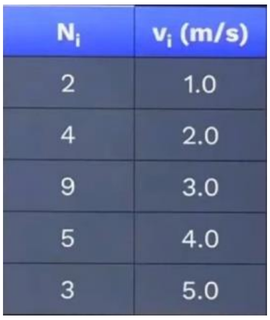
$$v_{avg} = \frac{v_1+v_2+......+v_n}{n}$$

$$v_{ag} = \frac{(2\times1)+(4\times2)+(9\times3)+(5\times4)+(3\times5)}{23}$$

$$= \frac{72}{23} = 3.13 \text{ m/s}$$

9. Given the following group of particles, N_i represents the number of molecules with speed v_i Calculate rms speed.

Ans till two places after decimal



N_i	v_i (m/s)
2	1.0
4	2.0
9	3.0
5	4.0
3	5.0

Solution:

Root means sq

$$v_{rms}^2 = \frac{N_1v_1^2 + N_2v_2^2 + ... + N_iv_i^2}{N_1 + N_2 + ... + N_i}$$

$$v_{rms} = \sqrt{\frac{2\times1^2 + 4\times2^2 + 9\times3^2 + 5\times4^2 + 3\times5^2}{23}} =$$

333

10. At a certain temperature, the r.m.s. velocity for O_2 is 400 m/sec. At the same temperature, the r.m.s. velocity for H_2 molecules will be

(a) 100 m/sec **(b) 25 m/sec**

(c) 1600 m/sec **(d) 6400 m/sec**

Solution:

The answer is C.

Given,

$$V_{O_2} = \frac{400m}{s}, Mass\ of\ O_2, M_{O_2} = 32\ and\ Mass\ of\ H_2, M_{H_2} = 2$$

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\frac{V_{H_2}}{V_{O_2}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}}$$

$$V_{H_2} = 400 \times \sqrt{\frac{32}{2}} = 1600m/s$$

11. At a pressure of 24 × 10^5 dyne/cm^2 , the volume of O_2 is 10 litre and mass is 20 gm. The r.m.s. velocity will be

(a) 800 m/sec **(b) 400 m/sec**

(c) 600 m/sec **(d) Data is incomplete**

Solution:

The correct answer is C.

$$v_{rms} = \sqrt{\frac{3PV}{m}}$$

$$= \sqrt{\frac{3\times24\times10^4\times10\times10^{-3}}{20\times10^{-3}}} = 600\text{m/s}$$

12. At what temperature will the oxygen molecules have the same root mean square speed as hydrogen molecules at 200 K

(a) 800 K **(b) 1600 K**

(c) 2400 K **(d) 3200 K**

Solution:

The correct answer is D.

$$rms\ speed\ v_{rms} = \sqrt{\frac{3RT}{M}}$$

Equating rms speed expression for oxygen and hydrogen molecules;

$$\sqrt{\frac{3R\times200}{2}} = \sqrt{\frac{3RT}{32}}$$

Solving for T,

$T = 3200K = (3200 - 273)℃ = 2927℃$

13. A sample of gas is at 0°C. To what temperature it must be raised in order to double the r.m.s. speed of the molecule

(a) 270°C **(b) 819°C**

(c) 1090°C **(d) 100°C**

Solution:

The correct answer is B.

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

$V_{rms} = \sqrt{T}$

To double the rms speed, temperature should be made four times is

$i.e. T_2 = 4T_1$

$\therefore T_2 = 4 \times 273 = 1092K$

$\therefore T_2 = 819℃$

14. The temperature of an ideal gas is increased from 27°C to 927°C. The root mean square speed of its molecules becomes

(a) Twice **(b) Half**

(c) Four times **(d) One-fourth**

Solution:

The correct answer is A.

$T_1 = 273 + 27 = 300K$

$T_2 = 273 + 927 = 1200K$

$V_{rms} \propto \sqrt{T}$

(T) has become four times.

Therefore, v_(rms) will become two times.

15. The speeds of 5 molecules of a gas (in arbitrary units) are as follows : 2, 3, 4, 5, 6. The root mean square speed for these molecules is

(a) 2.91 **(b) 3.52**

(c) 4.00 **(d) 4.24**

Solution:

The correct answer is D.

$$V_{rms} = \sqrt{\frac{{v_1}^2 + {v_2}^2 + {v_3}^2 + {v_4}^2 + {v_5}^2}{5}}$$

$$= \sqrt{\frac{2^2 + 3^2 + 4^2 + 5^2 + 6^2}{5}}$$

$$= \sqrt{\frac{100}{5}} = \sqrt{20} = 4.24$$

16. The root mean square speed of hydrogen molecules at 300 K is 1930 m/s. Then the root mean square speed of oxygen molecules at 900 K will be

(a) $1930\sqrt{3}$ **m/s** **(b) 836 m/s**

(c) 643 m/s **(d)** $\frac{1930}{\sqrt{3}}$ **m/s**

Solution:

The correct answer is B.

$$\frac{C_2}{C_1} = \sqrt{\frac{T_2}{T_1}\frac{M_1}{M_2}}$$

$$= \sqrt{\frac{900}{300} \times \frac{2}{32}} = \sqrt{\frac{3}{16}}$$

$$C_2 = \frac{\sqrt{3}}{4} \times C_1$$

$$= \frac{\sqrt{3}}{4} \times 1930$$

$$= 836\ m/s$$

17. The r.m.s. speed of a group of 7 gas molecules having speeds (6, 4, 2, 0, – 2, – 4, – 6) m/s is

(a) 1.5 m/s **(b) 3.4 m/s**

(c) 9 m/s **(d) 4 m/s**

Solution:

The correct answer is D.

Square-root of the mean square velocity of gas molecules, is called root-mean.

Square velocity (rms),

$$v_{rms} = \sqrt{\frac{{v_1}^2 + {v_2}^2 + \cdots {v_N}^2}{N}}$$

Given

$v_1 = 6\ m/s$

$v_2 = 4m/s, v_3 = 2m/s, v_4 = 0m/s$

$v_5 = -\frac{2m}{s}, v_6 = -\frac{4m}{s}, v_7 = -6m/s$

$$U_{rms} = \sqrt{\frac{\frac{6^2 + 4^2 + 2^2}{(-2)^2 + (-4)^2 + (-6)^2}}{7}}$$

$$= \sqrt{\frac{112}{7}} = \sqrt{16} = 4m/s$$

18. The velocities of three molecules are 3v, 4v and 5v. Their rms speed will be

(a) $\sqrt{\frac{50}{3}}v$ **(b)** $\sqrt{\frac{3}{50}}v$

(c) $\frac{50}{3}v$ **(d)** $\frac{3}{50}v$

Solution:

Correct option is A.

$V_{rms}(root\ mean\ square\ velocity)$

$= \sqrt{{v_1}^2 + {v_2}^2 + \cdots + {v_n}^2}(standard\ result)$

$v_1 = 3V$

$v_2 = 4V \quad (Given)$

$v_3 = 5V$

$$V_{rms} = \sqrt{(\frac{3^2+4^2+5^2}{3})}\text{v}$$

$$= \sqrt{\frac{50}{3}}V$$

19. The temperature of a gas is doubled (i) on absolute scale (ii) on centigrade scale. The increase in root mean square velocity of gas will be

(a) More in case (i)

(b) More in case (ii)

(c) Same in both case

(d) Information not sufficient

Solution:

Correct option is A.

The temperature of a gas is doubled (i) on absolute scale (ii) on centigrade scale. The increase in root mean square velocity of gas will be more in case (i)

The expression for RMS speed is

$$u = \sqrt{\frac{3RT}{M}}$$

Thus,

$$\frac{u'}{u} = \sqrt{\frac{T'}{T}}$$

(i)Temperature is doubled from 100 K to 200 K

$$\frac{u'}{u} = \sqrt{\frac{200}{100}} = 1.414$$

(ii)Temperature is doubled from 100 C to 200 C

$$\frac{u'}{u} = \sqrt{\frac{473}{373}} = 1.13$$

20. One mole of an ideal gas at STP is heated in an insulated closed container until the average speed of its molecules is doubled. Its pressure would therefore increase by factor.

(a) 1.5 **(b) $\sqrt{2}$**

(c) 2 **(d) 4**

Solution:

Correct option is D.

$$Average\ velocity = V_{AV} = \sqrt{\frac{2RT}{M}}$$

$V_{AV} \propto \sqrt{T}$

$V_{av1}/V_{av2} = \sqrt{T_1}/\sqrt{T_2}$

We know the average speed of its molecules is doubled (given)

$V_{av1}/V_{av2} = \sqrt{T_1}/\sqrt{T_2} = 1/2$

If we say initial temperature is $T_1 = T$,

$\sqrt{T_1}/\sqrt{T_2} = 1/2$

$\sqrt{T_2} = 2\sqrt{T}$

$T_2 = 4T$

We know ideal gas equation

$PV = nRT$

$P = (nR/V)T$

$P \propto T$

$\therefore P_1/P_2 \propto T/4T$

$\therefore P_2 = 4P_1$

21. Three particles have speeds of 2u, 10u and 11u. Which of the following statements is correct?

(a) The r.m.s. speed exceeds the mean speed by about u.

(b) The mean speed exceeds the r.m.s. speed by about u.

(c) The r.m.s. speed equals the mean speed.

(d) The r.m.s. speed exceeds the mean speed by more than 2u.

Solution:

Correct option is A.

$$V_{rms} = U\sqrt{\frac{4+100+121}{3}} = 5\sqrt{3}U = 8.66U$$

$$V_{avg} = \left(\frac{2+10+11}{3}\right)U = \frac{23}{3}U = 7.66U$$

$$V_{rms} - V_{avg} \cong IU$$

22. How many degrees of freedom do the molecules of a gas have if under standard conditions the gas density ρ = 1.3 kgm^{-3} and the velocity of sound propagation in it is v = 330 m/s?

Solution:

In standard condition, pressure of gas is P = 10^5N/m^2.

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3(10^5)}{1.3}} = 480m/s$$

Using

$$\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}}$$

Or

$$\frac{480}{330} = \sqrt{\frac{3}{\gamma}} \Rightarrow \gamma = 1.4$$

Also,

$$\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}} = \sqrt{\frac{3f}{f+2}}$$

Or

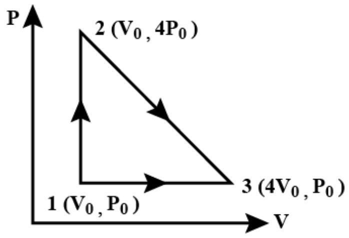
$$\frac{3}{\gamma} = \frac{3f}{f+2}$$

Or

$$f + 2 = \gamma f = 1.4f$$

$$\therefore f = \frac{2}{0.4} = 5$$

23. One mole of an ideal diatomic gas is taken through the cycle as shown in the figure.



1 → 2 : isochoric process

2 → 3 : straight line on P-V diagram

3 → 1 : isobaric process

The average molecular speed of the gas in the states 1, 2 and 3 are in the ratio

(a) 1 : 2 : 2 **(b) 1 : $\sqrt{2}$: $\sqrt{2}$**

(c) 1 : 1 : 1 **(d) 1 : 2 : 4**

Solution:

Correct option is A.

State 1

Let temperature be T_o

State 2

Since 1-2 is isochoric process $\frac{P}{T}$ = constant

Thus Temperature = $4T_o$

State 3

Since 3-1 is isochoric process $\frac{V}{T}$ = constant

Thus Temperature = $4T_o$

The average molecular speed is directly proportional to $\sqrt{\frac{RT}{M}}$

$v \propto \sqrt{T}$

Hence,

State 1 $v \propto T_\circ$

State 2 $v \propto \sqrt{4T_\circ} = 2\sqrt{T_\circ}$

State 3 $v \propto \sqrt{4T_\circ} = 2\sqrt{T_\circ}$

Ratio = 1 : 2 : 2

24. Three particles have speeds of 2u, 3u and 4u. Which of the following statements is correct?

(a) The r.m.s. speed exceeds the mean speed.

(b) The mean speed exceeds the r.m.s. speed.

(c) The r.m.s. speed equals the mean speed.

(d) The r.m.s. speed exceeds the mean speed by less than u.

Solution:

Correct options are A and D.

$$V_{mean} = \frac{24 + 34 + 44}{3}$$

$$= 3u$$

$$V_{rms} = \sqrt{\frac{(24)^2 + (34)^2 + (44)^2}{3}}$$

$$= \sqrt{\frac{44^2 + 54^2 + 164^2}{3}}$$

$$= \sqrt{\frac{29}{3}}\, u = 3.1\, u$$

$V_{rms} > V_{mean}$

25. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be:

(a) 100 m/s **(b) 80$\sqrt{5}$ m/s**

(c) 100$\sqrt{5}$ m/s **(d) 80 m/s**

Solution:

(c) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \frac{(273+127)}{(273+237)}$$

$$= \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore\ v_2 = \frac{\sqrt{5}}{2} v_1 = \frac{\sqrt{5}}{2} \times 200$$

$= 100\sqrt{5}$ m/s.

26. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to :

[Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K
Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg
Radius of Earth : 6.4×10^6 m Gravitational acceleration on Earth = 10 ms^{-2}]

(a) 800 K **(b) 3×10^5 K**

(c) 10^4 K **(d) 650 K**

Solution:

(c) $v_{rms} = v_e$

$$\sqrt{\frac{3RT}{M}} = 11.2 \times 10^3$$

or $\sqrt{\frac{3kT}{m}} = 11.2 \times 10^3$

or $\sqrt{\frac{3 \times 1.38 \times 10^{-23} T}{2 \times 10^{-3}}} = 11.2 \times 10^3$

$\therefore$ T = 10^4 K

27. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container.

The ratio of their rms speeds $\left[\frac{V_{rms}(helium)}{V_{rms}(\arg on)}\right]$ **is close to :**

(a) 3.16 **(b) 0.32**

(c) 0.45 **(d) 2.24**

Solution:

(a) Using $\frac{V_{1rms}}{V_{2rms}}=\sqrt{\frac{M_2}{M_1}}$

$$\frac{V_{rms}(He)}{V_{rms}(Ar)}=\sqrt{\frac{M_{Ar}}{M_{He}}}=\sqrt{\frac{40}{4}} = 3.16$$

28. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1}. The gas is:

(a) H_2 **(b) Cl_2**

(c) O_2 **(d) F_2**

Solution:

(a) $\because C=\sqrt{\frac{3RT}{M}}$

$$(1930)^2=\frac{3\times8.314\times300}{M}$$

$$M=\frac{3\times8.314\times300}{1930\times1930}$$

$\approx 2 \times 10^{-3}$ kg

The gas is H_2.

29. In the isothermal expansion of 10g of gas from volume V to 2V the work done by the gas is 575J. What is the root mean square speed of the molecules of the gas at that temperature?

(a) 398m/s **(b) 520m/s**

(c) 499m/s **(d) 532m/s**

Solution:

(c) $v_{rms}=\sqrt{\frac{3\rho v}{mass\,of\,the\,gas}}$

30. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is _____________. (Molar mass of N_2 gas 28 g);

Solution:

Room mean square speed is given by

$$v_{rms}=\sqrt{\frac{3RT}{M}}$$

Here, M = Molar mass of gas molecule

T = temperature of the gas molecule

We have given $v_{N_2}=v_{H_2}$

$$\therefore\sqrt{\frac{3RT_{N_2}}{M_{N_2}}}=\sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$$

$$\frac{T_{H_2}}{2}=\frac{573}{28}$$

T_{H_2} = 41 K

31. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{rms\ (helium)}{rms\ (argon)}\right)$ is

(a) 0.32 **(b) 0.45**

(c) 2.24 **(d) 3.16**

Solution:

(d) Using $V_{rms}=\sqrt{\frac{3RT}{M}}\Rightarrow V_{rms}\propto\frac{1}{\sqrt{M}}$

$$\frac{V_{rms}(helium)}{V_{rms}(argon)}=\sqrt{\frac{M_{argon}}{M_{helium}}}=\sqrt{\frac{40}{4}}=\sqrt{10}$$

≈ 3.16

32. The molecules of a given mass of a gas have r.m.s. velocity of 200ms^{-1} at 27°C and 1.0 × 10^5 Nm^{-2} pressure. When the temperature and pressure of the gas are respectively, 127°C and 0.05 × 10^5 Nm^{-2}, the rms velocity of its molecules in ms^{-1} is

(a) **(b)** $\frac{100\sqrt{2}}{3}$

(c) $\frac{100}{3}$ **(d)** $100\sqrt{2}$

Solution:

(a) It is given that

$\frac{400}{\sqrt{3}}$ V$_{rms}$ = 200 ms^{-1},T$_1$ = 300 K, P$_1$ = 10^5 N/m^2

T$_2$ = 400 K, P$_2$ = 0.05 × 10^5 N/m^2

As, rms velocity of gas molecules,

$\because\quad v_{rms}\propto\sqrt{T}\qquad\left(\because v_{rms}=\sqrt{\frac{3RT}{m}}\right)$

For two different cases

$$\frac{(v_{rms})_1}{(v_{rms})_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{200}{(v_{rms})_2} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$$(v_{rms})_2 = \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}} ms^{-1}$$

Expert **Various Speeds**

1. The root mean square velocity of the molecules in a sample of helium is 5/7th that of the molecules in a sample of hydrogen. If the temperature of hydrogen sample is 0°C, then the temperature of the helium sample is about

(a) 0°C **(b) 0 K**

(c) 273°C **(d) 100°C**

Solution:

The answer is A.

$$\frac{(v_{rms})_{He}}{(v_{rms})_{H_2}} = \sqrt{\frac{TH_e}{TH_2} . \frac{(M_0)H_2}{(M_0)He}}$$

$$\frac{5}{7} = \sqrt{\frac{TH_e}{(0+273)} . \frac{2}{4}} = \sqrt{\frac{TH_e}{546}}$$

$$TH_e = \left(\frac{5}{7}\right)^2 \times 546 = 278.6K$$

$$= 278.6 - 273 = 5.6°\text{C} \cong 0°\text{C}$$

2. The molecules of a given mass of a gas have a r.m.s. velocity of 200 m/sec at 27°C and 1.0 × 10^5 N/m^2 pressure. When the temperature is 127°C and pressure is 0.5 × 10^5 N/m^2 , the r.m.s. velocity in m/sec will be

(a) $\frac{100\sqrt{2}}{3}$ **(b)** $100\sqrt{2}$

(c) $\frac{400}{\sqrt{3}}$ **(d) None of the above**

Solution:

The answer is C.

We know that:

$$v \propto \sqrt{T}$$

$$\Rightarrow \frac{v}{200} = \sqrt{\frac{400}{300}}$$

$$\Rightarrow v = \frac{200 \times 2}{\sqrt{3}} = \frac{400}{\sqrt{3}} m/s$$

3. The r.m.s. speed of the molecules of a gas in a vessel is 400 ms^{-1} . If half of the gas leaks out, at constant temperature, the r.m.s. speed of the remaining molecules will be

(a) 800 ms^{-1} **(b)** $400\sqrt{2}$ **ms^{-1}**

(c) 400 ms^{-1} **(d) 200 ms^{-1}**

Solution:

The answer is C.

Root mean square velocity does not depends upon the quantity of gas. For a given gas and at constant temperature it always remains same.

4. A cubical box with porous walls containing an equal number of O_2 and H_2 molecules is placed in a large evacuated chamber. The entire system is maintained at constant temperature T. The ratio of v_{rms} of O_2 molecules to that of the v_{rms} of H_2 molecules, found in the chamber outside the box after a short interval is

(a) $\frac{1}{2\sqrt{2}}$ **(b)** $\frac{1}{4}$

(c) $\frac{1}{\sqrt{2}}$ **(d)** $\sqrt{2}$

Solution:

The correct answer is B.

According to equation,

$$\frac{(v_{rms})o_2}{(v_{rms})H_2}$$

$$= \sqrt{\frac{(M_0)H_2}{(M_0)O_2}} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$

5. N(<100) molecules of a gas have velocities 1, 2, 3........ N/km/s respectively. Then

(a) RMS speed and average speed of molecules is same.

(b) Ratio of RMS speed to Average speed is √(2N+1)(N+1)/6N

(c) Ratio of RMS speed to Average speed is √(2N+1)(N+1)/6

(d) Ratio of RMS speed to Average speed of a molecule is 2/√6 x √(2N+1)/(N+1)

Solution:

Correct option is D.

As we know, sum of the first n natural numbers is n[n + 1]/2 and sum of the squares of first n natural numbers is n[n + 1][2n + 1]/6.

$$So\ U_{rms} = \sqrt{\text{n[n + 1][2n + 1]/6}}\ and\ U_{avg} = \text{n[n + 1]/2}$$

So, ratio of rms speed to average speed of a molecule is $2/\sqrt{6}x\sqrt{(2N+1)/(N+1)}$

The root mean square speed (u_{rms})

$$u=\sqrt{\frac{{u_1}^2+{u_2}^2+\cdots+{u_n}^2}{n}}$$

Average speed (u_{avg}): It is arithmetic mean of the various speeds of the molecules.

$$u=\frac{{u_1}^2+{u_2}^2+\cdots+{u_n}^2}{n}$$

6. At what temperature is the root mean squared speed of an atom in an argon gas cylinder equal the rms speed of a helium gas atom at –20°C?

(Atomic mass of Ar = 39.9, of He = 4.0).

Solution:

Let C and C' be the r.m.s. velocity of argon and helium gas atoms at temperature TK and T'K respectively.

Here,

$$M=39.9, M'=4.0, T=?, T'=-20+273$$
$$=253K$$

Now,

$$C=\sqrt{\frac{3RT}{M}}=\sqrt{\frac{3RT}{39.9}}\ and$$

$$C'=\sqrt{\frac{3RT'}{M'}}=\sqrt{\frac{3R\times 253}{4}}$$

Since,

$$C=C'$$

$$\sqrt{\frac{3RT}{39.9}}=\sqrt{\frac{3R\times 253}{4}}\ or\ T=\frac{39.9\times 253}{4}$$
$$=2524K$$

7. A mixture of 4 gm helium and 28 gm of nitrogen in enclosed in a vessel of constant volume at 300°K. Find the quantity of heat absorbed by the mixture to double the root mean velocity of its molecules.

(R = Universal gas constant)

Solution:

Given,

$$T=300K\ , v'=2v$$

Root mean square velocity

$$v=\sqrt{\frac{3RT}{M}}$$

$$\frac{v'}{v}=\sqrt{\frac{T'}{T}}$$

$$\therefore \frac{2v}{v}=\sqrt{\frac{T'}{300}}\quad T'=1200K$$

Number of moles of H_e,

$$n_1=\frac{4}{4}=1$$

Number of moles N_2,

$$n_2=\frac{28}{28}=1$$

$$total\ number\ of\ moles\ n=1+1=2$$

For Helium,

$$C_{v_1}=\frac{3}{2}R$$

For Nitrogen,

$$C_{v_2}=\frac{5}{2}R$$

Specific heat capacity of the mixture

$$C_v=\frac{n_1\times C_{v_1}+n_2\times C_{v_2}}{n_1+n_2}$$

$$C_v=\frac{1\times\frac{3}{2}R+1\times\frac{5}{2}R}{1+1}=2R$$

Net heat absorbed at constant volume

$$Q_v=nC_v(T'-T)$$

$$\therefore Q_v=2\times 2R(1200-300)=3600R$$

8. The temperature of a gas consisting of rigid diatomic molecules is T = 300 K. Calculate the angular root mean square velocity of a rotating molecule if its moment of inertia is I = 2.1×10

Solution:

By formula,

If I is the moment of inertia then

$$\frac{1}{2}I\omega^2=\frac{1}{2}\times f\times K_BT$$

Where f = degree of freedom

K_B = Boltzmann constant

T = temperature

The root mean square angular velocity,

$$\omega^2=\frac{fK_BT}{I}$$

$$\omega=\sqrt{\frac{fK_BT}{I}}$$

For diatomic molecules, f = 2

$$\omega = \sqrt{\frac{2 \times 1.3 \times 10^{-23} \times 300}{2.1 \times 10^{-39}}}$$

$$= 6.3 \times 10^{12}\ rad/sec$$

$$\omega = 6.3 \times 10^{12}\ rad/sec$$

In angular root mean square velocity is

$$6.3 \times 10^{12}\ rad/sec$$

Beginner Maxwell's Distribution of Speed

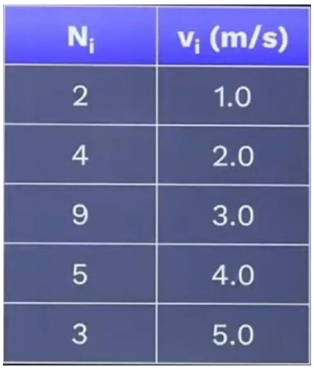
1. The average velocity of molecules of a gas must be zero if the gas as a whole and the container are not in translational motion. Explain how it can be that the average speed is not zero.

Solution:

Molecules of a gas move randomly; hence average velocity is zero since velocity is a vector quantity.

There is a speed distribution for the molecules, average speed will be non-zero.

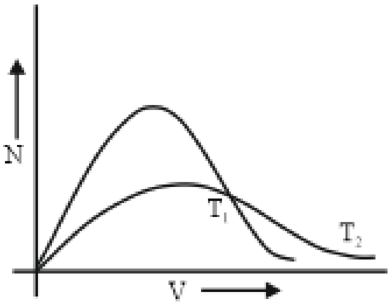
2. Given the following group of particles, N_i represents the number of molecules with speed v_i Calculate most probable speed.



N_i	v_i (m/s)
2	1.0
4	2.0
9	3.0
5	4.0
3	5.0

Solution:3m/s

3. Maxwell's velocity distribution curve is given for two different temperatures. For the given curves –



(a) $T_1 > T_2$ **(b) $T_1 < T_2$**

(c) $T_1 \leq T_2$ **(d) $T_1 = T_2$**

Solution:(b)

Beginner Freedom & Molar Heat

4. A monoatomic gas molecule has

(a) Three degrees of freedom

(b) Four degrees of freedom

(c) Five degrees of freedom

(d) Six degrees of freedom

Solution:

The answer is A.

Monoatomic gas molecule has three degrees of freedom.

5. A diatomic molecule has how many degrees of freedom

(a) 3 **(b) 4**

(c) 5 **(d) 6**

Solution:

The answer is C.

Number of degree of freedom d = 3N – 1

Where N is the number of atoms in a molecules

In diatomic molecules,

N = 2

$\Rightarrow d = 3(2) - 1 = 5$

Hence diatomic molecule has 5 degrees of freedom (3 translational and 2 rotational).

6. A cylinder rolls without slipping down an inclined plane, the number of degrees of freedom it has, is

(a) 2 **(b) 3**

(c) 5 **(d) 1**

Solution:

The answer is A.

If the body is rolling on a plane, then its degree of freedom is 2: one for rotation about the body's axis and one for translational of its center of gravity in forward and backward direction.

If there is no slipping, the translational can be calculated from the rotation and the radius of block. Thus, the degree of freedom is degenerated to 2.

7. If the degree of freedom of a gas are f, then the ratio of two specific heats C_P/C_V is given by

(a) $\frac{2}{f}+1$ **(b)** $1-\frac{2}{f}$

(c) $1+\frac{1}{f}$ **(d)** $1-\frac{1}{f}$

Solution:

The correct answer is A.

$(C_P/C_V) = r = 1 + (2/f)$

8. The degrees of freedom of a triatomic gas is

(a) 2 **(b) 4**

(c) 6 **(d) 8**

Solution:

The correct answer is C.

The degree of freedom of a triatomic gas is 3 translational, 3 rotational.

9. A diatomic gas molecule has translational, rotational and vibrational degrees of freedom. The C_P/C_V is

(a) 1.67 **(b) 1.4**

(c) 1.29 **(d) 1.33**

Solution:

The correct answer is D.

The diatomic molecule has total 6 degrees of freedom (3 translational, 2 rotational and 1 vibrational)

Now,

$$C_P \text{ is given as } \left(1+\frac{f}{2}\right)R = \left(1+\frac{6}{2}\right)R = 4R$$

$$\text{and } C_V \text{ is given as } \frac{f}{2}R = \frac{6}{2}R = 3R$$

Thus, we get

$$\frac{C_P}{C_V} = \frac{4}{3} = 1.33$$

10. For a gas $\frac{R}{C_V} = 0.67$. This gas is made up of molecules which are

(a) Diatomic

(b) Mixture of diatomic and polyatomic

(c) Monoatomic

(d) Polyatomic

Solution:

The correct answer is C.

$$\therefore R = C_P - C_V$$

$$\therefore \frac{R}{C_V} = \frac{C_P - C_V}{C_V} = 0.67$$

Or

$$\frac{C_P}{C_V} - 1 = 0.670 \text{ or } \frac{C_P}{C_V} = 1.67$$

The gas is made up of molecules which are monoatomic.

11. At constant volume, for different diatomic gases the molar specific heat is

(a) Same and 3 cal/mole/°C approximately

(b) Exactly equal and its value is 4 cal/mole/°C

(c) Will be totally different

(d) Approximately equal and its value is 5 cal/mole/°C

Solution:

The correct answer is D.

We know,

Molar specific heat,

$$C_V = \frac{fR}{2}$$

f is the degrees of freedom.

For diatomic gas, f = 5

The values of R is 1.98 cal/mol/^{o}C

Substitute,

$$C_V = \frac{5 \times (1.98)}{2} = 4.9s \approx 5 cal/mol/℃$$

12. At constant volume the specific heat of a gas is $\frac{3R}{2}$, then the value of 'γ' will be

(a) $\frac{3}{2}$ **(b)** $\frac{5}{2}$

(c) $\frac{5}{3}$ **(d) None of the above**

Solution:

The correct answer is C.

Specific heat at constant volume

$C_V = R/Y - 1 = 3R/2 \ (given)$

$\therefore y - 1 = 2/3$

$Y = 5/3$

13. Molar specific heat at constant volume is C_V for a monoatomic gas is

(a) $\frac{3}{2}R$ **(b)** $\frac{5}{2}R$

(c) 3R **(d) 2R**

Solution:

The correct answer is A.

For a monoatomic gas, the molar heat capacity at constant volume is given as,

$$C_V = \frac{3}{2}R$$

14. For a gas if γ = 1.4, then atomicity, C_P and C_V of the gas are respectively

(a) Monoatomic, $\frac{5}{2}R, \frac{3}{2}R$

(b) Monoatomic, $\frac{7}{2}R, \frac{5}{2}R$

(c) Diatomic, $\frac{7}{2}R, \frac{5}{2}R$

(d) Triatomic, $\frac{7}{2}R, \frac{5}{2}R$

Solution:

The correct answer is C.

Given,

$$r = 1.4 = \frac{14}{10} = \frac{7}{5}$$

We know,

$$C_P - C_V = R$$

$$r = \frac{7}{5} = \frac{C_P}{C_V} \qquad \therefore C_P = \frac{7}{5} C_V$$

$$\frac{7}{5} C_V - C_V = R$$

$$\frac{2}{5} C_V = R$$

$$C_P = \frac{7}{2} R$$

Diatomic gas.

15. For a gas the difference between the two specific heats is 4150 J/kg K. What is the specific heats at constant volume of gas if the ratio of specific heat is 1.4

(a) 8475 J/kg K **(b) 5186 J/kg**

(c) 1660 J/kg K **(d) 10375 J/kg K**

Solution:

The correct answer is D.

$$C_P - C_V = nR$$

$$C_P / C_V = 1.4$$

$$C_P - C_V = 4150$$

$$0.4 C_V = 4150$$

$$C_V = 4150/0.4$$

$$C_V = 10375 \text{ J/Kg K}$$

16. What is the ratio of specific heats of constant pressure and constant volume for NH_3

(a)1.33 **(b) 1.44**

(c) 1.28 **(d) 1.67**

Solution:

The correct answer is A.

Degree of freedom for NH_3 is 6.

$$\gamma = \frac{C_P}{C_V} = 1 + \frac{2}{f}$$

$$= 1 + \frac{2}{6} = \frac{4}{3} = 1.33$$

17. If two moles of diatomic gas and one mole of mono atomic gas are mixed then the ratio of specific heats is

(a) $\frac{7}{3}$ **(b)** $\frac{5}{4}$

(c) $\frac{19}{13}$ **(d)** $\frac{15}{19}$

Solution:

The correct answer is C.

$$C_{Peq} = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 + n_2} C_{Veq} = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 + n_2}$$

$$r_{eq} = \frac{C_{Peq}}{C_{Veq}} = \frac{2 \times \frac{7}{2} + 1 \times \frac{5}{2}}{2 \times \frac{5}{2} + 1 \times \frac{3}{2}} = \frac{19}{13}$$

18. For a gas if ratio of specific heats at constant pressure and volume is γ then value of degrees of freedom is

(a) $\frac{3\gamma - 1}{2\gamma - 1}$ **(b)** $\frac{2}{\gamma - 1}$

(c) $\frac{9}{2}(\gamma - 1)$ **(d)** $\frac{25}{2}(\gamma - 1)$

Solution:

The correct answer is B.

As

$$Y = 1 + (2/f)$$

$$Y - 1 = (2/f)$$

$$f = [2/(y - 1)]$$

19. The degrees of freedom of a stationary rigid body about its axis will be

(a) One **(b) Two**

(c) Three **(d) Four**

Solution:

The correct answer is C.

A stationary rigid body rotating about its axis has three degree of freedom.

20. A gaseous mixture consists of 16g of helium and 16g of oxygen. The ratio $\frac{C_P}{C_V}$ **of the mixture is**

(a) 1.4 **(b) 1.54**

(c) 1.59 **(d) 1.62**

Solution:

The correct answer is D.

$$M_{he} = 4, \qquad m_{he} = 16g$$

$M_{OX} = 32\,, \qquad m_{OX} = 16g$

Specific heat of mixture at constant volume is given by,

$$C_V = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} \quad ----(1)$$

For Helium gas,

No. Of moles,

$$n_1 = \frac{16}{4} = 4$$

$$\gamma 1 = \frac{5}{3}$$

For Oxygen gas,

No. Of moles,

$$n_2 = \frac{m_{OX}}{M_{OX}}$$

$$n_2 = \frac{16}{32} = \frac{1}{2}$$

$$\gamma_2 = \frac{7}{5}$$

And specific heat constant volume of helium gas,

$$C_{V1} = \frac{R}{\gamma 1 - 1} = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2} R$$

The specific heat of oxygen at constant volume,

$$C_{V1} = \frac{R}{\gamma 2 - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5}{2} R$$

From equation (1),

$$C_V = \frac{4 \times \frac{3}{2} R + \frac{1}{2} \times \frac{5}{2} R}{\frac{4}{1} + \frac{1}{2}}$$

$$C_V = \frac{29}{18} R = \frac{R}{\gamma - 1}$$

$$\gamma = \frac{18}{29} + 1$$

$$\frac{C_P}{C_V} = \gamma = 1.62$$

21. A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at a temperature. T. Neglecting all vibrational moles, the total internal energy of the system is

(a) 4RT **(b) 15RT**

(c) 9RT **(d) 11RT**

Solution:

Correct option is D.

Internal energy of 2 moles of oxygen

$$U_{O2} = \mu \left(\frac{5}{2} RT \right)$$

$$= 2.\frac{5}{2} RT = 5RT$$

Internal energy of 4 moles of Argon.

$$U_{AR} = \mu \left(\frac{3}{2} RT \right)$$

$$= 4.\frac{3}{2} RT = 6RT$$

Therefore, total internal energy

$U = U_{O2} + U_{AR}$=11 RT

22. The ratio of specific heats of a gas is $\frac{9}{7}$, then the number of degrees of freedom of the gas molecules for translational motion is :

(a) 7 **(b) 3**

(c) 6 **(d) none**

Solution:

Correct option is B.

The ratio of a specific heats of a gas is 9/7, then the number of degrees of freedom of the gas molecules for translational motion is 3. We will use the standard formula $\frac{(f+2)}{f} = \gamma$. On cross multiplication and solving we get f = 7, but total translational will be 3 only.

23. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma \left(= \frac{C_p}{C_v} \right)$ are given, respectively, by:

(a) U = $\frac{5}{2}$RT and $\gamma = \frac{6}{5}$

(b) U = 5RT and $\gamma = \frac{7}{5}$

(c) U = $\frac{5}{2}$RT and $\gamma = \frac{7}{5}$

(d) U = 5RT and $\gamma = \frac{6}{5}$

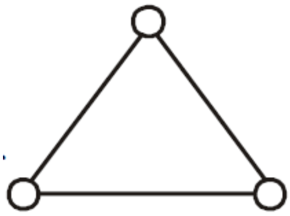
Solution:

(c) Total degree of freedom f = 3 + 2 = 5

Total energy, $U = \frac{nRT}{2} = \frac{5RT}{2}$

And $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$

24. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is:



(a) $\frac{5}{2}RT$ **(b)** $\frac{3}{2}RT$

(c) $\frac{9}{2}RT$ **(d) 3RT**

Solution:

(d) Here degree of freedom, f = 3 + 3 = 6 for triatomic non-linear molecule.

Internal energy of a mole of the gas at temperature T,

$$U=\frac{f}{2}nRT=\frac{6}{2}RT=3RT$$

25. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) of gas A and B, respectively is:

(a) 7:9 **(b) 5:9**

(c) 3:5 **(d) 5:7**

Solution:

(d) Specific heat of gas at constant volume

$C_v=\frac{1}{2}fR$; f = degree of freedom

For gas A (diatomic)

f = 5 (3 translational + 2 rotational)

$$\therefore\ C_v^A=\frac{5}{2}R$$

For gas B (diatomic) in addition to (3 translational + 2 rotational) 2 vibrational degrees of freedom.

$$\therefore\ C_v^B=\frac{7}{2}R$$

Hence $\frac{C_v^A}{C_v^B}=\frac{\frac{5}{2}R}{\frac{7}{2}R}=\frac{5}{7}$

26. Consider a mixture of n moles of helium gas and 2n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_P/C_V value will be:

(a) 19/13 **(b) 67/45**

(c) 40/27 **(d) 23/15**

Solution:

(a) Helium is a monoatomic gas and Oxygen is a diatomic gas.

For helium, $C_{V_1}=\frac{3}{2}R$ and $C_{P_1}=\frac{5}{2}R$

For oxygen, $C_{V_2}=\frac{5}{2}R$ and $C_{P_2}=\frac{7}{2}R$

$$\gamma=\frac{N_1C_{P_1}+N_2C_{p_2}}{N_1C_{V_1}+N_2C_{V_2}}$$

$$\gamma=\frac{n.\frac{5}{2}R+2n.\frac{7}{2}R}{n.\frac{3}{2}R+2n.\frac{5}{2}R}=\frac{19nR\times 2}{2(13nR)}$$

$$\therefore\ \left(\frac{C_P}{C_V}\right)_{mixture}=\frac{19}{13}$$

27. Two moles of an ideal gas with $\frac{C_p}{C_V}=\frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_V}=\frac{4}{3}$. The value of $\frac{C_p}{C_V}$ for the mixture is:

(a) 1.45 **(b) 1.50**

(c) 1.47 **(d) 1.42**

Solution:

(d) Using, $\gamma_{mixture}=\frac{n_1C_{p_1}+n_2C_{p_2}}{n_1C_{v_1}+n_2C_{v_2}}$

$$\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}=\frac{n_1+n_2}{\gamma_m-1}$$

$$\frac{3}{\frac{4}{3}-1}+\frac{2}{\frac{5}{3}-1}=\frac{5}{\gamma_m-1}$$

$$\frac{9}{1}+\frac{2\times 3}{2}=\frac{5}{\gamma_m-1}$$

$$\gamma_m-1=\frac{5}{12}$$

$$\gamma_m = \frac{17}{12} = 1.42$$

28. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? (R = 8.3 J/mol K)

(a) 19.7 J/mol L **(b) 15.7 J/mol K**

(c) 17.4J/mol K **(d) 21.6 J/mol K**

Solution:

(c) $$[C_v]_{\min} = \frac{n_1\left[C_{v_1}\right]+n_2\left[C_{v_2}\right]}{n_1+n_2}$$

$$= \left[\frac{2\times\frac{3R}{2}+3\times\frac{5R}{2}}{2+3}\right]$$

= 2.1 R = 2.1 × 8.3= 17.4 J/mol-k

29. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process ?

(a) 25 J **(b) 35 J**

(c) 30J **(d) 40J**

Solution:

(b) $F = \frac{C_v}{C_p} = \frac{1}{r} = \frac{1}{(7/5)} = \frac{5}{7}$

or $\frac{W}{Q} = 1-\frac{5}{7} = \frac{2}{7}$

or $Q = \frac{7}{2}W = \frac{7\times 10}{2}$ = 35 J

30. Two moles of helium are mixed with n with moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value of n is

(a) 3/2 **(b) 2**

(c) 1 **(d) 3**

Solution:

(b) Using formula,

$$\gamma_{mixture} = \left(\frac{C_p}{C_v}\right)_{mix} = \frac{\frac{n_1\gamma_1}{\gamma_1-1}+\frac{n_2\gamma_2}{\gamma_2-1}}{\frac{n_1}{\gamma_1-1}+\frac{n_2}{\gamma_2-1}}$$

Putting the value of

n 1 = 2, n2 = n, $\left(\frac{C_p}{C_v}\right)_{mix} = \frac{3}{2}$

$\gamma_1 = \frac{5}{3}$, $\gamma_2 = \frac{7}{5}$ and solving we get, n = 2

31. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v$ = a for hydrogen gas

$C_p - C_v$ = b for nitrogen gas

The correct relation between a and b is :

(a) a = 14b **(b) a = 28 b**

(c) $a = \frac{1}{14}b$ **(d) a = b**

Solution:

(a) As we know $C_p - C_v = R$

Where C_p and C_v are molar specific heat capacities

or, $C_p - C_v = \frac{R}{M}$

For hydrogen $(M = 2)\, C_p - C_v = a = \frac{R}{2}$

For nitrogen $(M = 28)\, C_p - C_v = b = \frac{R}{28}$

$\therefore\ \frac{a}{b} = 14$

or, a = 14b

32. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_p) and at constant volume (C_v) is :

(a) 6 **(b)** $\frac{7}{2}$

(c) $\frac{5}{2}$ **(d)** $\frac{7}{5}$

Solution:

(d) The ratio of specific heats at constant pressure (C_p) and constant volume (C_v)

$$\frac{C_p}{C_v} = \gamma = \left(1+\frac{2}{f}\right)$$

Where f is degree of freedom

$$\frac{C_p}{C_v} = \left(1+\frac{2}{5}\right) = \frac{7}{5}$$

33. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume Vis given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific

heat at constant pressure and constant volume, respectively):

(a) $n=\frac{C_p-C}{C-C_V}$ **(b)** $n=\frac{C-C_V}{C-C_P}$

(c) $n=\frac{C_P}{C_V}$ **(d)** $n=\frac{C-C_p}{C-C_V}$

Solution:

(d) For a polytropic process

$$C=C_v+\frac{R}{1-n}$$

$$\therefore\ C-C_v=\frac{R}{1-n}$$

$$\therefore\ 1-n=\frac{R}{C-C_v}$$

$$\therefore\ 1-\frac{R}{C-C_v}=n$$

$$\therefore\ n=\frac{C-C_v-R}{C-C_v}=\frac{C-C_v-C_p+C_v}{C-C_v}$$

$$\text{n}=\frac{C-C_p}{C-C_v}\quad\left(\because C_p-C_{v=R}\right)$$

34. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure,respectively. Then

(a) C_p-C_v is larger for a diatomic ideal gas than for a monatomic ideal gas

(b) C_p+C_v is larger for a diatomic ideal gas than for a monatomic ideal gas

(c) C_p/C_v is larger for a diatomic ideal gas than for a monatomic ideal gas

(d) $C_p\,.\,C_v$ is larger for a diatomic ideal gas than for a monatomic ideal gas

Solution:

(b, d)

We know for all gases $C_p.C_v=\frac{15}{4}and\ C_p+C_v=4$

For diatomic gas: $C_v=\frac{5}{2R}$; $C_p=\frac{7}{2}R$; $\gamma=\frac{7}{5}$

$$\therefore C_p.C_v=\frac{35}{4}and\ C_p+C_v=6$$

35. The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_p and C_v respectively. If $\gamma=\frac{C_p}{C_v}$ and R is the universal gas constant, then C_v is equal to

(a) $\frac{1+\gamma}{1-\gamma}$ **(b)** $\frac{R}{(\gamma-1)}$

(c) $\frac{(\gamma-1)}{R}$ **(d) γR**

Solution:

(b) As we know that

$C_p-C_v=R$

$C_p=R+C_v$

and $\frac{C_p}{C_v}=\gamma$ (given)

So, $\frac{R+C_v}{C_v}=\gamma\Rightarrow\gamma C_v=R+C_v$

$$\gamma C_v-C_v=R$$

$$C_v=\frac{R}{\gamma-1}$$

Expert Freedom & Molar Heat

1. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is

(a) $\frac{4}{3}$ **(b) 2**

(c) $\frac{5}{3}$ **(d)** $\frac{3}{2}$

Solution:

(d) According to question,

$p\propto T^3$ … (i)

$$\begin{pmatrix}p=pressure\\T=temperature\end{pmatrix}$$

and we know that

pV = nRT and pV ∝ T … (ii)

So, putting Eq. (ii) in (i),

$p\propto(pV)^3$

p^2V^3 = constant

$pV^{3/2}$ = constant ... (iii)

Comparing Eq. (iii) with

pv^y =constant

We have γ = 3/2.

2. The value of $\gamma\left(=\frac{C_p}{C_v}\right)$, for hydrogen helium and another ideal diatomic gas X (whose molecules are not rigid but have an additional vibrational mode), are respectively equal to

(a) $\frac{7}{5}, \frac{5}{3}, \frac{9}{7}$ **(b)** $\frac{5}{3}, \frac{7}{5}, \frac{9}{7}$

(c) $\frac{5}{3}, \frac{7}{5}, \frac{7}{5}$ **(d)** $\frac{7}{5}, \frac{5}{3}, \frac{7}{5}$

Solution:

(a) The Poisson's ratio,

$$\gamma = \frac{C_p}{C_v} \qquad \dots \text{(i)}$$

where, C_p =molar heat capacity constant pressure and C_v = molar heat capacity at constant volume

Also, $C_p = C_v + R$ (from Mayer's relation)

$C_v = \frac{f}{2}R$ (where, f = degree of freedom)

$$C_p = \left(\frac{f}{2} + 1\right) R$$

So, Eq. (i) becomes,

$$\gamma = 1 + \frac{2}{f}$$

For hydrogen gas, which is diatomic, the degree of freedom is 5 (3 translational, 2 rotational).

$$\therefore \quad \gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

For helium gas, which is monoatomic, the degree of freedom is 3(3 translational only).

$$\therefore \quad \gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

The diatomic gas X also have vibrational motion, so degree of freedom is 7(3 translational, 2 rotational and 2 vibrational).

$$\therefore \quad \gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

3. The molar heat capacity for an ideal gas

(a) Can be negative

(b) Can be equal to either C_V or C_P

(c) Can lie in the range $C_V \leq C \leq C_P$

(d) It may have any value between $-\infty$ to ∞.

Solution:

Correct options are A, B, C & D.

The molar heat capacity has the general definition

$$C = \frac{1}{n}\frac{\Delta Q}{\Delta T}$$

Where n = number of moles

ΔQ = heat absorbed by the gas

ΔT = rise in temperature of gas

It is possible to obtain almost any set of values for ΔQ and ΔT by proper selection of a process.

4. One mole of an ideal monatomic gas undergoes a process described by the equation pV^3 = constant. The heat capacity of the gas during this process is

(a) $\frac{3}{2}R$ **(b)** $\frac{5}{2}R$

(c) 2 R **(d) R**

Solution:

(a) As we know that for polytropic process of index α specific heat capacity

$$= C_v + \frac{R}{1-\alpha}$$

∵ Process, pV^3 = constant ⇒ α = 3

$$\therefore \quad C = C_v + \frac{R}{1-\alpha} = \frac{fR}{2} + \frac{R}{1-3}$$

where, $C_v = \frac{fR}{2} = \frac{3R}{2}$

For monatomic gas, $f = 3 = \frac{3R}{2}$

5. A mixture of ideal gases 7 kg of nitrogen and 11 kg of CO_2. Then

(a)

equivalent molecular weight of the mixture is 36.

(b)

equivalent molecular weight of the mixture is 18.

(c) γ for the mixture is 5/2

(d) γ for the mixture is 47/35.

(Take γ for nitrogen and CO_2 as 1.4 and 1.3 respectively)

Solution:

Correct options are A and D.

$$n_{N_2} = \frac{7}{28} = 0.25\ Kmoles$$

$$f_{N_2} = \frac{2}{Y-1} = \frac{2}{1.4-1} = 5$$

Similarly;

$$n_{Co_2} = 0.25\ k\ moles$$

$$f_{Co_2} = 20/3$$

Equivalent mol wt:

$$= \frac{n_1M_1 + n_2M_2}{n_1 + n_2}$$

$$= m_1 + m_2/n_1 + n_2$$

$$= 7 + 11/0.25 + 0.25$$

$$= 36\ gms$$

$$f_{min} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2}$$

$$= \frac{\left(0.25 \times 5 + 0.25 \times \frac{20}{3}\right)}{0.25 + 0.25}$$

$= 35/6$

$$Y_{min} = 1 + \frac{2}{f_{min}}$$

$$= 1 + \frac{2}{35/6}$$

$$Y_{min} = 47/35$$

Pro **Freedom & Molar Heat**

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6. For hydrogen gas C$_P$ – C$_V$ = a and for oxygen gas C$_P$ – C$_V$ = b . So the relation between a and b is given by

(a) a = 16b **(b) b = 16a**

(c) a = 4b **(d) a = b**

Solution:

The answer is D.

For any ideal gas, C$_P$ – C$_V$ = R, where R is the gas constant.

That is C$_P$ – C$_V$ per mole for any gas is a constant value.

So, a = b

7. One mole of monoatomic gas and three moles of diatomic gas are put together in a container. The molar specific heat (in JK^{-1} mol^{-1}) at constant volume is (R = 8.3 J K^{-1} mol^{-1})

(a) 18.7 **(b) 18.9**

(c) 19.2 **(d) None of the above**

Solution:

The answer is A.

As,

$$C_V = \frac{R}{r-1}$$

For monoatomic gas,

$$C_V = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2}R$$

For diatomic gas,

$$C'_V = \frac{R}{\frac{7}{5} - 1} = \frac{5}{2}R$$

For mixture C''_V,

$$= \frac{nC_V + n'C'_V}{n + n'}$$

$$= \frac{1 \times \frac{3}{2}R + 3 \times \frac{5}{2}R}{1 + 3}$$

$$= \frac{9}{4}R = \frac{9}{4} \times 8.3$$

$$= 18.7 JK^{-1}mol^{-1}$$

8. How many degrees of freedom have the gas molecules, if under standard conditions the gas density is ρ = 1.3 mg/cm^3 and the velocity of sound propagation in it is v = 330 m/s.

Solution:

The answer is 5.

In standard condition, pressure of gas is $P = 10^5 N/m^2$

$$v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3(10^5)}{1.3}} = 480\ m/s$$

Using

$$\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}}$$

Or

$$\frac{480}{330} = \sqrt{\frac{3}{\gamma}} \Rightarrow \gamma = 1.4$$

Also,

$$\frac{v_{rms}}{v_{sound}} = \sqrt{\frac{3}{\gamma}} = \sqrt{\frac{3f}{f+2}}$$

Or

$$\frac{3}{\gamma} = \frac{3f}{f+2}$$

Or

$$f + 2 = \gamma f = 1.4f$$

$$\therefore f = \frac{2}{0.4} = 5$$

9. The value of the ratio C$_p$/C$_v$ for hydrogen is 1.67 at 30 K but decreases to 1.4 at 300 K as more degrees of freedom become active. During this rise in temperature,

(a) C$_p$ remains constant but C$_v$ increases

(b) C$_p$ decreases but C$_v$ increases

(c) Both C$_p$ and C$_v$ decrease by the same amount

(d) Both C$_p$ and C$_v$ increase by the same amount

Solution:

The correct answer is D.

The value of the ratio $\frac{C_P}{C_V}$ for hydrogen is 1.67 at 30 K but decreases to 1.4 at 300K as more degrees of freedom become active. During this rise in temperature both C_P and C_V increase by the same amount.

For an ideal gas, $C_P = C_V + R$. If this is a molecular gas, increasing temperature enables vibrational degrees of freedom, so that C_V increases. Hence $\frac{C_P}{C_V} = 1 + \frac{R}{C_V}$ decreases.

10. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)?

(a) 5 **(b) 6**

(c) 3 **(d) 7**

<u>Solution:</u>

(b) Let C_p and C_v be the specific heat capacity of the gas at constant pressure and volume.

At constant pressure, heat required

$\Delta Q_1 = nC_p\Delta T$

$160 = nC_p \cdot 50$ ... (i)

At constant volume, heat required

$\Delta Q_2 = nC_v\Delta T$

$240 = nC_v \cdot 100$ ... (ii)

Dividing (i) by (ii), we get

$$\frac{160}{240} = \frac{C_p}{C_v} \cdot \frac{50}{100}$$

$$\frac{C_p}{C_v} = \frac{4}{3}$$

$$\gamma = \frac{C_p}{C_v} = \frac{4}{3} = 1 + \frac{2}{f}$$

(Here, f = degree of freedom)

f = 6.

11. The specific heats, C_p and C_v of a gas of diatomic molecules, A, are given (in units of $\text{Jmol}^{-1}\,\text{k}^{-1}$) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then:

(a) A is rigid but B has a vibrational mode

(b) A has a vibrational mode but B has none.

(c) A has one vibrational mode and B has two. (d) Both A and B have a vibrational mode each.

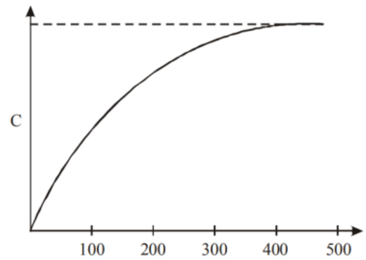
<u>Solution:</u>

(b) $\gamma_A = \frac{C_P}{C_v} = \frac{29}{22}$ = 1.32 < 1.4 (diatomic)

and $\gamma_B = \frac{30}{21} = \frac{10}{7}$ = 1.43 > 1.4

Gas A has more than 5-degrees of freedom.

12. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation.



(a) The rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T.

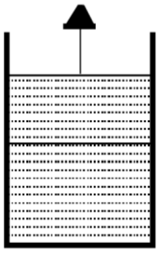
(b) Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K.

(c) There is no change in the rate of heat absorption in the range 400-500 K.

(d) The rate of heat absorption increases in the range 200-300 K.

In the figure, a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulated material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400K. The heat capacities per mole of an ideal monatomic

gas are $C_V = \frac{3}{2}R, C_P = \frac{5}{2}R,$ **and those for an ideal diatomic gas are** $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R.$



Solution:

(b, c, d)

(a) As we know, Q = mc ΔT

$\Rightarrow \frac{dQ}{dt} = mc\frac{dT}{dT} or, \frac{dQ}{dt} \propto$

$C\ i.e., rate\ of\ heat\ absorption\ \propto$ C.

In the range 0 to 100K from the graph, C increases with temperature but not linearly therefore the rate at which heat is absorbed varies with temperature. But not linearly.

(b) As the value of C is greater in the temperature range 400-500K, the heat absorbed in increasing the temperature from 0-100K is less than the heat required for increasing the temperature from 400-500K.

(c) From the graph the value of C does not change in the temperature range 400-500K, therefore there is no change in the rate of heat absorption in the range.

(d) As the value of C increases from 200-300K, the rate of heart absorption increases in the range 200-300K.

Beginner **Internal Energy & Law of Equipartition**

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1. The ratio of average translational kinetic energy to rotational kinetic energy of a diatomic molecule at temperature T is

(a) 3 **(b) 7/5**

(c) 5/3 **(d) 3/2**

Solution:

Correct option is D.

For a diatomic molecule, the three-translation direction contribute 1/2kT per molecule, and for each molecule there are also two axes of rotation, contributing rotational kinetic energy in the amount of 1/2Kt each.

So total translational kinetic energy= (3/2)kT & total rotational kinetic energy = (2/2)kT.

So, the ratio of average translational molecule at temperature T is (3/2).

2. A mass M = 15 g of nitrogen is enclosed in a vessel at temperature T = 300 K. what amount of heat to be transferred to the gas to increase the room mean square velocity of molecules n=2 times?

Solution:

$Q = nC_V\Delta T$ as gas in closed vessel.

$Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$

$Q = 10000J = 10kJ$

3. Two monoatomic ideal gas at temperature T₁ and T₂ are mixed. There is no loss of energy. If the masses of molecules of the two gases are m₁ and m₂ and number of their molecules are n₁ and n₂ respectively. The temperature of the mixture will be :

(a) $\frac{T_1 + T_2}{n_1 + n_2}$ **(b)** $\frac{T_1}{n_1} + \frac{T_2}{n_2}$

(c) $\frac{n_2T_1 + n_1T_2}{n_1 + n_2}$ **(d)** $\frac{n_1T_1 + n_2T_2}{n_1 + n_2}$

Solution:

Correct option is D.

Net P-V work done = 0 as no change in volume takes place

Also net heat absorbed = 0

Thus, by first law, net internal energy change = 0

$n_1C_{v_1}\Delta T_1 + n_2C_{v_2}\Delta T_2 = 0$

Both gases are monoatomic

$C_{v_1} + C_{v_2} = \frac{3}{2}R$

Let final temperature of mixture at equilibrium be T

Then,

$\frac{3}{2}R\big(n_1(T - T_1) + n_2(T - T_2)\big) = 0$

$T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$

4. Mean translational kinetic energy of each degree of freedom of one molecule of a gas will be –

(a) RT/2 **(b) kT/2**

(c) 3RT/2 **(d) 3kT/2**

Solution:(b)

5. Mean kinetic energy of one gram helium at 27°C will be –

(a) 3527 × 10^{-7} Joule (b) 6 × 10^{-18} Joule

(c) 933 × 10^{-3} Joule (d) 933.7 Joule

Solution:(d)

4gm = U = 3/2 RT

1gm of Helium = 1/4 U

$$=\frac{3}{8}\times 8314\times 300$$

6. The kinetic energy of gas molecules at 300 K is 75 joule. This energy at 500 K will be –

(a) 125 J **(b) 208 J**

(c) 270 J **(d) 375 J**

Solution:(a)

Kinetic intranet of temp

U ∝ T

$$\frac{U}{75}=\frac{500}{300}\quad =\frac{5}{3}\times 75 = 125 \text{ J}$$

7. The value of rotational K.E. at temperature T of one gram molecule of a diatomic gas will be –

(a) RT **(b) 3RT/2**

(c) 5RT **(d) 5RT/2**

Solution:(a)

$$U_{rot}=(2)\left(\frac{1}{2}RT\right) = \text{RT}$$

8. CO_2 is linear triatomic molecule.

The average K.E. at temperature T will be –

(a) 3kT/2 **(b) 5kT/2**

(c) 6kT/2 **(d) 7kT/2**

Solution:(b)

9. A closed vessel contains a mixture of two diatomic gases A and B. molar mass of A is 16 times that of B and mass of gas A contained in the vessel is 2 times that of B. which of the following statements are correct?

(a) average kinetic energy per molecule of A is equal to that of B

(b) Root mean square value of translational velocity of B is four times that of A.

(c) pressure exerted by B is eight times of that exerted by a

(d) Number of molecules of B, in the cylinder, is eight times that of A.

Solution:

Correct options are A,B,C and D.

A

$Molecular\ wt = 16M_0$

$mass = 2m_0$

$n_A = \dfrac{n_0}{8}$

B

$Molecular\ wt = M_0$

$mass = m_0$

$n_B = n_0$

(A) K.E/atom=f/2kT which will be same for both the gases as both are diatomic.

(B) $C_{rms_A} = \sqrt{\frac{3RT}{16M_0}}$

$$C_{rms_B} = \sqrt{\frac{3RT}{M_0}}$$

$(C_{rms})_B = 4(C_{rms})_A$

(C) $(P)_A = \frac{(n_0/8)RT}{V}$

$$(P)_B = \frac{(n_0)RT}{V}$$

$(P)_B = 8(P)_A$

(D) $n_B = 8n_A$

10. N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas?

(a) $\frac{1}{2}nRT$ **(b) 0**

(c) $\frac{3}{2}nRT$ **(d)** $\frac{5}{2}nRT$

Solution:

(a) Energy associated with N moles of diatomic gas,

$$U_i = N\frac{5}{2}RT$$

Energy associated with n moles of monoatomic gas

$$=n\frac{3}{2}RT$$

Total energy when n moles of diatomic gas converted into monoatomic

$$\left(U_f\right) = 2n\frac{3}{2}RT + (N-n)\frac{5}{2}RT$$

$$=\frac{1}{2}nRT + \frac{5}{2}NRT$$

Now, change in total kinetic energy of the gas

$$\Delta U = Q = \frac{1}{2}nRT$$

11. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is :

(a) 15 **(b) 13**

(c) 20 **(d) 11**

Solution:

(a) Total energy of the gas mixture,

$$E_{mix} = \frac{f_1 n_1 RT_1}{2} + \frac{f_2 n_2 RT_2}{2}$$

$$= 3\times\frac{5}{2}RT + \frac{5}{2}\times 3RT = 15RT$$

12. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is :

(a) $\frac{2}{3}Q$ **(b)** $\frac{5}{3}Q$

(c) $\frac{7}{5}Q$ **(d)** $\frac{3}{2}Q$

Solution:

(c) Amount of heat required (Q) to raise the temperature at constant volume

$Q = nC_v\Delta T$ ... (i)

Amount of heat required (Q_1) at constant pressure

$Q_1 = nC_p\Delta T$ ... (ii)

Dividing equation (ii) by (i), we get

$$\therefore \frac{Q_1}{Q} = \frac{C_p}{C_v}$$

$$Q_1 = (Q)\left(\frac{7}{5}\right) \qquad \left(\because \gamma = \frac{C_p}{C_v} = \frac{7}{5}\right)$$

13. An ideal gas occupies a volume of 2 m^3 at a pressure of 3 × 10^6 Pa. The energy of the gas:

(a) 9 × 10^6 J **(b) 6 × 10^4 J**

(c) 10^8 J **(d) 3 × 10^2 J**

Solution:

(a) Energy of the gas, E

$$= \frac{f}{2}nRT = \frac{f}{2}PV = \frac{f}{2}\left(3\times10^6\right)(2)$$

$= f \times 3 \times 10^6$

Considering gas is monoatomic i.e., f = 3

Energy, E = 9 × 10^6 J

14. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is :

(a) 15 RT **(b) 12 RT**

(c) 4RT **(d) 20 RT**

Solution:

(a) $U = \frac{f_1}{2}n_1RT + \frac{f_2}{2}n_2RT$

Considering translational and rotational modes, degrees of freedom $f_1 = 5$ and $f_2 = 3$

$$\therefore u = \frac{5}{2}(3RT) + \frac{3}{2}\times 5RT$$

U = 15RT

15. Two kg of a monoatomic gas is at a pressure of 4 × 10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion?

(a) 10^3 J **(b) 10^5 J**

(c) 10^4 J **(d) 10^6 J**

Solution:

(c) Thermal energy of N molecule

$$= N\left(\frac{3}{2}kT\right) = \frac{N}{N_A}\frac{3}{2}RT = \frac{3}{2}(nRT) = \frac{3}{2}PV$$

$$= \frac{3}{2}P\left(\frac{m}{\rho}\right) = \frac{3}{2}P\left(\frac{2}{8}\right) = \frac{3}{2}\times4\times10^4\times\frac{2}{8}$$

= 1.5 × 10^4 J

Therefore, order = 10^4 J

16. Using equipartition of energy, the specific heat (in J kg^{-1} K^{-1}) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27)

(a) 410 **(b) 25**

(c) 1850 **(d) 925**

Solution:

(d) Using equipartition of energy, we have

$$\frac{6}{2}KT = mCT$$

$$C = \frac{3\times1.38\times10^{-23}\times6.02\times10^{23}}{27\times10^{-3}}$$

∴ C = 925 J/kgK

17. Increase in temperature of a gas filled in a container would lead to

(a) increase in its kinetic energy
(b) decrease in its pressure
(c) decrease in intermolecular distance
(d) increase in its mass

Solution:

(a) As the temperature of gas in the container is increased, the kinetic energy also increases. This is because the average kinetic energy of a gas is given by

$$KE = \frac{f}{2} nRT \quad \dots \text{(i)}$$

where, f = degree of freedom,
n = number of moles of gas molecules,
R = universal gas constant, and
T = absolute temperature of the gas.

From Eq. (i),

$KE \propto T$

option (b) is incorrect as increase in temperature will lead to increase in pressure as $p \propto T$. Other options (c) and (d) are also incorrect as molecular distance increases while mass remains the same for increase in the temperature.

Expert Internal Energy & Law of Equipartition

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1. Two perfect monoatomic gases at absolute temperature T_1 and T_2 are mixed. There is no loss of energy. Find the temperature of the mixture if masses of the molecules are m_1 and m_2 and the number of molecules in the gases are n_1 and n_2.

Solution:

Net P-V work done = 0 as no change in volume takes place

Also, net heat absorbed = 0

Thus, by first law, net internal energy change = 0

$$n_1 C_{v_1} \Delta T_1 + n_2 C_{v_2} \Delta T_2 = 0$$

Both gases are monoatomic

$$C_{v_1} = C_{v_2} = \frac{3}{2} R$$

Let final temperature of mixture at equilibrium be T

Then,

$$\frac{3}{2} R\big(n_1(T - T_1) + n_2(T - T_2)\big) = 0$$

$$T = \frac{n_1 T_1 + n_2 T_2}{n_1 + n_2}$$

2. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take R = 8.3 J/K mole]

(a) 0.9 kJ **(b) 6kJ**
(c) 10kJ **(d) 14 kJ**

Solution:

(c) Heat transferred,

$Q = nC_v\Delta T$ as gas in closed vessel

To double the rms speed, temperature should be 4 times i.e., T'

= 4T as $v_{rms} = \sqrt{3RT/M}$

$$\therefore\ Q = \frac{15}{28} \times \frac{5 \times R}{2} \times (4T - T)$$

$$\left[\therefore \frac{CP}{CV} = \gamma_{diatomic} = \frac{7}{5}\ \&\ C_p - C_v = R\right]$$

or, Q = 10000 J = 10 kJ

3. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to __________.

Solution:

Here work done on gas and heat supplied to the gas are zero.

Let T be the final equilibrium temperature of the gas in the vessel.

Total internal energy of gases remains same.

i.e., $u_1 + u_2 = u_1' + u_2'$

or, $n_1 C_v \Delta T_1 + n_2 C_v \Delta T_2 = (n_1 + n_2) C_v T$

$(0.1)\, Cv(200) + (0.05) C_v(400) = (0.15) C_v T$

$$\therefore\ T = \frac{800}{3} = 266.67 \text{ k.}$$

4. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are)

(a) The average energy per mole of the gas mixture is 2RT

(b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$

(c) The ratio of the rms speed of helium atoms to that of hydrogen molecules is ½

(d) The ratio of the speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$

Solution:

(a, b, d)

According to question no. of mole of hydrogen = no. of mole of helium = 1

Total internal energy, u

$$= \frac{f_1}{2}nRT + \frac{f_2}{2}nRT \Rightarrow u = \frac{3}{2}RT + \frac{5}{2}RT = 4RT$$

$\therefore$ Average internal energy per mole $= \frac{u}{2n} = \frac{4RT}{2} = 2RT$

We know that $V_{sound} = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \Rightarrow \frac{2}{\gamma_{mix} - 1} = \frac{1}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1}$$

$$\frac{2}{\gamma_{mix} - 1} = \frac{3}{2} + \frac{5}{2} = 4 \Rightarrow \gamma_{mix} - 1 = \frac{1}{2} \therefore \gamma_{mix} = \frac{3}{2}$$

$$\frac{(V_s)_{mix}}{(V_s)_{He}} = \sqrt{\frac{\gamma_{mix}}{M_{mix}} \times \frac{M_{He}}{\gamma_{He}}} = \sqrt{\frac{\frac{3}{2} \times 4}{3 \times \frac{5}{3}}} = \sqrt{\frac{6}{5}}$$

$$\left[\because M_{mix} = \frac{1 \times 2 + 1 \times 4}{2} = 3\right]$$

We know that $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\therefore \frac{(V_{rms})_{He}}{(V_{rms})_{H2}} = \sqrt{\frac{M_{H2}}{M_{H_E}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

5. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J mo$l^{-1}K^{-1}$, the decrease in its internal energy, in Joule, is______.

Solution:

(900J) Given: $T_i = 100K, V_f = 8V_i$

For and adiabatic process, $TV^{\gamma-1} = constant$

Or, $T_iV_i^{\gamma-1} = T_fV_f^{\gamma-1}$

$$\Rightarrow \frac{T_i}{T_f} = \left(\frac{V_f}{V_i}\right)^{\gamma-1} \qquad \Rightarrow \frac{T_i}{T_f} = \left(\frac{8V_i}{V_i}\right)^{\gamma-1}$$

For monoatomic gas $\gamma = \frac{5}{3}$

$$\therefore T_f = \frac{T_i}{(8^{\frac{5}{3}} - 1)} = \frac{T_i}{4}$$

Change in internal energy $\Delta u = nC_v\Delta T$

$$= 1 \times \frac{3}{2}R\left(\frac{T_i}{4} - T_i\right) = \frac{3}{2} \times 8\left(-\frac{3}{4}\right) \times 100 = -900J = 900j$$

Beginner	**Mean Free Path**

1. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule varies with T is:

(a) T **(b)** $\frac{1}{\sqrt{T}}$

(c) $\frac{1}{T}$ **(d)** $\sqrt{T}$

Solution:

(b) Mean free path, $\lambda = \frac{1}{\sqrt{2}\pi nd^2}$

Where, d = diameter of the molecule

n = number of molecules per unit volume

But, mean time fo collision, $\tau = \frac{\lambda}{v_{rms}}$

But $v_{rms} = \sqrt{\frac{3kT}{R}}$

$$\therefore \tau = \frac{\lambda}{\sqrt{\frac{3kT}{m}}} \Rightarrow \tau \propto \frac{1}{\sqrt{T}}$$

2. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?

(1) The mean free path of the molecules decreases

(2) The mean collision time between the molecules decreases

(3) The mean free path remains unchanged

(4) The mean collision time remains unchanged

(a) (2) and (3) **(b) (1) and (2)**

(c) (3) and (4) **(d) (1) and (4)**

Solution:

As we know mean free path

$$\lambda = \frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$$

Here,

N = no. of molecule

V = volume of container

d = diameter of molecule

But PV = nRT = nNKT

$$\frac{N}{V} = \frac{P}{KT} = n$$

$$\lambda = \frac{1}{\sqrt{2}}\frac{KT}{\pi d^2 P}$$

For constant volume and hence constant number density n of gas molecules $\frac{P}{T}$ is constant.

So, mean free path remains same.

As temperature increases no. of collision increases so relaxation time decreases.

3. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to:

(a) 3.67 **(b) 1.83**

(c) 2.3 **(d) 4.67**

Solution:

Mean free path of a gas molecule is given by

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$$

Here, n = number of collisions per unit volume

d = diameter of the molecule

If average speed of molecule is v then

Mean free time, $\tau = \frac{\lambda}{v}$

$$\tau = \frac{1}{\sqrt{2}\pi n d^2 v} = \frac{1}{\sqrt{2}\pi n d^2}\sqrt{\frac{M}{3RT}}$$

$$\left(\because v = \sqrt{\frac{3RT}{M}}\right)$$

$$\therefore \ \tau \propto \frac{\sqrt{M}}{d^2}$$

$$\therefore \ \frac{\tau_1}{\tau_2} = \frac{\sqrt{M}}{d_1^2} \times \frac{d_2^2}{\sqrt{M_2}}$$

$$= \sqrt{\frac{40}{140}} \times \left(\frac{0.1}{0.07}\right)^2 = 1.09$$

4. Two gases Ar (40) and Xe (131) at same temperature have same number density. Their diameters are 0.07 nm and 0.10 nm respectively. Find the ratio of their mean free time

(a) 1.03 **(b) 2.04**

(c) 1.13 **(d) 2.40**

Solution:(b)

$n_{Xe} = n_{Ar}$

$$\lambda \propto \frac{1}{d^2}$$

d_{Ar} = 007 nm

d_{Xe} = 010 nm

$$\frac{\lambda_{Ar}}{\lambda_{Xe}} = \frac{d_{Xe}^2}{d_{Ar}^2} = \frac{100}{49} = 2.04$$

5. Estimate the mean free path of nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 27°C. Take the radius of a nitrogen molecule to be roughly 1.0° Å.

$k = 1.38 \times 10^{-23}$ J/K

Ans is 1.16×10^{-X} Find X

Solution:

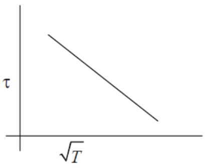
$$\lambda = \frac{kT}{\sqrt{2}\pi d^2 P}$$

$$= \frac{\left(1.38\times10^{-23}\right)\times 300}{\sqrt{2}\pi\left(4\times10^{-20}\right)\times\left(2\times10^{5}\right)}$$

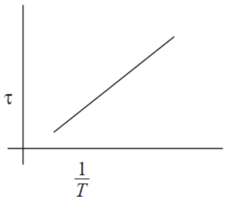
$$= \frac{13.8\times3}{8\times1.414\times314}\times10^{-22+20-5}$$

X = 7

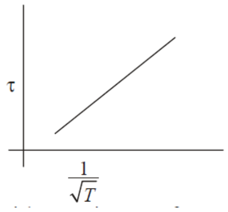
6. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale)



(a)

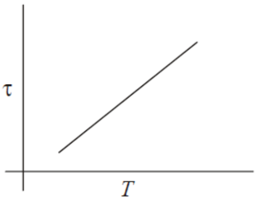


(b)



(c)

(d)



Solution:

(c) Relaxation time $\left(\tau\right) \propto \frac{mean\ freepath}{speed}$

$$\tau \propto \frac{1}{v}$$

and, $v \propto \sqrt{T}$

$$\therefore\ \tau \propto \frac{1}{\sqrt{T}}$$

Hence graph between τ v/s $\frac{1}{\sqrt{T}}$ is a straight line which is correctly depicted by graph shown in option (c).

7. The mean free path l for a gas molecule depends upon diameter, d of the molecule as

(a) $l \propto \frac{1}{d^2}$ **(b) l ∝ d**

(c) l ∝ d^2 **(d)** $l \propto \frac{1}{d}$

Solution:

(a) The mean free path l for a gas molecule is given as

$$l = \frac{1}{\sqrt{2}\pi n d^2} \Rightarrow l \propto \frac{1}{d^2}$$

where, d = diameter of molecule of gas.

8. The mean free path λ for a gas, with molecular diameter d and number density n can be expressed as

(a) $\frac{1}{\sqrt{2}n\pi d^2}$ **(b)** $\frac{1}{\sqrt{2}n^2\pi d^2}$

(c) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$ **(d)** $\frac{1}{\sqrt{2}n\pi d}$

Solution:

(a) The mean free path λ for a gas, with molecular diameter and number density n is given by the relation

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$$

Hence, correct option is (a).

Expert Mean Free Path

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1. A 25 × 10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300 K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule?

(a) ~10^{12} **(b) ~10^{11}**

(c) ~10^{10} **(d) ~10^{13}**

Solution:

(c) V = 25 × 10^{-3} m^3, N = 1 mole of O_2

T = 300 K

V_{rms} = 200 m/s

$$\therefore\ \lambda = \frac{1}{\sqrt{2}N\pi r^2}$$

Average time $\frac{1}{\tau} = \frac{<V>}{\gamma} = 200.N\pi r^2.\sqrt{2}$

$$= \frac{\sqrt{2}\times 200\times 6.023\times 10^{23}}{25\times 10^{-3}}.\pi\times 10^{-18}\times 0.09$$

The closest value in the given option is = 10^{10}

2. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6 × 10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to:

(a) 2 × 10^{-7} s **(b) 4 × 10^{-8} s**

(c) 0.5 × 10^{-8} s **(d) 3 × 10^{-6} s**

Solution:

(b) Using, $\tau = \frac{1}{2n\pi d^2 V_{avg}}$

$$\therefore\ t \propto \frac{\sqrt{T}}{P}$$

$$\left[\therefore n = \frac{no.of\ molecules}{Volume}\right]$$

or, $\frac{t_1}{6\times 10^{-8}} = \frac{\sqrt{500}}{2P}\times\frac{P}{\sqrt{300}}$ ≈ 4 × 10^{-8} s

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Beginner Ideal Gas Equation

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1. Find the approx. number of molecules contained in a vessel of volume 7 litres at 0°C at 1.3 × 10^5 pascal

(a) 2.4 × 10^{23} **(b) 3 × 10^{23}**

(c) 6 × 10^{23} **(d) 4.8 × 10^{23}**

Solution:

Correct option is A.

The number of moles is,

$$n = \frac{PV}{RT} = \frac{1.3\times 10^5\times 7\times 10^{-3}}{8.314\times 273} = 0.4\,mol$$

The approximate number of molecules contained in a vessel= $0.4 \times 6.023 \times 10^{23} = 2.4 \times 10^{23}$

2. When 2 gms of a gas are introduced into an evacuated flask kept at 25°C the pressure is found to be one atmosphere. If 3 gms of another gas added to the same flask the pressure becomes 1.5 atmospheres. The ratio of the molecular weights of these gases will be

(a) 1 : 3 **(b) 3 : 1**

(c) 2 : 3 **(d) 3 : 2**

Solution:

Correct option is A.

When 2 grams of a gas A is introduced in an evacuated flask kept at 25°C; the pressure was found to be 1 atm i.e.

Partial Pressure of gas A = 1atm.

And let mole be

$$n_A = \frac{2}{M_A}$$

If 3 g of another gas B is then added to the same flask;

Partial Pressure of gas B = 1.5atm – 1atm = 0.5 atm

$$Mole\ of\ gas\ B = \frac{3}{M_B}$$

Again, the temp and volume are constant therefore,

$$\frac{P_1}{n_1} = \frac{P_2}{n_2}$$

Hence,

$$\frac{1}{\frac{2}{M_A}} = \frac{0.5}{\frac{3}{M_B}}$$

$$M_A = \frac{0.5M_B}{3} \times 2$$

$$\frac{M_A}{1} = \frac{M_B}{3}$$

$$M_A = \frac{M_B}{3}$$

Or

$$\frac{M_A}{M_B} = \frac{1}{3} \Rightarrow 1:3$$

3. A vessel of volume 0.02 m^3 contains a mixture of hydrogen and helium at 20°C and 2 atmospheric pressure. The mass of mixture is 5 gms. Find the ratio of mass of hydrogen to that of helium in the mixture.

(a) 1 : 2 **(b) 1 : 3**

(c) 2 : 3 **(d) 3 : 2**

Solution:

Correct option is A.

PV = nRT

$$n_T = \frac{(2)(20lit)}{0.0821 \times 293}$$

$$n_T = 1.66 \quad \Rightarrow \frac{m_1}{2} + \frac{m_2}{4} = 1.66$$

$$2m_1 + m_2 = 6.65$$

$$given\ m_1 + m_2 = 5\ \Rightarrow m_1 = 1.65$$

$$m_2 = 3.35$$

$$m_1 : m_2 = 1 : 2$$

4. 28 gm of N_2 gas is contained in a flask at a pressure of 10 atm and at a temperature of 57°. It is found that due to leakage in the flask, the pressure is reduced to half and the temperature reduced to 27°C. The quantity of N_2 gas that leaked out is

(a) 11/20 gm **(b) 20/11 gm**

(c) 5/63 gm **(d) 63/5 gm**

Solution:

Correct option is D.

The ideal gas law must hold both before and after the leakage of the gas.

Thus,

$$\frac{P_iV}{n_iT_i} = \frac{P_fV_f}{n_fT_f}$$

$$n_f = \frac{P_f}{P_i}\frac{T_i}{T_f}n_i$$

$$n_f = \frac{1}{2}\frac{273+57}{273+27}n_i$$

$$= \frac{11}{20}n_i$$

$$m_f = \frac{11}{20}m_i$$

Thus,

$$leaked\ out\ mass = \frac{9}{20}m_i = \frac{63}{5}g$$

5. If a mixture of 28 g of Nitrogen, 4 g of Hydrogen and 8 gm of Helium is contained in a vessel at temperature 400 K and pressure 8.3 × 10^5 Pa, the density of the mixture will be :

(a) 3 kg/m^3 **(b) 0.2 kg/m^3**

(c) 2 g/litre **(d) 1.5 g/litre**

Solution:

Correct option is C.

Moles,

$$N_2 = \frac{28}{28} = 1$$

$$H_2 = \frac{4}{2} = 2$$

$$H_e = \frac{8}{4} = 2$$

$$molar\ mass = \frac{1\times 28 + 2\times 4 + 8\times 2}{5} = \frac{40}{4}$$

$$= 5$$

$$P = \frac{PM}{RT} = \frac{8.3\times 8}{0.08\times 400} = 2\ gm/l$$

6. A closed vessel of volume V_0 contains oxygen at a pressure P_0 and temperature T_0. Another closed vessel of the same volume V_0 contains helium at a pressure of P_0 and temperature $T_0/2$. Find the ratio of the masses of oxygen to the helium.

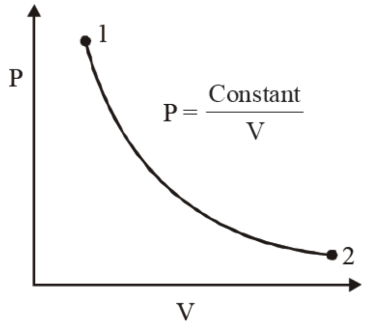
Solution:

$$w = \frac{PVM}{RT}$$

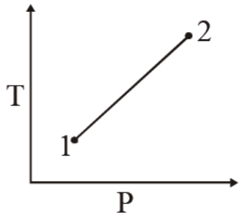
$$\frac{w_{O_2}}{w_{He}} = \frac{P_{O_2}V_{O_2}M_{O_2}}{RT_{O_2}} \times \frac{RT_{He}}{P_{He}V_{He}M_{He}}$$

$$\frac{w_{O_2}}{w_{He}} = \frac{P_0V_0 \times 32}{T_0} \times \frac{\frac{T_0}{2}}{P_0V_0 \times 4} = \frac{4}{1}$$

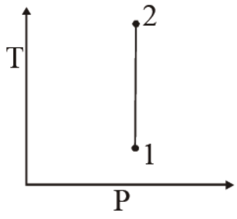
7. For the P-V diagram given for an ideal gas,



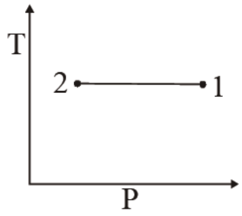
Out of the following which one correctly represents the T-P diagram ?



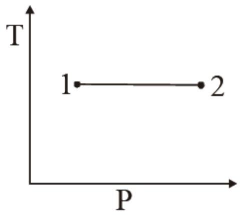
(a)



(b)



(c)



(d)

Solution:

(c) From P-V graph,

$$P \propto \frac{1}{V},$$

T = constant and Pressure is increasing from 2 to 1 so option (c) represents correct T-P graph.

8. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is ___________.

Solution:

Using ideal gas equation, PV = nRT

$P_1V_1 = nR \times 250$ [∵ T_1 = 250 K] ... (i)

$P_2\left(2V_1 = \frac{5n}{4}R \times 2000\right)$ [∵ T_2 = 2000 K] ... (ii)

Dividing eq. (i) by (ii),

$$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000}$$

$$\frac{P_1}{P_2} = \frac{1}{5}$$

$$\therefore \frac{P_2}{P_1} = 5$$

9. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm. respectively. If $|\Delta T| = C|\Delta P|$, then value of C in (K/atm.) is ____________.

Solution:

In first case,

From ideal gas equation

PV = nRT

$P\Delta V + V\Delta P = 0$

(As temperature is constant)

$$\Delta V = -\frac{\Delta P}{P}V \qquad \text{... (i)}$$

In second case, using ideal gas equation again

$P\Delta V = -nR\Delta T$

$$\Delta V = -\frac{nR\Delta T}{P} \qquad \text{... (ii)}$$

Equation (i) and (ii), we get

$$-\frac{nR\Delta T}{P} = -\frac{\Delta P}{P}V$$

$$\Delta T = \Delta P\frac{V}{nR}$$

Comparing the above equation with $|\Delta T| = C|\Delta P|$, we have

$$C = \frac{V}{nR} = \frac{\Delta T}{\Delta P} = \frac{300K}{2atm}$$

= 150 K/atm

10. A jar contains a gas and a few drops of water at T°K. The pressure in the jar is 830 mm of Hg. The temperature of the jar is reduced by 1%. The saturated vapor pressures of water at the two temperatures are 30 and 25 mm of Hg. Calculate the new pressure in the jar.

Solution:

$P_1 = 830 - 30 = 800 mm\ Hg\ \ ; P_2?$

$V_1 = V \quad ; \ V_2 = V \ ; \ \ T_1 = T \ \ ; T_2 = T - 0.01\ T = 0.99\ T$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \therefore P_2 = \frac{P_1T_2}{T_1} = \frac{800 \times 0.099T}{T} = 792 mmHg$$

∴ Total pressure in the jar = 792 + 25 = 817 mm Hg

11. Given samples of 1 c.c. of hydrogen and 1c.c. of oxygen, both at N.T.P. which sample has a larger number of molecules?

Solution:

From, PV = nRT

When P,T are same n ∝ V

As volumes are same, i.e., 1c.c of each hydrogen and oxygen, So both samples will have equal number of molecules.

12. An ideal gas equation can be written as

$$p = \frac{\rho RT}{M_0}$$

where, ρ and M_0 are respectively,

(a) mass density, mass of the gas

(b) number density, molar mass

(c) mass density, molar mass

(d) number density, mass of the gas

Solution:

(a) Ideal gas equation is given as

$$p = \frac{\rho RT}{M_0} \Rightarrow p \cdot \frac{M_0}{\rho} = RT$$

pV = RT

where, $V = \frac{M_0}{\rho}$

Hence, ρ and M_0 are mass density and mass of gas, respectively.

13. A cylinder contains hydrogen gas at pressure of 249 kPa and temperature 27°C.

Its density is (R = 8.3 J mol^{-1} K^{-1}).

(a) 0.2 kg/m^3 **(b) 0.1 kg/m^3**

(c) 0.02 kg/m^3 **(d) 0.5 kg/m^3**

Solution:

(a) Given,

pressure p = 249 kPa = 249 × 10^3 Pa

Temperature, T = 27°C

= 273 + 27 K = 300K

Density, ρ = ?

As, from ideal gas equation,

pV = nRT

$$pV = \frac{m}{M}RT \qquad \left[as\ n = \frac{m}{M}\right]$$

pVM = mRT

$$pM = \frac{m}{V}RT = \rho RT \ \left[as\ \frac{m}{V} = \rho\right]$$

$$\rho = \frac{pM}{RT}$$

$$= \frac{249 \times 10^3 \times 2 \times 10^{-3}}{8.3 \times 300}$$

[∵ for hydrogen gas, M = 2g = 2 × 10^{-2} kg]

ρ = 0.2kg/m^3

Hence, correct option is (a).

Pro Ideal Gas Equation

1. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1, and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by: (R is universal gas constant and g is the acceleration due to gravity)

(a) $\frac{RT}{ng}\left[\frac{l_1 - 3l_2}{l_1l_2}\right]$ **(b)** $\frac{RT}{g}\left[\frac{2l_1 + l_2}{l_1l_2}\right]$

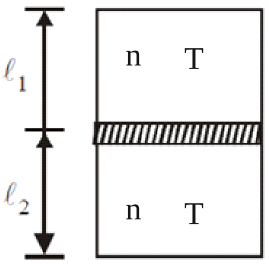
(c) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{1}{l_1}\right]$ **(d)** $\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1l_2}\right]$

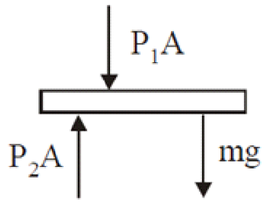
Solution:

(d) Clearly from figure,

$P_2A = P_1A + mg$

or, $\frac{nRT \cdot A}{Al_2} = \frac{nRT \cdot A}{Al_1} + mg$





$$nRT\left(\frac{1}{l_2}-\frac{1}{l_1}\right)=mg$$

$$\therefore\ m=\frac{nRT}{g}\left(\frac{l_1-l_2}{l_1\cdot l_2}\right)$$

2. The temperature of an open room of volume 30m^3increases from 17℃ to 27℃ due to sunshine. The atmospheric pressure in the room remains 1× 10^5Pa. If n_i and $n_f$are the number of molecules in the room before and after heating, then $n_f - n_i$ will be:

(a) 2.5 × 10^{25} **(b) -2.5 × 10^{25}**

(c) -1.61 × 10^{23} **(d) 1.38 × 10^{23}**

Solution:

(b) Given: Temperature $T_1 = 17 + 273 = 290\ K$

Temperature $T_f = 27 + 273 = 300K$

Atmospheric pressure, $P_0 = 1 \times 10^5 Pa$

Volume of room, $V_0 = 30m^3$

Difference in number of molecules, $n_f - n_i = ?$

Using ideal gas equation, $PV = nRT(N_0)$,

$N_0 = Avogadro's\ number$

$\Rightarrow n = \frac{PV}{RT}(N_0)$

$$\therefore n_f - n_i = \frac{P_0V_0}{R}\left(\frac{1}{T_f}-\frac{1}{T_i}\right)N_0$$

$$= \frac{1\times10^5\times30}{8.314}\times6.023\times10^{23}\left(\frac{1}{300}-\frac{1}{290}\right)$$

$= -2.5\times10^{25}$

3. Two non-reactive monatomic ideal gases have their atomic masses in the ratio 2:3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4:3. The ratio of their densities is

(a) 1:4 **(b) 1:2**

(c) 6:9 **(d) 8:9**

Solution:

(d) $P_1M_1 = P_1RT\ and\ P_2M_2 = P_2RT$

$$\therefore \frac{P_1}{P_2}\times\frac{M_1}{M_2}=\frac{P_1}{P_2}$$

$$\frac{4}{3}\times\frac{2}{3}=\frac{P_1}{P_2}\qquad \therefore \frac{P_1}{P_2}=\frac{8}{9}$$

Beginner Miscellaneous Problems

1. Two vessels of the same volume contain the same gas at same temperature. If the pressure in the vessels be in the ratio of 1 : 2, then

(a) the ratio of the average kinetic energy is 1 : 2

(b)

the ratio of the root mean square velocity is 1 : 1

(c) the ratio of the average velocity is 1 : 2

(d) the ratio of number of molecules is 1 : 2

Solution:

Correct options are B and D.

As we know,

$$P = \frac{1}{3}\frac{mnC^2}{V}$$

So,

$P \propto nC^2$

When m and V are constant.

But

$C \propto \sqrt{T}$

As the temperature of two vessel is the same, the rms velocity of gas molecules in two vessels will be the same.

When temperature is same for two vessels then

$P \propto n$

So,

$$\frac{P_1}{P_2}=\frac{n_1}{n_2}=\frac{1}{2}$$

2. A gas is enclosed in a vessel at a constant temperature at a pressure of 5 atmosphere and volume 4 litre. Due to a leakage in the vessel, after some time, the pressure is reduced to 4 atmosphere. As a result, the

(a) volume of the gas decreased by 20%

(b) average K.E. of gas molecule decreases by 20%

(c) 20% of the gas escaped due to the leakage

(d) 25% of the gas escaped due to the leakage

Solution:

Correct option is C.

$$\boldsymbol{n} = \frac{P}{P}\times n = \frac{4}{5}\times n = 0.8n$$

Thus, amount of gas escaped is n – 0.8n = 0.2n or 20%.

3. Modern vacuum pumps can evacuate a vessel down to a pressure of 4.0 × 10^{-15} atm. at room temperature(300K). Taking R = 8.0 JK^{-1}

mole^{-1}, 1 atm = 10^5 Pa and $N_{Avogadro}$ = 6 × 10^{23} mole^{-1}, the mean distance between molecules of gas in an evacuated vessel will be of the order of:

(a) 0.2 μm	**(b) 0.2 mm**
(c) 0.2 cm	**(d) 0.2 nm**

Solution:

(b)

4. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R=8.31 j/mol.K)

(a) 62 J	**(b) 104 J**
(c) 124 J	**(d) 208 J**

Solution:

(d) The heat is supplied at constant pressure. i.e., the process is isobaric

$$\therefore Q = nC_p\Delta t$$

$$= 2\left[\frac{5}{2}R\right] \times \Delta t = 2 \times \frac{5}{2} \times 8.31 \times 5 = 208\,J$$

$$\left(\because C_p = \frac{5}{2}R\ for\ mono - atomic\ gas\right)$$

5. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is –

(a) 1/T	**(b) 2/T**
(c) 3/T	**(d) 4/T**

Solution:

(c) $pT = constant\ (given)$

$$\therefore \left(\frac{nRT}{V}\right)T_2 = constant\ or\ T^3V^{-1} = constant$$

$(\therefore PV = nRT)$

Differentiating the equation, we get

$$\frac{3T^2}{V}.dT - \frac{T^3}{V^2}dV = 0\ or\ 3dT = \frac{T}{V}.dV \qquad ...(i)$$

From the equation

$dV = V\gamma dT$

γ = coefficient of volume expansion of gas = $\frac{dV}{V.dT}$

from eq. (i) $\gamma = \frac{dV}{V.dT} = \frac{3}{T}$

6. The amount of heat energy required to raise the temperature of 1 g of helium at NTP, from T_1K to T_2K is

(a) $\frac{3}{8}N_aK_B(T_2 - T_1)$ **(b)** $\frac{3}{2}N_aK_B(T_2 - T_1)$

(c) $\frac{3}{4}N_aK_B(T_2 - T_1)$ **(d)** $\frac{3}{4}N_aK_B\left(\frac{T_2}{T_1}\right)$

Solution:

(a) We know that, $Q = \frac{F}{2}nR\Delta T$

∴ Amount of heat required,

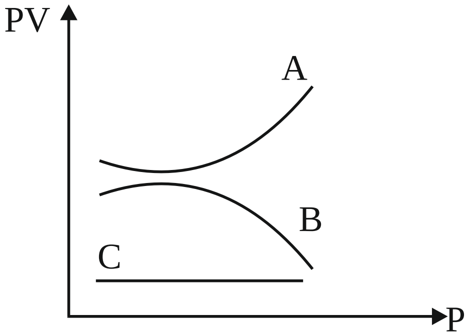
$$Q = \frac{3}{2} \times \frac{1}{4} \times K_BN_a\Delta T$$

$$= \frac{3}{8}N_aK_B(T_2 - T_1)$$

Test Yourself

Beginner Test - I

1. Under what conditions, real gases behave as an ideal gas?
2. The given graph shows the variation of p-V versus p graph for different gases at constant temperature. Which of the following gas is ideal and why?



3. The volume of a given mass of a gas at 27°C, 1 atm is 100 cc. What will be its volume at 327°C?
4. If a vehicle runs on the road for a long time, then the air pressure in the tyres increases. Explain.
5. Two molecules of a gas have speeds of 9×10^6 ms^{-1} and 1×10^6 ms^{-1}, respectively. What is the root mean square speed of these molecules?
6. If a gas is heated, its temperature increases. On the basis of kinetic theory of gases, explain.
7. Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27°C and 1 atm pressure. ($k_B = 1.38 \times 10^{-23}$ JK^{-1}).
8. A flask contains argon and chlorine in the ratio Of 2:1 by mass. The temperature of the mixture is 27°C. Obtain the ratio of
   (i) Average kinetic energy per molecule and
   (ii) Root mean square speed (V_{rms}) of the molecules of the two gases. Atomic mass of argon = 39.9 u; Molecular mass of chlorine = 70.9 u.
9. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules?
   Is the root mean square speed of molecules, the same in the three cases? If not, in which case is V_{rms}, the largest?
10. At what temperature, is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at –20°C? (Atomic mass of Ar = 39.9 u, He = 4.0 u).
11. A gas is contained in a closed vessel. How pressure due to the gas will be affected if for of attraction between the molecules disappear suddenly?
12. Estimate the average thermal energy of a helium atom at (i) room temperature (27°C) (ii) the temperature on the surface of the sun (6000 K), (iii) the temperature of 10 million Kelvin (the typical core temperature in the case of a star).
    Thermal energy represents that part of internal energy which is translational kinetic in nature. This is equal to $\frac{3}{2}k_B T$; which only depends on absolute temperature of the gas.
13. Explain, why
    (i) There is no atmosphere on moon.
    (ii) There is fall in temperature with altitude.
14. The container shown in figure has two chambers, separated by a partition of volumes $V_1 = 2.0$ L and $V_2 = 3.0$ L. The chambers contain $\mu_1 = 4.0$ and $\mu_2 = 5.0$ moles of a gas at pressures $P_1 = 1.00$ atm and $p_2 = 2.00$ atm. Calculate the pressure after the partition is removed and the mixture attains equilibrium.

V_1	V_2
μ_1	μ_2

p_1	p_2

15. A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, e.g. does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so called law of atmosphere.

$$n_2 = n_1 \exp[-mg(h_2 - h_1)/k_BT]$$

where, n_2 and n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column.

$$n_2 = n_1 \exp[-mg N_A(\rho - \rho')(h_2 - h_1)/(\rho RT)]$$

where, ρ is the density of the suspended particle and ρ', that of surrounding medium.

[∵ N_A is Avogadro's number and R is the universal gas constant.]

16. You are given, the following data about a group of particles, where n_i represents the number of molecules with speed v_i

n_i	2	4	8	6	3
$v_i(ms^{-1})$	1.0	2.0	3.0	4.0	5.0

Calculate

(i) average speed

(ii) rms speed

(iii) most probable speed.

17. Consider an ideal gas with following distribution of speeds.

Speed (m/s)	**% of molecules**
200	10
400	20
600	40
800	20
1000	10

(i) Calculate V_{rms} and hence T($m = 3.0 \times 10^{-26}$ kg)

(ii) If all the molecules with speed 1000 m/s escape from the system, calculate new v_{rms} and hence T.

18. What is the number of degree of freedom of a bee flying in a room?
19. Is molar specific heat of a solid, a constant quantity?
20. Name an experimental evidence in support of random motion of gas molecules.
21. How is mean free path depends on number density of the gas?
22. If a molecule having N atoms has k number of constraints, how many degrees of freedom does the gas possess?
23. Equal masses of monoatomic and diatomic gases are supplied heat at the same temperature, pressure and volume. If same amount of heat is supplied to both the gases, which of them will undergo greater temperature rise?
24. Calculate the number of degrees of freedom in 15 cm^3 of nitrogen at NTP.
25. What will be the internal energy of 8 g of oxygen at STP?
26. What is basic law followed by equipartition of energy?
27. Calculate the mean free path of a molecule of a gas at a room temperature and one atmospheric pressure. The radius of the gas molecules (avg) is 2×10^{-10} m?

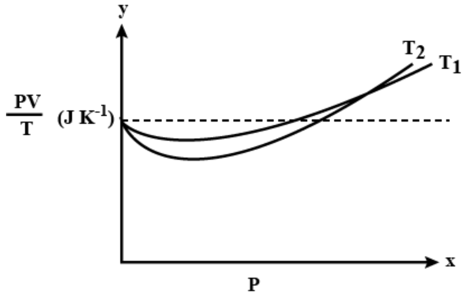
28. Calculate the temperature of atoms at which rms speed of Argon gas is equal to the rms speed of Helium gas atoms at –10°C? (Atomic mass of Ar = 39.9 u, that of He = 4u)
29. A cylinder of fixed capacity contains 44.8 L of helium gas at STP. Calculate the amount of heat required to raise the temperature of container by 15°C?
    [given R = 8.31 J mol^{-1} K^{-1}]
30. Three moles of a diatomic gas is mixed with two moles of monoatomic gas. What will be the molecular specific heat of the mixture at constant volume? [given, R = 8.31 J mol^{-1} K^{-1}]
31. A box of 1.00 m^3 is filled with nitrogen at 1.50 atm at 300 K. The box has a hole of an area is 0.010 mm^2. How much time is required for the pressure to reduce by 0.10 atm, if the pressure outside is 1 atm.
32. (i) What do you understand by specific heat capacity of water?
    (ii) If one mole of ideal monoatomic gas ($\gamma = 5/3$) is mixed with one mole of diatomic gas ($\gamma = 7/5$). What is the value of γ for the mixtures? (here, γ represents the ratio of specific heat at constant pressure to that at constant volume)

Answer Key

Refer. *Solutions*

Beginner Test - II

1. Estimate the fraction of molecular volume to the actual volume occupied by oxygen gas at STP. Take the diameter of an oxygen molecule to be 3Å.
2. Molar volume is the volume occupied by 1 mol of any (ideal) gas at standard temperature and pressure (STP: 1 atmospheric pressure, 0 °C). Show that it is 22.4 litres.
3. Figure shows plot of PV/T versus P for 1.00×10^{-3} kg of oxygen gas at two different temperatures.



What does the dotted plot signify?

Which is true: T1 > T2 or T1 < T2?

What is the value of PV/T where the curves meet on the y-axis?

If we obtained similar plots for 1.00×10^{-3} kg of hydrogen, would we get the same value of PV/T at the point where the curves meet on the y-axis? If not, what mass of hydrogen yields the same value of PV/T (for low pressure high temperature region of the plot)? (Molecular mass of H_2 = 2.02 u, of O_2 = 32.0 u, R = 8.31 J mol^{-1} K^{-1}.)

4. An oxygen cylinder of volume 30 litres has an initial gauge pressure of 15 atm and a temperature of 27 °C. After some oxygen is withdrawn from the cylinder, the gauge pressure drops to 11 atm and its temperature drops to 17 °C. Estimate the mass of oxygen taken out of the cylinder (R = 8.31 J mol^{-1} K^{-1}, molecular mass of O_2 = 32 u).
5. An air bubble of volume 1.0 cm^3 rises from the bottom of a lake 40 m deep at a temperature of 12 °C. To what volume does it grow when it reaches the surface, which is at a temperature of 35 °C?
6. Estimate the total number of air molecules (inclusive of oxygen, nitrogen, water vapour and other constituents) in a room of capacity 25.0 m^3 at a temperature of 27 °C and 1 atm pressure.
7. Estimate the average thermal energy of a helium atom at (i) room temperature (27 °C), (ii) the temperature on

the surface of the Sun (6000 K), (iii) the temperature of 10 million Kelvin (the typical core temperature in the case of a star).

8. Three vessels of equal capacity have gases at the same temperature and pressure. The first vessel contains neon (monatomic), the second contains chlorine (diatomic), and the third contains uranium hexafluoride (polyatomic). Do the vessels contain equal number of respective molecules? Is the root mean square speed of molecules the same in the three cases? If not, in which case is vrms the largest?
9. At what temperature is the root mean square speed of an atom in an argon gas cylinder equal to the rms speed of a helium gas atom at – 20 °C? (Atomic mass of Ar = 39.9 u, of He = 4.0 u).
10. Estimate the mean free path and collision frequency of a nitrogen molecule in a cylinder containing nitrogen at 2.0 atm and temperature 17 °C. Take the radius of a nitrogen molecule to be roughly 1.0 Å. Compare the collision time with the time the molecule moves freely between two successive collisions (Molecular mass of N_2 = 28.0 u).
11. A metre long narrow bore held horizontally (and closed at one end) contains a 76 cm long mercury thread, which traps a 15 cm column of air. What happens if the tube is held vertically with the open end at the bottom?
12. From a certain apparatus, the diffusion rate of hydrogen has an average value of 28.7 $cm^3 s^{-1}$. The diffusion of another gas under the same conditions is measured to have an average rate of 7.2 $cm^3 s^{-1}$. Identify the gas. [Hint: Use Graham's law of diffusion: $R_1/R_2 = (M_1/M_2)$ 1/2, where R_1, R_2 are diffusion rates of gases 1 and 2, and M_1 and M_2 their respective molecular masses. The law is a simple consequence of kinetic theory.]
13. A gas in equilibrium has uniform density and pressure throughout its volume. This is strictly true only if there are no external influences. A gas column under gravity, for example, does not have uniform density (and pressure). As you might expect, its density decreases with height. The precise dependence is given by the so-called law of atmospheres $n_2 = n_2 \exp[-mg(h_2 - h_1)/k_BT]$

    Where n_2, n_1 refer to number density at heights h_2 and h_1 respectively. Use this relation to derive the equation for sedimentation equilibrium of a suspension in a liquid column: $n_2 = n_1 \exp[-mg N_A (\rho - \rho')(h_2 - h_1)/(\rho RT)]$

    Where ρ is the density of the suspended particle, and ρ' that of surrounding medium. [NA is Avogadro's number, and R the universal gas constant.] [Hint: Use Archimedes principle to find the apparent weight of the suspended particle.]
14. Given below are densities of some solids and liquids. Give rough estimates of the size of their atoms:

Substance	Atomic Mass(u)	Density (10^3 Kg m^{-3})
Carbon (diamond)	12.01	2.22
Gold	197.00	19.32
Nitrogen (liquid)	14.01	1.00
Lithium	6.94	0.53
Fluorine (liquid)	19.00	1.14

[Hint: Assume the atoms to be 'tightly packed' in a solid or liquid phase, and use the known value of Avogadro's number. You should, however, not take the actual numbers you obtain for various atomic sizes too literally. Because of the crudeness of the tight packing approximation, the results only indicate that atomic sizes are in the range of a few Å].

Answer Key

1. 3.8×10^{-4} 2. 22.4 litres 3. 6.3×10^{-5} kg 4. 0.131 kg
5. 5.263 cm^3 6. 6.11×10^{26} molecules 7. 2.07×10^{-16} J 8. *Refer Solution*
9. 2.52×10^3 K 10. 500 11. 47.8 cm 12. 32.09 g, oxygen
14. 1.29 Å. , 1.59 Å. , 1.77 Å, 1.73 Å. , 1.88 Å

Expert Test - I

1. The mean free path l for a gas molecule depends upon diameter, d of the molecule as

(a) $l \propto \frac{1}{d^2}$ (b) $l \propto d$

(c) $l \propto d^2$ (d) $l \propto \frac{1}{d}$

2. An ideal gas equation can be written as $p = \frac{\rho RT}{M_0}$

where, ρ and M_0 are respectively,

(a) mass density, mass of the gas (b) number density, molar mass

(c) mass density, molar mass (d) number density, mass of the gas

3. The mean free path λ for a gas, with molecular diameter d and number density n can be expressed as

(a) $\frac{1}{\sqrt{2}n\pi d^2}$ (b) $\frac{1}{\sqrt{2}n^2\pi d^2}$

(c) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$ (d) $\frac{1}{\sqrt{2}n\pi d}$

4. A cylinder contains hydrogen gas at pressure of 249 kPa and temperature 27°C.
Its density is ($R = 8.3\ \text{J mol}^{-1}\ \text{K}^{-1}$).

(a) $0.2\ \text{kg/m}^3$ (b) $0.1\ \text{kg/m}^3$

(c) $0.02\ \text{kg/m}^3$ (d) $0.5\ \text{kg/m}^3$

5. Increase in temperature of a gas filled in a container would lead to

(a) increase in its kinetic energy (b) decrease in its pressure

(c) decrease in intermolecular distance (d) increase in its mass

6. At what temperature will the rms speed of oxygen molecules become just sufficient for escaping from the Earth's atmosphere?
(Given: mass of oxygen molecule, $m = 2.76 \times 10^{-26}$ kg, Boltzmann's constant $k_B = 1.38 \times 10^{-23}\ \text{JK}^{-1}$)

(a) 5.016×10^4 K (b) 8.362×10^4 K

(c) 2.508×10^4 K (d) 1.254×10^4 K

7. The molecules of a given mass of a gas have r.m.s. velocity of 200ms^{-1} at 27°C and $1.0 \times 10^5\ \text{Nm}^{-2}$ pressure. When the temperature and pressure of the gas are respectively, 127°C and $0.05 \times 10^5\ \text{Nm}^{-2}$, the rms velocity of its molecules in ms^{-1} is

(a) $\frac{400}{\sqrt{3}}$ (b) $\frac{100\sqrt{2}}{3}$

(c) $\frac{100}{3}$ (d) $100\sqrt{2}$

8. A given sample of an ideal gas occupies a volume V at a pressure p and absolute temperature T. The mass of each molecule of the gas is m. Which of the following gives the density of the gas?

(a) p/(kT) (b) pm/(kT)

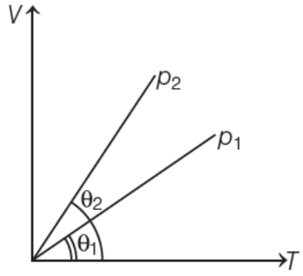
(c) p/(kTV) (d) mkT

9. The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_p and C_v respectively. If $\gamma = \frac{C_p}{C_v}$ and R is the universal gas constant, then C_v is equal to

(a) $\frac{1+\gamma}{1-\gamma}$ (b) $\frac{R}{(\gamma-1)}$

(c) $\frac{(\gamma - 1)}{R}$ (d) γR

10. In the given (V-T) diagram, what is the relation between pressures p_1 and p_2 ?



(a) $p_2 = p_1$ (b) $p_2 > p_1$
(c) $p_2 < p_1$ (d) Cannot be predicted

11. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is

[NEET 2013]

(a) $\frac{4}{3}$ (b) 2
(c) $\frac{5}{3}$ (d) $\frac{3}{2}$

12. Match Column I with Column II and choose the correct match from the given choices.

	Column I		**Column II**
A.	Root mean square of gas molecules	1.	$\frac{1}{3}nmv^{-2}$
B.	Pressure exerted by ideal gas	2.	$\sqrt{\frac{3RT}{M}}$
C.	Average kinetic energy of a molecule	3.	$\frac{5}{2}RT$
D.	Total internal energy of 1 mole of a diatomic gas	4.	$\frac{3}{2}k_BT$

Codes

	A	**B**	**C**	**D**
(a)	3	1	4	2
(b)	2	3	4	1
(c)	2	1	4	3
(d)	3	2	1	4

13. The average thermal energy for a monoatomic gas is (where, k_B is Boltzmann constant and T is absolute temperature.)

(a) $\frac{3}{2}k_BT$ (b) $\frac{5}{2}k_BT$
(c) $\frac{7}{2}k_BT$ (d) $\frac{1}{2}k_BT$

14. The value of $\gamma\left(=\frac{C_p}{C_v}\right)$, for hydrogen helium and another ideal diatomic gas X (whose molecules are not rigid but have an additional vibrational mode), are respectively equal to

(a) $\frac{7}{5},\frac{5}{3},\frac{9}{7}$
(b) $\frac{5}{3},\frac{7}{5},\frac{9}{7}$
(c) $\frac{5}{3},\frac{7}{5},\frac{7}{5}$
(d) $\frac{7}{5},\frac{5}{3},\frac{7}{5}$

15. A gas mixture consists of 2 moles of O_2 and 4 moles of Ar at temperature T. Neglecting all vibrational modes, the total internal energy of the system is

(a) 4 RT
(b) 15 RT
(c) 9 RT
(d) 11 RT

16. One mole of an ideal monatomic gas undergoes a process described by the equation pV^3 = constant. The heat capacity of the gas during this process is

(a) $\frac{3}{2}R$
(b) $\frac{5}{2}R$
(c) 2 R
(d) R

17. The amount of heat energy required to raise the temperature of 1 g of helium at NTP, from T_1K to T_2K is

(a) $\frac{3}{8}N_aK_B\left(T_2-T_1\right)$
(b) $\frac{3}{2}N_aK_B\left(T_2-T_1\right)$
(c) $\frac{3}{4}N_aK_B\left(T_2-T_1\right)$
(d) $\frac{3}{4}N_aK_B\left(\frac{T_2}{T_1}\right)$

Answer Key

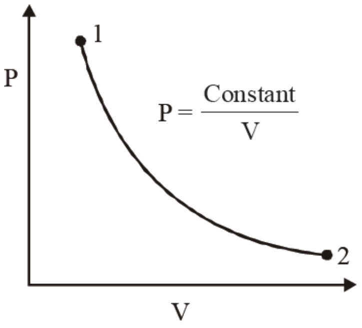
1. (a) 2. (a) 3. (a) 4. (a)
5. (a) 6. (b) 7. (a) 8. (b)
9. (b) 10. (c) 11. (d) 12. (c)
13. (a) 14. (a) 15. (d) 16. (a)
17. (a)

Expert Test - II

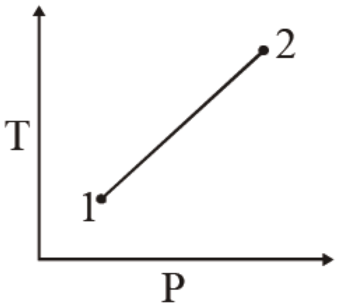
1. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston is l_1, and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass, m, will be given by: (R is universal gas constant and g is the acceleration due to gravity)

(a) $\frac{RT}{ng}\left[\frac{l_1-3l_2}{l_1l_2}\right]$
(b) $\frac{RT}{g}\left[\frac{2l_1+l_2}{l_1l_2}\right]$
(c) $\frac{nRT}{g}\left[\frac{1}{l_2}+\frac{1}{l_1}\right]$
(d) $\frac{nRT}{g}\left[\frac{l_1-l_2}{l_1l_2}\right]$

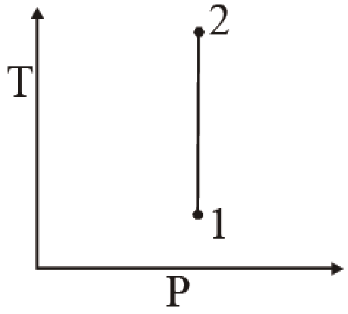
2. For the P-V diagram given for an ideal gas,



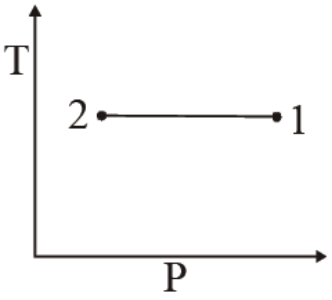
Out of the following which one correctly represents the T-P diagram ?



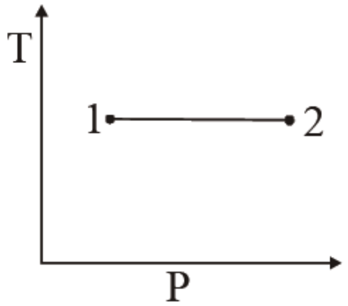
(a)



(b)



(c)



(d)

3. Initially a gas of diatomic molecules is contained in a cylinder of volume V_1 at a pressure P_1 and temperature 250 K. Assuming that 25% of the molecules get dissociated causing a change in number of moles. The pressure of the resulting gas at temperature 2000 K, when contained in a volume $2V_1$ is given by P_2. The ratio P_2/P_1 is ___________.

4. The change in the magnitude of the volume of an ideal gas when a small additional pressure ΔP is applied at a constant temperature, is the same as the change when the temperature is reduced by a small quantity ΔT at constant pressure. The initial temperature and pressure of the gas were 300 K and 2 atm. respectively. If $|\Delta T| = C|\Delta P|$, then value of C in (K/atm.) is ____________.

5. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to? (Given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, g = 980 cm/s^2, density of mercury = 13.6 g/cm^3)

(a) 4.0×10^{18} (b) 4.0×10^{16}

(c) 5.8×10^{16} (d) 5.8×10^{18}

6. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be:

(a) 100 m/s (b) $80\sqrt{5}$ m/s

(c) $100\sqrt{5}$ m/s (d) 80 m/s

7. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of:

(a) 10^4 N/m^2 (b) 10^8 N/m^2

(c) 10^3 N/m^2 (d) 10^{16} N/m^2

8. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to :
   [Boltzmann Constant $k_B = 1.38 \times 10^{-23}$ J/K
   Avogadro Number $N_A = 6.02 \times 10^{26}$ /kg
   Radius of Earth : 6.4×10^6 m
   Gravitational acceleration on Earth = 10 ms^{-2}]
   (a) 800 K (b) 3×10^5 K
   (c) 10^4 K (d) 650 K
9. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(helium)}{V_{rms}(\arg on)}\right]$ is close to :
   (a) 3.16 (b) 0.32
   (c) 0.45 (d) 2.24
10. N moles of a diatomic gas in a cylinder are at a temperature T. Heat is supplied to the cylinder such that the temperature remains constant but n moles of the diatomic gas get converted into monoatomic gas. What is the change in the total kinetic energy of the gas?
   (a) $\frac{1}{2}nRT$ (b) 0
   (c) $\frac{3}{2}nRT$ (d) $\frac{5}{2}nRT$
11. In an ideal gas at temperature T, the average force that a molecule applies on the walls of a closed container depends on T as T^q. A good estimate for q is:
   (a) $\frac{1}{2}$ (b) 2
   (c) 1 (d) $\frac{1}{4}$
12. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1}. The gas is:
   (a) H_2 (b) Cl_2
   (c) O_2 (d) F_2
13. In the isothermal expansion of 10g of gas from volume V to 2V the work done by the gas is 575J. What is the root mean square speed of the molecules of the gas at that temperature?
   (a) 398m/s (b) 520m/s
   (c) 499m/s (d) 532m/s
14. Nitrogen gas is at 300°C temperature. The temperature (in K) at which the rms speed of a H_2 molecule would be equal to the rms speed of a nitrogen molecule, is _____________. (Molar mass of N_2 gas 28 g);
15. Molecules of an ideal gas are known to have three translational degrees of freedom and two rotational degrees of freedom. The gas is maintained at a temperature of T. The total internal energy, U of a mole of this gas, and the value of $\gamma\left(=\frac{C_p}{C_v}\right)$ are given, respectively, by:
   (a) U = $\frac{5}{2}$RT and $\gamma = \frac{6}{5}$ (b) U = 5RT and $\gamma = \frac{7}{5}$
   (c) U = $\frac{5}{2}$RT and $\gamma = \frac{7}{5}$ (d) U = 5RT and $\gamma = \frac{6}{5}$
16. In a dilute gas at pressure P and temperature T, the mean time between successive collisions of a molecule

varies with T is:

(a) T

(b) $\frac{1}{\sqrt{T}}$

(c) $\frac{1}{T}$

(d) $\sqrt{T}$

17. Match the C_p/C_v ratio for ideal gases with different type of molecules:

Column-I	**Column-II**
Molecule Type	Cp/Cv
(A) Monatomic	(I) 7/5
(B) Diatomic rigid molecules	(II) 9/7
(C) Diatomic non-rigid molecules	(III) 4/3
(D) Triatomic rigid molecules	(IV) 5/3

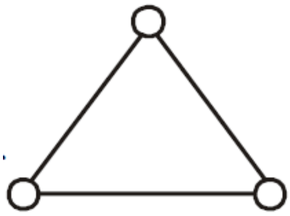
(a) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(b) (A)-(III), (B)-(IV),(C)-(II), (D)-(I)

(c) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(d) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

18. Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of massless rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is:



(a) $\frac{5}{2}RT$

(b) $\frac{3}{2}RT$

(c) $\frac{9}{2}RT$

(d) 3RT

19. To raise the temperature of a certain mass of gas by 50°C at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100°C at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)?

(a) 5

(b) 6

(c) 3

(d) 7

20. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Assuming the gases to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is :

(a) 15

(b) 13

(c) 20

(d) 11

21. An ideal gas in a closed container is slowly heated. As its temperature increases, which of the following statements are true?

(1) The mean free path of the molecules decreases

(2) The mean collision time between the molecules decreases

(3) The mean free path remains unchanged

(4) The mean collision time remains unchanged

(a) (2) and (3)

(b) (1) and (2)

(c) (3) and (4)

(d) (1) and (4)

22. Consider two ideal diatomic gases A and B at some temperature T. Molecules of the gas A are rigid, and have

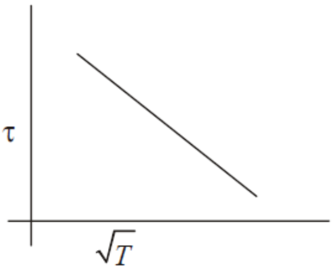
a mass m. Molecules of the gas B have an additional vibrational mode, and have a mass $\frac{m}{4}$. The ratio of the specific heats (C_V^A and C_V^B) of gas A and B, respectively is:

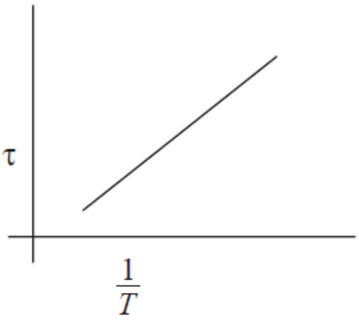
(a) 7:9
(b) 5:9
(c) 3:5
(d) 5:7

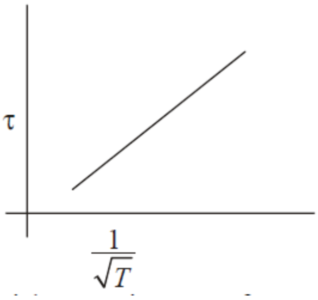
23. Two gases-argon (atomic radius 0.07 nm, atomic weight 40) and xenon (atomic radius 0.1 nm, atomic weight 140) have the same number density and are at the same temperature. The ratio of their respective mean free times is closest to:

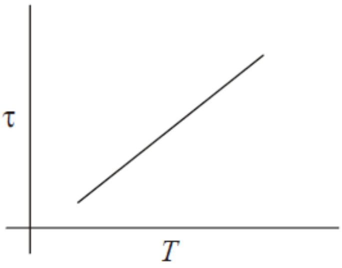
(a) 3.67
(b) 1.83
(c) 2.3
(d) 4.67

24. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) for the molecules of an ideal gas, as a function of temperature (T), qualitatively, is: (Graphs are schematic and not drawn to scale)

(a)


(b)


(c)


(d)


25. Consider a mixture of n moles of helium gas and 2n moles of oxygen gas (molecules taken to be rigid) as an ideal gas. Its C_P/C_V value will be:

(a) 19/13
(b) 67/45
(c) 40/27
(d) 23/15

26. Two moles of an ideal gas with $\frac{C_p}{C_V} = \frac{5}{3}$ are mixed with 3 moles of another ideal gas with $\frac{C_p}{C_V} = \frac{4}{3}$. The value of $\frac{C_p}{C_V}$ for the mixture is:

(a) 1.45
(b) 1.50
(c) 1.47
(d) 1.42

27. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume? (R = 8.3 J/mol K)

(a) 19.7 J/mol L
(b) 15.7 J/mol K
(c) 17.4J/mol K
(d) 21.6 J/mol K

28. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process ?

(a) 25 J
(b) 35 J

(c) 30J (d) 40J

29. A 25×10^{-3} m^3 volume cylinder is filled with 1 mol of O_2 gas at room temperature (300 K). The molecular diameter of O_2, and its root mean square speed, are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule?

(a) ~10^{12} (b) ~10^{11}

(c) ~10^{10} (d) ~10^{13}

30. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is :

(a) $\frac{2}{3}Q$ (b) $\frac{5}{3}Q$

(c) $\frac{7}{5}Q$ (d) $\frac{3}{2}Q$

31. The specific heats, C_p and C_v of a gas of diatomic molecules, A, are given (in units of Jmol^{-1} k^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules, B, has the corresponding values 30 and 21. If they are treated as ideal gases, then:

(a) A is rigid but B has a vibrational mode. (b) A has a vibrational mode but B has none.

(c) A has one vibrational mode and B has two. (d) Both A and B have a vibrational mode each.

32. An ideal gas occupies a volume of 2 m^3 at a pressure of 3×10^6 Pa. The energy of the gas:

(a) 9×10^6 J (b) 6×10^4 J

(c) 10^8 J (d) 3×10^2 J

33. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to:

(a) 2×10^{-7} s (b) 4×10^{-8} s

(c) 0.5×10^{-8} s (d) 3×10^{-6} s

34. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is :

(a) 15 RT (b) 12 RT

(c) 4RT (d) 20 RT

35. Two kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion?

(a) 10^3 J (b) 10^5 J

(c) 10^4 J (d) 10^6 J

36. A 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled, is about: [Take R = 8.3 J/K mole]

(a) 0.9 kJ (b) 6kJ

(c) 10kJ (d) 14 kJ

37. Two moles of an ideal monoatomic gas occupies a volume Vat 27°C. The gas expands adiabatically to a volume 2 V. Calculate (1) the final temperature of the gas and (2) change in its internal energy.

(a) (1) 189 K (2) 2.7 kJ

(b) (1) 195K (2) –2.7 kJ

(c) (1) 189K (2) –2.7 kJ

(d) (1) 195K (2) 2.7 kJ

38. Two moles of helium are mixed with n with moles of hydrogen. If $\frac{C_P}{C_V} = \frac{3}{2}$ for the mixture, then the value of n is

(a) 3/2 (b) 2
(c) 1 (d) 3

39. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is :
(a) a = 14b (b) a = 28 b
(c) $a = \frac{1}{14} b$ (d) a = b

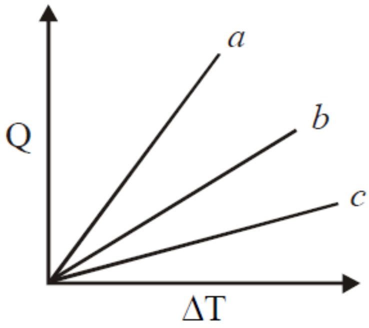
40. An ideal gas has molecules with 5 degrees of freedom. The ratio of specific heats at constant pressure (C_p) and at constant volume (C_v) is :
(a) 6 (b) $\frac{7}{2}$
(c) $\frac{5}{2}$ (d) $\frac{7}{5}$

41. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume Vis given by PV^n = constant, then n is given by (Here C_P and C_V are molar specific heat at constant pressure and constant volume, respectively):
(a) $n = \frac{C_p - C}{C - C_V}$ (b) $n = \frac{C - C_V}{C - C_P}$
(c) $n = \frac{C_P}{C_V}$ (d) $n = \frac{C - C_p}{C - C_V}$

42. Using equipartition of energy, the specific heat (in J kg^{-1} K^{-1}) of aluminium at room temperature can be estimated to be (atomic weight of aluminium = 27)
(a) 410 (b) 25
(c) 1850 (d) 925

43. Modern vacuum pumps can evacuate a vessel down to a pressure of 4.0×10^{-15} atm. at room temperature (300K). Taking R = 8.0 JK^{-1} mole^{-1}, 1 atm = 10^5 Pa and $N_{\text{Avogadro}} = 6 \times 10^{23}$ mole^{-1}, the mean distance between molecules of gas in an evacuated vessel will be of the order of:
(a) 0.2 μm (b) 0.2 mm
(c) 0.2 cm (d) 0.2 nm

44. Figure shows the variation in temperature (ΔT) with the amount of heat supplied (Q) in an isobaric process corresponding to a monoatomic (M), diatomic (D) and a polyatomic (P) gas. The initial state of all the gases are the same and the scales for the two axes coincide. Ignoring vibrational degrees of freedom, the lines a, b and c respectively correspond to :



(a) P, M and D (b) M, D and P
(c) P, D and M (d) D, M and P

45. A closed vessel contains 0.1 mole of a monatomic ideal gas at 200 K. If 0.05 mole of the same gas at 400 K

is added to it, the final equilibrium temperature (in K) of the gas in the vessel will be close to __________.

Answer Key

1. (d) 2. (c) 3. (5) 4. (150)
5. (a) 6. (c) 7. (Bonus) 8. (c)
9. (a) 10. (a) 11. (c) 12. (a)
13. (c) 14. (41) 15. (c) 16. (b)
17. (c) 18. (d) 19. (b) 20. (a)
21. (a) 22. (d) 23. (Bonus) 24. (c)
25. (a) 26. (d) 27. (c) 28. (b)
29. (c) 30. (c) 31. (b) 32. (a)
33. (b) 34. (a) 35. (c) 36. (c)
37. (c) 38. (b) 39. (a) 40. (d)
41. (d) 42. (d) 43. (b) 44.(b)
45. (266.67)

Pro Test - I

1. The temperature of an open room of volume $30m^3$increases from 17°C to 27°C due to sunshine. The atmospheric pressure in the room remains 1×10^5Pa. If n_i and $n_f$are the number of molecules in the room before and after heating, then $n_f - n_i$ will be:

(a) 2.5×10^{25} (b) -2.5×10^{25}

(c) -1.61×10^{23} (d) 1.38×10^{23}

2. Two non-reactive monatomic ideal gases have their atomic masses in the ratio 2:3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4:3. The ratio of their densities is

(a) 1:4 (b) 1:2 (c) 6:9 (d) 8:9

3. A real gas behaves like an ideal gas if its

(a) Pressure and temperature are both high

(b) Pressure and temperature are both low

(c) Pressure is high and temperature is low

(d) Pressure is low and temperature is high

4. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds

$\left(\frac{rms\ (helium)}{rms\ (argon)}\right)$ is

(a) 0.32 (b) 0.45 (c) 2.24 (d) 3.16

5. A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are)

(a) The average energy per mole of the gas mixture is 2RT

(b) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$

(c) The ratio of the rms speed of helium atoms to that of hydrogen molecules is ½

(d) The ratio of the speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$

6. C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then

(a) $C_p - C_v$ is larger for a diatomic ideal gas than for a monatomic ideal gas

(b) $C_p + C_v$ is larger for a diatomic ideal gas than for a monatomic ideal gas

(c) C_p/C_v is larger for a diatomic ideal gas than for a monatomic ideal gas

(d) $C_p\,.\,C_v$ is larger for a diatomic ideal gas than for a monatomic ideal gas

7. **Statemet-1:** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is d1.5 times the product of its pressure and its volume. Because

Statement-**2:** The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.

(a) Statement-1 is True; Statement-2 is true;
Statement-2 is a correct explanation for Statement-1
(b) Statement-1 is True; Statement-2 is True;
Statement-2 is NOT a correct explanation for Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-2 is False, Statement-2 is True

8. Two moles of ideal helium gas are in a rubber balloon at 30℃. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35℃. The amount of heat required in raising the temperature is nearly (take R=8.31 j/mol.K)

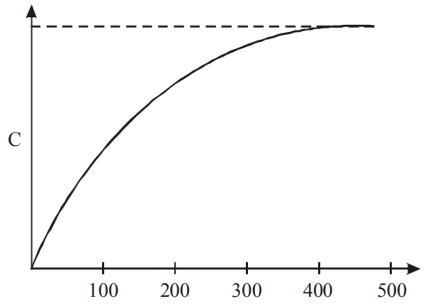
(a) 62 J (b) 104 J (c) 124 J (d) 208 J

9. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is –

(a) 1/T (b) 2/T (c) 3/T (d) 4/T

10. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant R = 8.0 J $mol^{-1}K^{-1}$, the decrease in its internal energy, in Joule, is______.

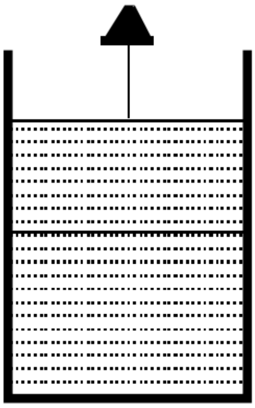
11. The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation.



(a) The rate at which heat is absorbed in the range 0-100 K varies linearly with temperature T.
(b) Heat absorbed in increasing the temperature from 0-100 K is less than the heat required for increasing the temperature from 400-500 K.
(c) There is no change in the rate of heat absorption in the range 400-500 K.
(d) The rate of heat absorption increases in the range 200-300 K.

Passage-1

In the figure, a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulated material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat. The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R, C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R, C_P = \frac{7}{2}R$.



12. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be

(a) 550 K (b) 525 K

(c) 513 K (d) 490 K

13. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. The total work done by the gases till the time they achieve equilibrium will be

(a) 250 R (b) 200 R

(c) 100 R (d) -100 R

14. A jar contains a gas and a few drops of water at T°K. The pressure in the jar is 830 mm of Hg. The temperature of the jar is reduced by 1%. The saturated vapor pressures of water at the two temperatures are 30 and 25 mm of Hg. Calculate the new pressure in the jar.

15. Given samples of 1 c.c. of hydrogen and 1c.c. of oxygen, both at N.T.P. which sample has a larger number of molecules?

Answer Key

1. (b) 2. (d) 3. (d) 4. (d)
5. (a,b,d) 6. (b,d) 7. (b) 8. (d)
9. (c) 10. (900J) 11. (b,c,d) 12. (d)
13. (d) 14. $P_2 = 792$mm Hg ∴ total pressure in the jar = 792 + 25 = 817mm Hg
15. From, PV = nRT, When P, T are same n ∝ V as volumes are same i.e., 1c.c of each hydrogen and oxygen, so both samples will have equal number of molecules.

Solution:
Test Papers

Solutions: Beginner Test - I

1.

At low pressure and high temperature, real gases behave as an ideal gas.

2.

Gas A is ideal because PV is constant for gas A so, gas A obeys Boyle's law for all values of pressure.

3.

Keeping p constant, we have

$$V_2 = \frac{V_1T_2}{T_1} = \frac{100\times 600}{300}$$

= 200 cc

4.

Due to the presence of friction between the road and tyres, the tyres get heated as a result of which temperature of air inside the tyre increases and hence pressure in tyre also increases.

5.

$$v_{rms} = \sqrt{\frac{v_1^2 + v_2^2}{2}} = \sqrt{\frac{\left(9\times 10^6\right)^2 + \left(1\times 10^6\right)^2}{2}}$$

$$= \sqrt{\frac{\left(81+1\right)\times 10^{12}}{2}}$$

= $\sqrt{41}$ × 10^6 ms^{-1}

6.

If a gas is heated, then the root mean square velocity of its molecules is increased.

$\because v_{rms} \propto \sqrt{T}$

∴ The temperature of the gas increases.

7.

Given, V = 25.0 m^3, T = 273 + 27 = 300 K

$k_B = 1.38 \times 10^{-23}$ JK^{-1}

Now, pV = μRT

pV = $\mu(N_Ak_B)T$

pV = $N'k_BT$

[N' is total number of molecules]

$$\therefore \quad N' = \frac{pV}{k_BT}$$

$$= \frac{\left(1.01\times 10^5\right)\times 25}{\left(1.38\times 10^{-23}\right)\times 300}$$

= 6.10 × 10^{26}

8.

(i) The average kinetic energy per molecule of any gas is $\frac{3}{2}$ k_Bt. It depends only on temperature and not on the nature of the gas. As both argon and chlorine have the same temperature in the flask, the ratio of average KE per molecule of the two gases is 1:1.

(ii) If m is mass of single molecule and M is the molecular mass, then

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}k_BT$$

= constant at a given temperature

$$\therefore \quad \frac{\left(v_{rms}^2\right)_{Ar}}{\left(v_{rms}^2\right)_{Cl_2}} = \frac{\left(m\right)_{Cl_2}}{\left(m\right)_{Ar}} = \frac{M_{Cl_2}}{M_{Ar}} = \frac{70.9}{39.9} = 1.777$$

$$\text{or} \quad \frac{\left(v_{rms}\right)_{Ar}}{\left(v_{rms}\right)_{Cl_2}} = \sqrt{1.777} = 1.333$$

9.

As three vessels are identical i.e. they have same volume. Now at constant pressure, temperature and volume, the three vessels will contain equal number of molecules (by Avogadro's law) and is equal to Avogadro's number, $N_A = 6.023 \times 10^{23}$.

$$\because \quad v_{rms} = \sqrt{\frac{3k_BT}{m}} \Rightarrow v_{rms} \propto \frac{1}{\sqrt{m}}$$

where, m is mass of single gas molecule as neon has the smallest mass, so rms speed will be greatest in case of neon.

10.

Let C and C' be the rms velocity of argon and a helium gas atom at temperatures T in K and T' in K, respectively.

Here, M = 39.9, M' = 4.0, T = ?,

T' = – 20 + 273 = 253 K

$$\text{Now,} \quad v = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{39.9}}$$

$$\text{and} \quad v' = \sqrt{\frac{3RT'}{M'}} = \sqrt{\frac{3R\times 253}{4}}$$

Since, v = v’, therefore $\sqrt{\frac{3RT}{39.9}} = \sqrt{\frac{3R\times 253}{4}}$

$$\text{or} \quad T = \frac{39.9\times 253}{4} = 2523.7K$$

11.

As force of attraction between molecules disappears, then molecules will hit the wall with more speeds, hence, rate of change of momentum will increase.

As we know $F = \frac{\Delta p}{\Delta t}$,

where F is average force on the wall due to molecules.

Δp is change in momentum and Δt is the time duration. Due to increase in Δp, force F will also increase, hence pressure, p = $\frac{F}{A}$ will increase.

Here, A is area of one wall.

12.

(i) Given, T = 27°C = (273.15 + 27)= 300.15 K

Average thermal energy, E = $\frac{3}{2}k_BT$

(where, k_B = Boltzmann constant
= 1.38×10^{-23} J K^{-1})

E = $\frac{3}{2} \times 1.38 \times 10^{-23} \times 300.15 = 6.21 \times 10^{-21}$ J

(ii) At the temperature, T = 6000 K
(Surface of the sun)

Average thermal energy, E = $\frac{3}{2}k_BT$

= $\frac{3}{2} \times 1.38 \times 10^{-23} \times 6000 = 1.241 \times 10^{-19}$ J

(iii) At temperature, T = 10^7 K
Average thermal energy,

E = $\frac{3}{2}k_BT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7$
= 2.07×10^{-16} J

13.

(i) The moon has small gravitational force and hence the escape velocity is small. As the moon is in the proximity of the earth as seen from the sun, the moon has the same amount of heat per unit area as that of the earth. The air molecules have large range of speeds.

Even though the rms speed of the air molecules is smaller than escape velocity on the moon, a significant number of molecules have speed greater than escape velocity and they escape.

Now, rest of the molecules arrange the speed distribution for the equilibrium temperature. Again, a significant number of molecules escape as their speeds exceed escape speed. Hence, over a long time the moon has lost most of its atmosphere.

(ii) As the molecules move higher, their potential energy increases and hence kinetic energy decreases and hence temperature reduces.

At greater height, more volume is available and gas expands and hence some cooling takes place.

14.

Given, V_1 = 2.0 L, V_2 = 3.0 L, μ_1 = 4.0 moles, μ_2 = 5.0 moles

p_1 = 1.00 atm, p_2 = 2.00 atm

$p_1V_1 = \mu_1RT_1$, $p_2V_2 = \mu_2RT_2$

$\mu = \mu_1 + \mu_2$, $V = V_1 + V_2$

For 1 mole, $pV = \frac{2}{3}E$

For μ_1 mole, $p_1V_1 = \frac{2}{3}\mu_1E_1$

For μ_2 mole, $p_2V_2 = \frac{2}{3}\mu_2E_2$

Total energy is $\left(\mu_1E_1 + \mu_2E_2\right) = \frac{3}{2}\left(p_1V_1 + p_2V_2\right)$

$$pV = \frac{2}{3}E_{total} = \frac{2}{3}\mu E_{per\,mole}$$

$$p\left(V_1 + V_2\right) = \frac{2}{3} \times \frac{3}{2}\left(p_1V_1 + p_2V_2\right)$$

$$p = \frac{p_1V_1 + p_2V_2}{V_1 + V_2} \qquad \text{... (i)}$$

$$= \left(\frac{1.00 \times 2.0 + 2.00 \times 3.0}{2.0 + 3.0}\right)$$

$$= \frac{8.0}{5.0} = 1.60 \text{ atm}$$

15.

According to the law of atmospheres,

$$n_2 = n_1 \exp.\left[-\frac{mg}{k_BT}\left(h_2 - h_1\right)\right] \text{ ... (i)}$$

where, n_2 and n_1 refer to number density of particles at heights h_2 and h_1 respectively.

If we consider the sedimentation equilibrium of suspended particles in a liquid, then in place of mg, we will have to take effective weight of the suspended particles.

Let, V = average volume of a suspended particle, ρ = density of suspended particle, ρ' = density of liquid, m = mass of one suspended particle, m' = mass of equal volume of liquid displaced.

According to Archimedes' principle, effective weight of one suspended particle

= Actual weight – weight of liquid displaced

= mg – m'g

$$= mg - V\rho' g = mg - \left(\frac{m}{\rho}\right)\rho' g = mg\left(1-\frac{\rho'}{\rho}\right)$$

Also, Boltzmann constant, $k_B = \frac{R}{N_A}$

Where, R is gas constant and N_A is Avogadro's number.

Putting, $mg\left(1-\frac{\rho'}{\rho}\right)$ in place of mg and value of k_B in Eq. (i), we get

$$n_2 = n_1 \exp\left[-\frac{mgN_A}{RT}\left(1-\frac{\rho'}{\rho}\right)(h_2-h_1)\right]$$, which is required relation.

16.

(i) Average speed

$$= \frac{n_1v_1 + n_2v_2 + n_3v_3 + n_4v_4 + n_5v_5}{n_1+n_2+n_3+n_4+n_5}$$

$$= \frac{2\times1 + 4\times2 + 8\times3 + 6\times4 + 3\times5}{2+4+8+6+3}$$

= 3.17 m/s

(ii) Root mean square speed

$$= \sqrt{\frac{n_1v_1^2 + n_2v_2^2 + n_3v_3^2 + n_4v_4^2 + n_5v_5^2}{n_1+n_2+n_3+n_4+n_5}}$$

$$= \sqrt{\frac{2\times1^2 + 4\times2^2 + 8\times3^2 + 6\times4^2 + 3\times5^2}{2+4+8+6+3}}$$

= 3.36 m/s

(iii) The most probable speed is that speed which is possessed by maximum number of molecules. Most probable speed

$$(v_{mp}) = \sqrt{\frac{2k_BT}{m}} = \sqrt{\frac{3k_BT}{m}\times 2/3}$$

$$v_{mp} = \sqrt{\frac{2}{3}}\times\sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{2}{3}}v_{rms}$$

$$= \sqrt{\frac{2}{3}}\times 3.36 \text{ m/s}$$

= 0.816 × 3.36 m/s = 2.74 m/s

17.

This problem is designed to give an idea about cooling by evaporation

(i) $v_{rms}^2 = \frac{\sum_i n_i v_i^2}{\sum n_i}$

$$= \frac{\begin{bmatrix}10\times(200)^2 + 20\times(400)^2 \\ +40\times(600)^2 + 20\times(800)^2 + 10\times(1000)^2\end{bmatrix}}{100}$$

$$= \frac{10\times100^2\times(1\times4 + 2\times16 + 4\times36 + 2\times64 + 1\times100)}{100}$$

= 1000 × (4 + 32 + 144 + 128 + 100)

= 408 × 1000 m^2/s^2

∴ v_{rms} = 639 m/s

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$$

$$\therefore \quad T = \frac{1}{3}\frac{mv_{rms}^2}{k}$$

$$= \frac{1}{3}\times\frac{3.0\times10^{-26}\times4.08\times10^5}{1.38\times10^{-23}}$$

= 2.96 × 10^2 = 296 K

(ii) $v_{rms}^2 = \frac{\begin{bmatrix}10\times(200)^2 + 20\times(400)^2 \\ +40\times(600)^2 + 20\times(800)^2\end{bmatrix}}{90}$

$$= \frac{10\times100^2\times(1\times4 + 2\times16 + 4\times36 + 2\times64)}{90}$$

$$v_{rms}^2 = 1000\times\frac{308}{9}$$

v^2_{rms} = 342 × 1000 m^2/s^2

v_{rms} = 584 m/s

$$T = \frac{1}{3}\frac{mv_{rms}^2}{k} = 248 \text{ K}$$

18.

Three, because bee is free to move along x-direction or y-direction or z-direction.

19.

Yes, the molar specific heat of a solid is a constant quantity as its value is 3R J/mol K

20.

Brownian motion and diffusion of gases provide experimental evidence in support of random motion of gas molecules.

21.

The mean free path is inversely proportional to the number density of the gas.

22.

Degree of freedom, f = 3N – K.

23.

For monoatomic gas, temperature rise will be greater because monoatomic gas possesses only translational degree of freedom whereas diatomic gas translation, rotation and vibrational (at higher temperature), os temperature rise for diatomic gases is lower.

24.

Number of nitrogen molecules in 22400 cm3 of gas at NTP = 6.023×10^{23}

∴ Number of molecules in 15 cm^3 of gas at NTP

$$= \frac{6.023 \times 10^{23} \times 15}{22400}$$

$= 4.03 \times 10^{20}$

Number of degrees of freedom of nitrogen (diatomic) molecule at 273 K = 5

∴ Total degrees of freedom of 15 cm^3 of gas

$= 4.03 \times 10^{20} \times 5 = 2.015 \times 10^{21}$

25.

Oxygen is a diatomic gas.

Number of moles of O_2 gas

$$= \frac{\text{Atomic wt.}}{\text{Molecular wt.}} = \frac{8}{32} = n$$

$$= \frac{1}{4} = 0.25$$

∴ Energy associated with 1 mole of oxygen

$$U = \frac{5}{2}RT$$

∴ Internal energy of 8g of oxygen = $0.25 \times \frac{5}{2}RT$

$$= 0.25 \times \frac{5}{2} \times 8.31 \times 273 = 1417.9 \text{ J}$$

26.

The law of equipartition of energy for any dynamical system in thermal equilibrium, the total energy is distributed equally amongst all the degrees of freedom.

The energy associated with each molecule per degree of freedom is $\frac{1}{2}$ k_BT, where k_B is Boltzmann's constant and T is temperature of the system.

27.

Given. T = 27° C = 273 + 27 = 300 K,

P = 1 atm = 1.01×10^5 N/m^2

d = $2 \times 2 \times 10^{-10}$ m = 4×10^{-10} m

∵ Mean free path, $\lambda = K_BT / \sqrt{2}\pi d^2p$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.414 \times 3.14\left(4 \times 10^{-10}\right)^2 1.013 \times 10^5}$$

= 5.75×10^{-8} m

28.

As we know that, $V_{rms} = \sqrt{\frac{3RT}{M}}$

Thus, $V_{rms}/Ar = V_{rms}/He$

$$\sqrt{\frac{T_{Ar}}{M_{Ar}}} = \sqrt{\frac{T_{He}}{M_{He}}}$$

T_{Ar} = ? T_{He} = 273 – 10 = 263 K

M_{Ar} = 39.9 u, M_{He} = 4 u

Thus, $\frac{T_{Ar}}{39.9} = \frac{263}{4}$

$$T_{Ar} = \frac{263 \times 39.9}{4} = 2623.43K$$

29.

At STP, 1 mole of gas occupy 22.4 L of volume.

∴ Moles of helium in container, $\mu = \frac{44.8}{22.4}$ = 2 moles

Now, helium is monoatomic, so, $C_V = \frac{3}{2}R$

Change in temperature,

$\Delta T = T_2 - T_1 = 15°C$

∴ Volume of gas remain constant

∴ $\Delta W = p\Delta V = 0 \Rightarrow \Delta Q = \Delta U + \Delta W$

Amount of heat required, $\Delta Q = \Delta U = \mu C_V \Delta T$

$= 2 \times \frac{3}{2} R \times 15 = 45$ R

= 45 × 8.31 = 374 J

30.

For a monoatomic gas, i.e. $\gamma = \frac{5}{3}$

$$C_{V_\gamma} = \frac{R}{\gamma - 1} = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2}R$$

For a diatomic gas, i.e. $\gamma = \frac{7}{5}$

$$C_V = \frac{R}{\frac{7}{5} - 1} = \frac{5}{2}R$$

By conservation of energy,

$$C_{V_{mixture}} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2}}{\mu_1 + \mu_2}$$

$$= \frac{2\times\frac{3}{2}R + 3\times\frac{5}{2}R}{2+3}$$

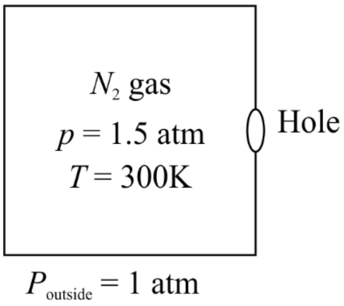
$$= \frac{3R + 7.5R}{5}$$

= 2.1 R

31.

Given, volume of the box, V = 1.00 m^3

Area, a = 0.010 mm^2 = 10^{-8} m^2



Temperature outside = Temperature inside

Initial pressure inside the box = 1.50 atm.

Final pressure inside the box = 0.10 atm.

Assuming,

v_{ix} = Speed of nitrogen molecule inside the box along x-direction.

n_1 = Number of molecules per unit volume in a time interval of Δt, all the particles at a distance (v_{ix} Δt) will collide the hole and the wall, the particle colliding along the hole will escape out reducing the pressure in the box.

Let area of the wall is A, number of particles colliding in time, Δt

$$= \frac{1}{2} n_1 (v_{ix}\Delta t) A$$

$\frac{1}{2}$ is the factor because all the particles along x-direction are behaving randomly. Hence, half of these are colliding against the walls on either side.

Inside the box, $v_{ix}^2 + v_{iy}^2 + v_{iz}^2 = v_{rms}^2$

$$v_{ix}^2 = \frac{v_{rms}^2}{3} \qquad [\because v_{ix} = v_{iy} = v_{iz}]$$

If particles collide along hole, they move out. Similarly, outer particles colliding along hole will move in. If a = area of hole

Then, net particle flow in time,

$$\Delta t = \frac{1}{2}(n_1 - n_2)\frac{k_B T}{m}\Delta ta$$

$$\left[\because v_{rms} = \sqrt{\frac{3k_B T}{m}}\right]$$

[Temperature inside and outside the box are equal]

Let n = number of densities of nitrogen

$$n = \frac{\mu N_A}{V} = \frac{pN_A}{RT} \qquad \left[\because \frac{\mu}{V} = \frac{p}{RT}\right]$$

Where, N_A = Avogadro's number

If after time Δt, pressure inside changes from p to p_t'

$$\therefore \quad n_1' = \frac{p_1' N_A}{RT}$$

Now, number of molecules gone out = $n_1 V - n_1' V$

$$= \frac{1}{2}(n_1 - n_2)\sqrt{\frac{k_B T}{m}}\Delta ta$$

$$\therefore \frac{p_1 N_A}{RT}V - \frac{p_1' N_A}{RT}V = \frac{1}{2}(p_1 - p_2)\frac{N_A}{RT}\sqrt{\frac{k_B T}{m}}\Delta ta$$

$$\Delta t = 2\left(\frac{p_1 - p_1'}{p_1 - p_2}\right)\frac{V}{a}\sqrt{\frac{m}{k_B T}}$$

Putting the values from the data given,

$$\Delta t = 2\left(\frac{1.5-1.4}{1.5-1.0}\right)\frac{1\times1.00}{0.01\times10^{-6}}\sqrt{\frac{46.7\times10^{-27}}{1.38\times10^{-23}\times300}}$$

$$= \frac{2}{5}\times3.358\times10^5$$

$$= \frac{6.717}{5}\times10^5$$

= 1.343 × 10^5 s

32.

(i) Refer to text on pages 531 and 532.

(ii) For monoatomic gas, $C_V = \frac{3}{2}R$

For diatomic gas, $C_V' = \frac{5}{2}R$

Let, μ and μ' be moles of mono and diatomic gases then, C_V (mixture) $= \frac{\mu C_V + \mu' C_V'}{\mu + \mu'}$

$$C_V = \frac{1\times\frac{3}{2}R + 1\times\frac{5}{2}R}{1+1} = 2R$$

γ(mixture) = $1 + \frac{R}{C_{V(mixture)}}$

$=1+\frac{R}{2R}$ = 1.5

Solutions: Beginner Test - II

1.

Diameter of an oxygen molecule, d = 3Å

Radius, r= $\frac{d}{2}=\frac{3}{2}$ = 1.5 Å = 1.5 × 10^{-8} cm

Actual volume occupied by 1 mole of oxygen gas at STP = 22400 cm^3

Molecular volume of oxygen gas,

$$V=\frac{4}{3}\pi r^3.N$$

Where, N is Avogadro's number = 6.023 × 10 ^{23}molecules/mole

$$\therefore V=\frac{4}{3}\times 3.14\times(1.5\times10^{-8})^3\times 6.023\times10^{23}=8.51cm^3$$

Ratio of the molecular volume to the actual volume of oxygen $\frac{8.51}{22400}$

= 3.8 × 10^{-4}

2.

The ideal gas equation relating pressure (P), volume (V), and absolute temperature (T) is given as:

PV = nRT

Where,

R is the universal gas constant = 8.314 J mol^{-1} K^{-1}

n = Number of moles = 1

T = Standard temperature = 273 K

P = Standard pressure = 1 atm = 1.013 × 10^5 Nm^{-2}

$\therefore V=\frac{nRT}{p}$

$=\frac{1\times8.314\times273}{1.013\times10^5}$

= 0.0224 m^3

= 22.4 liters.

Hence, the molar volume of a gas at STP is 22.4 liters.

3.

The dotted plot in the graph signifies the ideal behavior of the gas, i.e., the ratio $\frac{PV}{T}$ is equal. μR (μ is the number of moles and R is the universal gas constant) is a constant quality. It is not dependent on the pressure of the gas.

The dotted plot in the given graph represents an ideal gas. The curve of the gas at temperature T_1 is closer to the dotted plot than the curve of the gas at temperature T_2. A real gas approaches the behaviour of an ideal gas when its temperature increases. Therefore, T_1 > T_2 is true for the given plot.

The value of the ratio PV/T, where the two curves meet, is μR. This is because the ideal gas equation is given as:

$PV=\mu RT$

$\frac{PV}{T}=\mu R$

Where,

P is the pressure

T is the temperature V
is the volume μ is the
number of moles

R is the universal constant

Molecular mass of oxygen = 32.0 g

Mass of oxygen = 1 × 10^{-3} kg = 1 g

R = 8.314 J mol^{-1} K^{-1}

$\therefore\frac{PV}{T}=\frac{1}{32}\times8.314$

$=0.26JK^{-1}$

Therefore, the value of the ratio PV/T, where the curves meet on the y-axis, is 0.26 J K^{-1}.

If we obtain similar plots for 1.00 × 10^{-3} kg of hydrogen, then we will not get the same value of PV/T at the point where the curves meet the y-axis. This is because the molecular mass of hydrogen (2.02 u) is different from that of oxygen (32.0 u).

We have:

$\frac{PV}{T}=0.26JK^{-1}$

R = 8.314 J mol^{-1} K^{-1}

Molecular mass (M) of H_2 = 2.02 u

$\frac{PV}{T}=\mu R\ at\ constant\ temperature$

$Where,\mu=\frac{m}{M}$

$m=Mass\ of\ H_2$

$\therefore m=\frac{PV}{T}\times\frac{M}{R}$

$=\frac{0.26\times2.02}{8.31}$

= 6.3 × 10^{-2} g = 6.3 × 10^{-5}kg

Hence, 6.3 × 10^{-5}kg of H2 will yield the same value of PV/T.

4.

Volume of oxygen, V_1 = 30 litres = 30 × $10^{-3} m^3$

Gauge pressure, P_1 = 15 atm = 15 × 1.013 × 10^5Pa

Temperature, T_1 = 27°C = 300 K

Universal gas constant, R = 8.314 J $mol^{-1}\ K^{-1}$

Let the initial number of moles of oxygen gas in the cylinder be n_1.

The gas equation is given as:

$$P_1V_1 = n_1RT_1$$

$$\therefore n_1 = \frac{P_1V_1}{RT_1}$$

$$\frac{15.195 \times 10^5 \times 30 \times 10^{-3}}{(8.314) \times 300}$$

$$= 18.276$$

$But, n_1 = \frac{m_1}{M} Where, m_1 =$

Initial mass of oxygen

M = Molecular mass of oxygen = 32 g∴

m_1 = n_1M = 18.276 × 32 = 584.84 g

After some oxygen is withdrawn from the cylinder, the pressure and temperature reduce

Volume, V_2 = 30 litres = 30 × $10^{-3}\ m^3$

Gauge pressure, P_2 = 11 atm = 11 × 1.013 × 10^5 Pa

Temperature, T_2= 17°C = 290 K

Let n_2 be the number of moles of oxygen left in the cylinder.

The gas equation is given as:

$$P_2V_2 = n_2RT_2$$

$$\therefore n_2 = \frac{P_2V_2}{RT_2}$$

$$= \frac{11.143 \times 10^5 \times 30 \times 10^{-3}}{8.314 \times 290}$$

$$= 13.86$$

$$But, n_2 = \frac{m_2}{M} Where, m_2\ is\ the\ mass\ of\ oxygen$$

remaining in the cylinder m_2 = n_1M = 13.86 × 32

= 453.1 g

The mass of oxygen taken out of the cylinder is given by the relation:

Initial mass of oxygen in the cylinder – Final mass of oxygen in the cylinder

$= m_1 - m_2$

= 584.84 g – 453.1 g

= 131.74 g

= 0.131 kg

Therefore, 0.131 kg of oxygen is taken out of the cylinder.

5.

Volume of the air bubble, V_1 = 1.0 cm^3 = 1.0 × $10^{-6}\ m^3$

Bubble rises to height, d = 40 m

Temperature at a depth of 40 m, T_1 = 12°C = 285 K

Temperature at the surface of the lake, T_2 = 35°C = 308 K

The pressure on the surface of the lake:

P_2 = 1 atm = 1 ×1.013 × 10^5 Pa

The pressure at the depth of 40 m:

P_1 = 1 atm + dρg Where, ρ is the density of water = 10^3 kg/10^5 g is the acceleration due to gravity = 9.8 m/s^2

∴ P_1 = 1.013 × 10^5+ 40 × 10^3 × 9.8 = 493300 Pa

$$We\ have: \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

Where, V_2 is the volume of the air bubble when it reaches the surface

$$V_2 = \frac{P_1V_1T_2}{T_1P_2}$$

$$= \frac{(493300)(1.0 \times 10^{-6})308}{285 \times 1.013 \times 10^5}$$

$$= 5.263 \times 10^{-6} m^3 or\ 5.263\ cm^3$$

Therefore, when the air bubble reaches the surface, its volume becomes 5.263 cm^3

6.

Volume of the room, V = 25.0 m^3

temperature of the room, T = 27°C = 300 K

Pressure in the room, P = 1 atm = 1 × 1.013 × 10^5 Pa

The ideal gas equation relating pressure (P), Volume (V), and absolute temperature (T) can be written as:

PV = k_BNT

Where ,

k_B is Boltzmann constant = 1.38 × $10^{-23}\ m^2$ kg $s^{-2}\ K^{-1}$

N is the number of air molecules in the room

$$\therefore N = \frac{PV}{k_B T}$$

$$= \frac{1.013 \times 10^5 \times 25}{1.38 \times 10^{-23} \times 300}$$

$$= 6.11 \times 10^{26}\ molecules$$

Therefore, the total number of air molecules in the given room is 6.11 × $10^{26} molecules$.

7.

(i) At room temperature, T = 27°C = 300 K

Average thermal energy $=\frac{3}{2}$kT

Where k is Boltzmann constant = 1.38 × 10^{-23} m^2 kg $s^{-2}K^{-1}$

$$\therefore \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-38} \times 300$$

$$= 6.21 \times 10^{-21} J$$

Hence, the average thermal energy of a helium atom at room temperature (27°C) is 6.21 × 10^{-21} J.

(ii) On the surface of the sun, T = 6000 K

Average thermal energy$=\frac{3}{2}$kT

$$= \frac{3}{2} \times 1.38 \times 10^{-38} \times 6000$$

$$= 1.241 \times 10^{-19} J$$

Hence, the average thermal energy of a helium atom on the surface of the sun is 1.241 × 10^{-19} J.

(iii) At temperature, T = 10^7 K

Average thermal energy$=\frac{3}{2}$kT

$$= \frac{3}{2} \times 1.38 \times 10^{-23} \times 10^7$$

$$= 2.07 \times 10^{-16} J$$

Hence, the average thermal energy of a helium atom at the core of a star is 2.07 × 10^{-16} J.

8.

Yes. All contain the same number of the respective molecules.

No. The root mean square speed of neon is the largest.

Since the three vessels have the same capacity, they have the same volume

Hence, each gas has the same pressure, volume, and temperature.

According to Avogadro's law, the three vessels will contain an equal number of the respective molecules. This number is equal to Avogadro's number, N = 6.023 × 10^{23} .

The root mean square speed (v_{rms}) of a gas of mass m, and temperature T, is given by the relation:

$$v_{rms} = \sqrt{\frac{3kT}{m}}$$

Where, k is Boltzmann constant

For the given gases, k and T are constants

Hence v_{rms} depends only on the mass of the atoms, i.e.,

$$v_{rms} \propto \sqrt{\frac{1}{m}}$$

Therefore, the root mean square speed of the molecules in the three cases is not the same.

Among neon, chlorine, and uranium hexafluoride, the mass of neon is the smallest. Hence, neon has the largest root mean square speed among the given gases.

9.

Temperature of the helium atom, T_{He} = –20°C= 253 K

Atomic mass of argon, M_{Ar} = 39.9 u

Atomic mass of helium, M_{He} = 4.0 u

Let, (v_{rms})Ar be the rms speed of argon.

Let (v_{rms})He be the rms speed of helium.

The rms speed of argon is given by:

$$(v_{rms})_{Ar} = \sqrt{\frac{3RT_{Ar}}{M_{Ar}}} \ldots \text{(i)}$$

Where,

R is the universal gas constant

T_{Ar} is temperature of argon gas The rms speed of helium is given by:

$$(v_{rms})_{Hr} = \sqrt{\frac{3RT_{He}}{M_{He}}} \ldots \text{(ii)}$$

It is given that: (v_{rms})Ar = (vrms)He

$$\sqrt{\frac{3RT_{Ar}}{M_{Ar}}} = \sqrt{\frac{3RT_{He}}{M_{He}}}$$

$$\frac{T_{Ar}}{M_{Ar}} = \frac{T_{He}}{M_{He}}$$

$$T_{Ar} = \frac{T_{He}}{M_{He}} \times M_{Ar}$$

$$= \frac{253}{4} \times 39.9$$

= 2523.675 = 2.52 × 10^3 K

Therefore, the temperature of the argon atom is 2.52 × 10^3 K.

10.

Pressure inside the cylinder containing nitrogen, P = 2.0 atm = 2.026 × 10^5Pa

Temperature inside the cylinder, T = 17°C =290 K

Radius of a nitrogen molecule, r = 1.0 Å = 1 × 10^{-10} m

Diameter, d = 2 × 1 × 10 $^{-10}$ = 2 × 10^{-10}m

Molecular mass of nitrogen, M = 28.0 g = 28 × 10^{-3} kg

The root mean square speed of nitrogen is given by the relation:

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

Where,

R is the universal gas constant = 8.314 J $mole^{-1}\ K^{-1}$

$$\therefore v_{rms} = \sqrt{\frac{3 \times 8.314 \times 290}{28 \times 10^{-3}}} = 508.26\ m/s$$

The mean free path (l) is given by the relation:

$$l = \frac{kT}{\sqrt{2} \times \pi \times d^2 \times P}$$

Where, k is the Boltzmann constant = 1.38 × 10^{-23} kg $m^2\ s^{-2} k^{-1}$

$$\therefore l$$

$$= \frac{1.38 \times 10^{-23} \times 290}{\sqrt{2} \times 3.14 \times (2 \times 10^{-10})^2 \times 2.026 \times 10^5}$$

= 1.11 × 10^{-7} m

$$= \frac{v_{rms}}{l}$$

Collision frequency

$$= \frac{508.26}{1011 \times 10^{-7}} = 4.58 \times 10^9 s^{-1}$$

Collision time is given as:

$$T = \frac{d}{v_{rms}}$$

$$= \frac{2 \times 10^{-10}}{508.26} = 3.93 \times 10^{-13} s$$

Time taken between successive collisions:

$$T' = \frac{l}{v_{rms}}$$

$$= \frac{1.11 \times 10^{-7} m}{508.26 m/s} = 2.18 \times 10^{-10} s$$

$$\therefore \frac{T'}{T} = \frac{2.18 \times 10^{-10}}{3.93 \times 10^{-13}} = 500$$

Hence, the time taken between successive collisions is 500 times the time taken for a collision.

11.

Length of the narrow bore, L = 1 m = 100 cm

Length of the mercury thread, l = 76 cm

Length of the air column between mercury and the closed end, l_a = 15 cm

Since the bore is held vertically in air with the open end at the bottom, the mercury length that occupies the air space is: 100 – (76 + 15) = 9 cm

Hence, the total length of the air column = 15 + 9 = 24 cm

Let h cm of mercury flow out as a result of atmospheric pressure.

∴Length of the air column in the bore = 24 + h cm

And, length of the mercury column = 76 – h cm

Initial pressure, P_1 = 76 cm of mercury

Initial volume, V_1 = 15 cm^3

Final pressure, P_2 = 76 – (76 – h) = h cm of mercury

Final volume, V_2 = (24 + h) m^3

$P_1 V_1$ = $P_2 V_2$

76 × 15 = h (24 + h)

h^2 + 24h – 1140 = 0

$$\therefore h = \frac{-24 \pm \sqrt{(24)^2 + 4 \times 1 \times 1140}}{2 \times 1}$$

= 23.8 cm or 47.8 cm

Height cannot be negative. Hence, 23.8 cm of mercury will flow out from the bore and 52.2 cm of mercury will remain in it. The length of the air column will be 24 + 23.8 = 47.8 cm.

12.

Rate of diffusion of hydrogen, R1 = 28.7 $cm^3 s^{-1}$

Rate of diffusion of another gas, R2 = 7.2 $cm^3 s^{-1}$

According to Graham's Law of diffusion, we have:

$$\frac{R_1}{R_2} = \sqrt{\frac{M_2}{M_1}}$$

Where,

M_1 is the molecular mass of hydrogen = 2.020 g

M_2 is the molecular mass of the unknown gas

$$\therefore M_2 = M_1\left(\frac{R_1}{R_2}\right)^2$$

$$= 2.02\left(\frac{28.7}{7.2}\right)^2 = 32.09\ g$$

32 g is the molecular mass of oxygen. Hence, the unknown gas is oxygen.

13.

According to the law of atmospheres, we have:

$n_2 = n_1$ exp [-mg (h_2-h_1)/ k_BT] … (i)

Where, n_1 is the number density at height h_1, and n_2 is the number density at height h_2 mg is the weight of the particle suspended in the gas column

Density of the medium = ρ'

Density of the suspended particle = ρ

Mass of one suspended particle = m'

Mass of the medium displaced = m

Volume of a suspended particle = V

According to Archimedes' principle for a particle suspended in a liquid column, the effective weight of the suspended particle is given as:

Weight of the medium displaced – Weight of the suspended particle

$$= mg - m'g$$

$$= mg - V\rho' g = mg - \left(\frac{m}{\rho}\right)\rho' g$$

$$= mg\left(1 - \frac{\rho'}{\rho}\right) \text{ ... (ii)}$$

$$Gas\ constant, R = k_B N$$

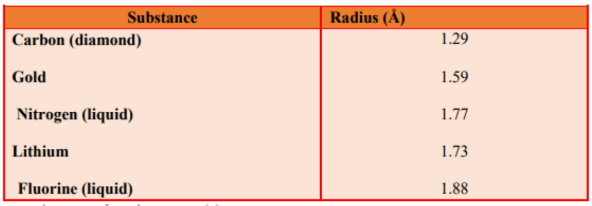
$$k_B = \frac{R}{N} \text{ ... (iii)}$$

Substituting equation (ii) in place of mg in equation (i) and then using equation (iii), we get: n_2 = $n_1$exp [-mg (h_2 –h_1)/ k_B T]

$$= n_1 \exp\left[-mg\left(1 - \frac{\rho'}{\rho}\right)(h_2 - h_1)\frac{N}{RT}\right]$$

$$= n_1 \exp\left[-mg(\rho - \rho')(h_2 - h_1)\frac{N}{RT\rho}\right]$$

14.



Substance	Radius (Å)
Carbon (diamond)	1.29
Gold	1.59
Nitrogen (liquid)	1.77
Lithium	1.73
Fluorine (liquid)	1.88

Atomic mass of a substance = M

Density of the substance = ρ

Avogadro's number = N = 6.023 × 10^{23}

Volume of each atom $=\frac{4}{3}\pi r^3$

Volume of N number of molecules$=\frac{4}{3}\pi r^3$…….(i)

Volume of one mole of a substance $=\frac{M}{P}$…….(ii)

$$\frac{4}{3}\pi r^3 N = \frac{M}{\rho}$$

$$\therefore r = \sqrt[3]{\frac{3M}{4\pi\rho N}}$$

For carbon: M = 12.01 × 10^{-3}kg, ρ = 2.22 × 10^{3} kg m^{-3}

$$\therefore r = \left(\frac{3 \times 12.01 \times 10^{-3}}{4\pi \times 2.22 \times 10^3 \times 6.023 \times 10^{23}}\right)^{\frac{1}{3}}$$

$$= 1.29\ \text{Å}$$

Hence, the radius of a carbon atom is 1.29 Å.

For gold: M = 197.00× 10^{-3} kg

ρ = 19.32 ×10^{3} kg m^{-3}

$$\therefore r = \left(\frac{3 \times 197 \times 10^{-3}}{4\pi \times 19.32 \times 10^3 \times 6.023 \times 10^{23}}\right)^{\frac{1}{3}}$$

$$= 1.59\ \text{Å}$$

Hence, the radius of a gold atom is 1.59 Å.

For liquid nitrogen:

M = 14.01 × 10^{-3} kg

ρ = 1.00 × 10^{3} kg m^{-3}

$$\therefore r = \left(\frac{3 \times 14.01 \times 10^{-3}}{4\pi \times 1.00 \times 10^3 \times 6.23 \times 10^{23}}\right)^{\frac{1}{3}}$$

$$= 1.77\ \text{Å}$$

Hence, the radius of a liquid nitrogen atom is 1.77 Å.

For lithium:

M = 6.94 × 10^{-3} kg ρ = 0.53 × 10^{3} kg m^{-3}

$$\therefore r = \left(\frac{3 \times 6.94 \times 10^{-3}}{4\pi \times 0.53 \times 10^3 \times 6.23 \times 10^{23}}\right)^{\frac{1}{3}}$$

$$= 1.73\ \text{Å}$$

Hence, the radius of a lithium atom is 1.73 Å.

For liquid fluorine:

M = 19.00 × 10^{-3} kg

ρ = 1.14 × 10^{3} kg m^{-3}

$$\therefore r = \left(\frac{3 \times 19 \times 10^{-3}}{4\pi \times 1.14 \times 10^{3} \times 6.023 \times 10^{23}}\right)^{\frac{1}{3}}$$

$$= 1.88\ \text{Å}$$

Hence, the radius of a liquid fluorine atom is 1.88 Å

Solutions: Expert Test - I

1.

(a) The mean free path l for a gas molecule is given as

$$l = \frac{1}{\sqrt{2}\pi n d^2} \Rightarrow l \propto \frac{1}{d^2}$$

where, d = diameter of molecule of gas.

2.

(a) Ideal gas equation is given as

$$p = \frac{\rho RT}{M_0} \Rightarrow p \cdot \frac{M_0}{\rho} = RT$$

$pV = RT$

where, $V = \frac{M_0}{\rho}$

Hence, ρ and M_0 are mass density and mass of gas, respectively.

3.

(a) The mean free path λ for a gas, with molecular diameter and number density n is given by the relation

$$\lambda = \frac{1}{\sqrt{2}n\pi d^2}$$

Hence, correct option is (a).

4.

(a) Given,

pressure $p = 249\text{ kPa} = 249 \times 10^3\text{ Pa}$

Temperature, T = 27°C

= 273 + 27 K = 300K

Density, ρ = ?

As, from ideal gas equation,

$pV = nRT$

$$pV = \frac{m}{M}RT \qquad \left[as\ n = \frac{m}{M}\right]$$

$pVM = mRT$

$$pM = \frac{m}{V}RT = \rho RT \qquad \left[as\ \frac{m}{V} = \rho\right]$$

$$\rho = \frac{pM}{RT}$$

$$= \frac{249 \times 10^3 \times 2 \times 10^{-3}}{8.3 \times 300}$$

[∵ for hydrogen gas, $M = 2g = 2 \times 10^{-2}$ kg]

$\rho = 0.2\text{kg/m}^3$

Hence, correct option is (a).

5.

(a) As the temperature of gas in the container is increased, the kinetic energy also increases. This is because the average kinetic energy of a gas is given by

$$KE = \frac{f}{2}nRT \qquad \ldots \text{(i)}$$

where, f = degree of freedom,

n = number of moles of gas molecules,

R = universal gas constant, and

T = absolute temperature of the gas.

From Eq. (i),

$KE \propto T$

option (b) is incorrect as increase in temperature will lead to increase in pressure as p ∝ T. Other options (c) and (d) are also incorrect as molecular distance increases while mass remains the same for increase in the temperature.

6.

(b) **Key Concept** The minimum velocity with which the body must be projected vertically upwards, so that it could escape from the Earth's atmosphere, is its escape velocity (v_e).

As, $v_e = \sqrt{2gR}$

Substituting the value of g (9.8 ms^{-2}) and radius of Earth ($R = 6.4 \times 10^6$ m), we get

$$v_e = \sqrt{2 \times 9.8 \times 6.4 \times 10^6}$$

$\cong 11.2\text{ km s}^{-1} = 11200\text{ m s}^{-1}$

Let the temperature of molecule be T when it attains v_e.

According to the question,

$V_{rms} = v_e$

where, V_{rms} is the rms speed of the oxygen molecule.

$$\sqrt{\frac{3k_BT}{m_{0_2}}} = 11.2 \times 10^3$$

or $$T = \frac{\left(11.2 \times 10^3\right)^2 \left(m_{0_2}\right)}{\left(3k_B\right)}$$

Substituting the given values, i.e.,

$R_B = 1.38 \times 10^{-23}$ JK^{-1} and

m_{0_2} = m = 2.76×10^{-26} kg

We get,

$$T = \frac{\left(11.2\times10^{3}\right)^{2}\left(2.76\times10^{-26}\right)}{\left(3\times1.38\times10^{-23}\right)}$$

= 8.3626×10^4 K

7.

(a) It is given that

$V_{rms} = 200$ ms^{-1}, $T_1 = 300$ K, $P_1 = 10^5$ N/m^2

$T_2 = 400$ K, $P_2 = 0.05 \times 10^5$ N/m^2

As, rms velocity of gas molecules,

$$\because \quad v_{rms} \propto \sqrt{T} \qquad \left(\because v_{rms} = \sqrt{\frac{3RT}{m}}\right)$$

For two different cases

$$\frac{\left(v_{rms}\right)_1}{\left(v_{rms}\right)_2} = \sqrt{\frac{T_1}{T_2}}$$

$$\frac{200}{\left(v_{rms}\right)_2} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$$\left(v_{rms}\right)_2 = \frac{2}{\sqrt{3}}\times200 = \frac{400}{\sqrt{3}}ms^{-1}$$

8.

(b) As we know that

Pressure, $p = \frac{1}{3}\cdot\frac{m}{V}v_{rms}^{2}$

∵ m = mass of the gas, V = volume of the gas

∴ $\frac{mn}{V}$ = density of the gas. Thus,

$$p = \frac{1}{2}pv_{rms}^{2} = \frac{1}{3}\rho\frac{3RT}{M_0} = \frac{\rho RT}{M_0}$$

$$\left(\because v_{rms} = \sqrt{\frac{3RT}{M_0}}\right)$$

$$\rho = \frac{pM_0}{RT} = \frac{pmN_A}{kN_AT}$$

[∵ $R = N_Ak$ and $M_0 = mN_A$]

$$\rho = \frac{pm}{kT}$$

9.

(b) As we know that

$C_p - C_v = R$

$C_p = R + C_v$

and $\quad \frac{C_p}{C_v} = \gamma \qquad$ (given)

So, $\quad \frac{R+C_v}{C_v} = \gamma \Rightarrow \gamma C_v = R + C_v$

$$\gamma C_v - C_v = R$$

$$C_v = \frac{R}{\gamma-1}$$

10.

(c) According to question,

Slope of the graph ∝ $\frac{1}{\text{Pr}essure\ p}$

So, $p_2 < p_1$

11.

(d) According to question,

$p \propto T^3 \qquad$ … (i)

$$\begin{pmatrix} p = pressure \\ T = temperature \end{pmatrix}$$

and we know that

$pV = nRT$ and $pV \propto T \qquad$ … (ii)

So, putting Eq. (ii) in (i),

$p \propto (pV)^3$

p^2V^3 = constant

$pV^{3/2}$ = constant $\qquad$ ... (iii)

Comparing Eq. (iii) with

pv^{y}=constant

We have $\gamma = 3/2$.

12.

(c) We know that, the rms speed of the gas molecules,

$$v_{rms} = \sqrt{3RT/M}$$

Here, T is the temperature of the gas,

R is the universal gas constant,

M is the molar mass of the gas.

Pressure exerted by an ideal gas,

$$p = \frac{1}{3}nm\bar{v}^{2}$$

Here, n is the number of moles,

m is the mass of the gas,

$\bar{v}$ is the average speed of the gas molecules.

The average kinetic energy of a molecule,

$$KE_{av} = \frac{3}{2}k_BT$$

Here, k_B is the Boltzmann constant,

T is the temperature of the gas.

For diatomic gas, degree of freedom,

f = 5

As, total internal energy of 1 mole of diatomic gas,

$$\Delta U = \frac{nfRT}{2}$$

$$\Delta U = \frac{1(5)RT}{2} \Rightarrow \Delta U = \frac{5RT}{2}$$

The correct match is A→2, B→1, C→4 and D→3.

13.

(a) The average thermal energy of a system with degree of freedom f is equals to its average energy, which is given as

$$= \frac{f}{2} k_B \cdot T$$

For monoatomic gas, f = 3

∴ Average thermal energy = $\frac{3}{2} k_B \cdot T$

Hence, correct option is (a).

14.

(a) The Poisson’s ratio,

$$\gamma = \frac{C_p}{C_v} \qquad \ldots \text{(i)}$$

where, C_p =molar heat capacity constant pressure and C_v = molar heat capacity at constant volume

Also, $C_p = C_v + R$ (from Mayer's relation)

$C_v = \frac{f}{2}R$ (where, f = degree of freedom)

$$C_p = \left(\frac{f}{2} + 1\right)R$$

So, Eq. (i) becomes,

$$\gamma = 1 + \frac{2}{f}$$

For hydrogen gas, which is diatomic, the degree of freedom is 5 (3 translational, 2 rotational).

$$\therefore \quad \gamma = 1 + \frac{2}{5} = \frac{7}{5}$$

For helium gas, which is monoatomic, the degree of freedom is 3(3 translational only).

$$\therefore \quad \gamma = 1 + \frac{2}{3} = \frac{5}{3}$$

The diatomic gas X also have vibrational motion, so degree of freedom is 7(3 translational, 2 rotational and 2 vibrational).

$$\therefore \quad \gamma = 1 + \frac{2}{7} = \frac{9}{7}$$

15.

(d) Total internal energy of system = Internal energy of oxygen molecules + Internal energy of argon molecules

$$= \frac{f_1}{2} n_1 RT + \frac{f_2}{2} n_2 RT = \frac{5}{2} \times 2RT + \frac{3}{2} \times 4RT$$

= 11 RT

16.

(a) As we know that for polytropic process of index α specific heat capacity

$$= C_v + \frac{R}{1-\alpha}$$

∵ Process, pV^3 = constant ⇒ α = 3

$$\therefore \quad C = C_v + \frac{R}{1-\alpha} = \frac{fR}{2} + \frac{R}{1-3}$$

where, $C_v = \frac{fR}{2} = \frac{3R}{2}$

For monatomic gas, $f = 3 = \frac{3R}{2}$

17.

(a) We know that, $Q = \frac{F}{2} nR\Delta T$

∴ Amount of heat required,

$$Q = \frac{3}{2} \times \frac{1}{4} \times K_B N_a \Delta T$$

$$= \frac{3}{8} N_a K_B (T_2 - T_1)$$

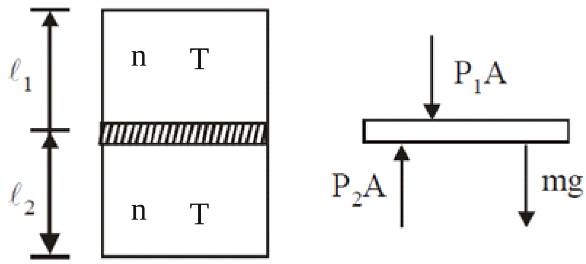
Solutions: Expert Test - II

1.

(d) Clearly from figure,

$P_2A = P_1A + mg$

or, $\frac{nRT \cdot A}{Al_2} = \frac{nRT \cdot A}{Al_1} + mg$



$$nRT\left(\frac{1}{l_2} - \frac{1}{l_1}\right) = mg$$

$$\therefore \; m = \frac{nRT}{g}\left(\frac{l_1 - l_2}{l_1 \cdot l_2}\right)$$

2.

(c) From P-V graph,

$P \propto \frac{1}{V}$,

T = constant and Pressure is increasing from 2 to 1 so option (c) represents correct T-P graph.

3.

Using ideal gas equation, PV = nRT

$P_1V_1 = nR \times 250$ [∵ $T_1 = 250$ K] ... (i)

$P_2\left(2V_1 = \frac{5n}{4}R \times 2000\right)$ [∵ $T_2 = 2000$ K] ...

(ii)

Dividing eq. (i) by (ii),

$$\frac{P_1}{2P_2} = \frac{4 \times 250}{5 \times 2000}$$

$$\frac{P_1}{P_2} = \frac{1}{5}$$

$$\therefore \frac{P_2}{P_1} = 5$$

4.

In first case,

From ideal gas equation

PV = nRT

$P\Delta V + V\Delta P = 0$

(As temperature is constant)

$$\Delta V = -\frac{\Delta P}{P}V \quad \text{... (i)}$$

In second case, using ideal gas equation again

$P\Delta V = -nR\Delta T$

$$\Delta V = -\frac{nR\Delta T}{P} \quad \text{... (ii)}$$

Equation (i) and (ii), we get

$$-\frac{nR\Delta T}{P} = -\frac{\Delta P}{P}V$$

$$\Delta T = \Delta P\frac{V}{nR}$$

Comparing the above equation with $|\Delta T| = C|\Delta P|$, we have

$$C = \frac{V}{nR} = \frac{\Delta T}{\Delta P} = \frac{300K}{2atm}$$

= 150 K/atm

5.

(a) Given : $K.E._{mean} = \frac{3}{2}kT = 4 \times 10^{-14}$

P = 2 cm of Hg, V = 4 cm^3

$$N = \frac{PV}{KT} = \frac{P\rho gV}{KT}\frac{2 \times 13.6 \times 980 \times 4}{\frac{8}{3} \times 10^{-14}}$$

$\simeq 4 \times 10^{18}$

6.

(c) $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} = \frac{(273+127)}{(273+237)}$$

$$= \sqrt{\frac{400}{500}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$$

$$\therefore v_2 = \frac{\sqrt{5}}{2}v_1 = \frac{\sqrt{5}}{2} \times 200$$

$= 100\sqrt{5}$ m/s.

7.

(Bonus) Rate of change of momentum during collision

$$= \frac{mv-(-mv)}{\Delta t} = \frac{2mv}{\Delta t}N$$

So, pressure $P = \frac{N \times (2mv)}{\Delta t \times A}$

$$= \frac{10^{22} \times 2 \times 10^{-26} \times 10^{4}}{1 \times 1}$$

= 2 N/m^2

8.

(c) $v_{rms} = v_e$

$$\sqrt{\frac{3RT}{M}} = 11.2 \times 10^3$$

or $\sqrt{\frac{3kT}{m}} = 11.2 \times 10^3$

or $\sqrt{\frac{3 \times 1.38 \times 10^{-23}T}{2 \times 10^{-3}}} = 11.2 \times 10^3$

∴ T = 10^4 K

9.

(a) Using $\frac{V_{1rms}}{V_{2rms}} = \sqrt{\frac{M_2}{M_1}}$

$$\frac{V_{rms}(He)}{V_{rms}(Ar)} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}}$$

= 3.16

10.

(a) Energy associated with N moles of diatomic gas,

$U_i = N\frac{5}{2}RT$

Energy associated with n moles of monoatomic gas

$= n\frac{3}{2}RT$

Total energy when n moles of diatomic gas converted into monoatomic

$(U_f) = 2n\frac{3}{2}RT + (N-n)\frac{5}{2}RT$

$= \frac{1}{2}nRT + \frac{5}{2}NRT$

Now, change in total kinetic energy of the gas

$\Delta U = Q = \frac{1}{2}nRT$

11.

(c) Pressure, $P = \frac{1}{3}\frac{mN}{V}V_{rms}^2$

or, $P = \frac{(mN)T}{V}$

If the gas mass and temperature are constant then

$P \propto (V_{rms})^2 \propto T$

So, force $\propto (V_{rms})^2 \propto T$

i.e., Value of q = 1

12.

(a) $\because C = \sqrt{\frac{3RT}{M}}$

$(1930)^2 = \frac{3\times 8.314\times 300}{M}$

$M = \frac{3\times 8.314\times 300}{1930\times 1930}$

$\approx 2 \times 10^{-3}$ kg

The gas is H_2.

13.

(c) $v_{rms} = \sqrt{\frac{3\rho v}{mass\,of\ the\,gas}}$

14.

Room mean square speed is given by

$v_{rms} = \sqrt{\frac{3RT}{M}}$

Here, M = Molar mass of gas molecule

T = temperature of the gas molecule

We have given $v_{N_2} = v_{H_2}$

$\therefore \sqrt{\frac{3RT_{N_2}}{M_{N_2}}} = \sqrt{\frac{3RT_{H_2}}{M_{H_2}}}$

$\frac{T_{H_2}}{2} = \frac{573}{28}$

$T_{H_2} = 41$ K

15.

(c) Total degree of freedom f = 3 + 2 = 5

Total energy, $U = \frac{nRT}{2} = \frac{5RT}{2}$

And $\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f} = 1 + \frac{2}{5} = \frac{7}{5}$

16.

(b) Mean free path, $\lambda = \frac{1}{\sqrt{2}\pi n d^2}$

Where, d = diameter of the molecule

n = number of molecules per unit volume

But, mean time fo collision, $\tau = \frac{\lambda}{v_{rms}}$

But $v_{rms} = \sqrt{\frac{3kT}{R}}$

$\therefore \tau = \frac{\lambda}{\sqrt{\frac{3kT}{m}}} \Rightarrow \tau \propto \frac{1}{\sqrt{T}}$

17.

(c) As we know,

$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$,

Where f = degree of freedom

(A) Monatomic, f = 3

$\therefore \gamma = 1 + \frac{2}{3} = \frac{5}{3}$

(B) Diatomic rigid molecules, f = 5

$\therefore \gamma = 1 + \frac{2}{5} = \frac{7}{5}$

(C) Diatomic non-rigid molecules, f = 7

$\therefore \gamma = 1 + \frac{2}{7} = \frac{9}{7}$

(D) Triatomic rigid molecules, f = 6

$\therefore \gamma = 1 + \frac{2}{6} = \frac{4}{3}$

18.

(d) Here degree of freedom, f = 3 + 3 = 6 for triatomic non-linear molecule.

Internal energy of a mole of the gas at temperature T,

$$U=\frac{f}{2}nRT=\frac{6}{2}RT=3RT$$

19.

(b) Let C_p and C_v be the specific heat capacity of the gas at constant pressure and volume.

At constant pressure, heat required

$\Delta Q_1 = nC_p\Delta T$

$160 = nC_p\cdot 50$ ... (i)

At constant volume, heat required

$\Delta Q_2 = nC_v\Delta T$

$240 = nC_v\cdot 100$ ... (ii)

Dividing (i) by (ii), we get

$$\frac{160}{240}=\frac{C_p}{C_v}\cdot\frac{50}{100}$$

$$\frac{C_p}{C_v}=\frac{4}{3}$$

$$\gamma=\frac{C_p}{C_v}=\frac{4}{3}=1+\frac{2}{f}$$

(Here, f = degree of freedom)

f = 6.

20.

(a) Total energy of the gas mixture,

$$E_{mix}=\frac{f_1n_1RT_1}{2}+\frac{f_2n_2RT_2}{2}$$

$$=3\times\frac{5}{2}RT+\frac{5}{2}\times 3RT=15RT$$

21.

As we know mean free path

$$\lambda=\frac{1}{\sqrt{2}\left(\frac{N}{V}\right)\pi d^2}$$

Here,

N = no. of molecule

V = volume of container

d = diameter of molecule

But PV = nRT = nNKT

$$\frac{N}{V}=\frac{P}{KT}=n$$

$$\lambda=\frac{1}{\sqrt{2}}\frac{KT}{\pi d^2P}$$

For constant volume and hence constant number density n of gas molecules $\frac{P}{T}$ is constant.

So, mean free path remains same.

As temperature increases no. of collision increases so relaxation time decreases.

22.

(d) Specific heat of gas at constant volume

$C_v=\frac{1}{2}fR$; f = degree of freedom

For gas A (diatomic)

f = 5 (3 translational + 2 rotational)

$$\therefore\ C_v^A=\frac{5}{2}R$$

For gas B (diatomic) in addition to (3 translational + 2 rotational) 2 vibrational degrees of freedom.

$$\therefore\ C_v^B=\frac{7}{2}R$$

Hence $\frac{C_v^A}{C_v^B}=\frac{\frac{5}{2}R}{\frac{7}{2}R}=\frac{5}{7}$

23.

Mean free path of a gas molecule is given by

$$\lambda=\frac{1}{\sqrt{2}\pi d^2n}$$

Here, n = number of collisions per unit volume

d = diameter of the molecule

If average speed of molecule is v then

Mean free time, $\tau=\frac{\lambda}{v}$

$$\tau=\frac{1}{\sqrt{2}\pi nd^2v}=\frac{1}{\sqrt{2}\pi nd^2}\sqrt{\frac{M}{3RT}}$$

$$\left(\because v=\sqrt{\frac{3RT}{M}}\right)$$

$$\therefore\ \tau\propto\frac{\sqrt{M}}{d^2}$$

$$\therefore\ \frac{\tau_1}{\tau_2}=\frac{\sqrt{M}}{d_1^2}\times\frac{d_2^2}{\sqrt{M_2}}$$

$$=\sqrt{\frac{40}{140}}\times\left(\frac{0.1}{0.07}\right)^2=1.09$$

24.

(c) Relaxation time $(\tau)\propto\frac{mean\ freepath}{speed}$

$$\tau\propto\frac{1}{v}$$

and, $v \propto \sqrt{T}$

$$\therefore\ \tau \propto \frac{1}{\sqrt{T}}$$

Hence graph between τ v/s $\frac{1}{\sqrt{T}}$ is a straight line which is correctly depicted by graph shown in option (c).

25.

(a) Helium is a monoatomic gas and Oxygen is a diatomic gas.

For helium, $C_{V_1} = \frac{3}{2}R$ and $C_{P_1} = \frac{5}{2}R$

For oxygen, $C_{V_2} = \frac{5}{2}R$ and $C_{P_2} = \frac{7}{2}R$

$$\gamma = \frac{N_1 C_{P_1} + N_2 C_{p_2}}{N_1 C_{V_1} + N_2 C_{V_2}}$$

$$\gamma = \frac{n.\frac{5}{2}R + 2n.\frac{7}{2}R}{n.\frac{3}{2}R + 2n.\frac{5}{2}R} = \frac{19nR \times 2}{2(13nR)}$$

$$\therefore \left(\frac{C_P}{C_V}\right)_{mixture} = \frac{19}{13}$$

26.

(d) Using, $\gamma_{mixture} = \frac{n_1 C_{p_1} + n_2 C_{p_2}}{n_1 C_{v_1} + n_2 C_{v_2}}$

$$\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} = \frac{n_1 + n_2}{\gamma_m - 1}$$

$$\frac{3}{\frac{4}{3} - 1} + \frac{2}{\frac{5}{3} - 1} = \frac{5}{\gamma_m - 1}$$

$$\frac{9}{1} + \frac{2 \times 3}{2} = \frac{5}{\gamma_m - 1}$$

$$\gamma_m - 1 = \frac{5}{12}$$

$$\gamma_m = \frac{17}{12} = 1.42$$

27.

(c) $[C_v]_{\min} = \frac{n_1\left[C_{v_1}\right] + n_2\left[C_{v_2}\right]}{n_1 + n_2}$

$$= \left[\frac{2 \times \frac{3R}{2} + 3 \times \frac{5R}{2}}{2 + 3}\right]$$

= 2.1 R = 2.1 × 8.3

= 17.4 J/mol-k

28.

(b) $F = \frac{C_v}{C_p} = \frac{1}{r} = \frac{1}{(7/5)} = \frac{5}{7}$

or $\frac{W}{Q} = 1 - \frac{5}{7} = \frac{2}{7}$

or $Q = \frac{7}{2}W = \frac{7 \times 10}{2}$

= 35 J

29.

(c) V = 25 × 10^{-3} m^3, N = 1 mole of O_2

T = 300 K

V_{rms} = 200 m/s

$$\therefore\ \lambda = \frac{1}{\sqrt{2}N\pi r^2}$$

Average time $\frac{1}{\tau} = \frac{<V>}{\gamma} = 200.N\pi r^2.\sqrt{2}$

$$= \frac{\sqrt{2} \times 200 \times 6.023 \times 10^{23}}{25 \times 10^{-3}}.\pi \times 10^{-18} \times 0.09$$

The closest value in the given option is = 10^{10}

30.

(c) Amount of heat required (Q) to raise the temperature at constant volume

Q = n$C_v\Delta$T ... (i)

Amount of heat required (Q_1) at constant pressure

Q_1 = n$C_p\Delta$T ... (ii)

Dividing equation (ii) by (i), we get

$$\therefore\ \frac{Q_1}{Q} = \frac{C_p}{C_v}$$

$$Q_1 = (Q)\left(\frac{7}{5}\right) \qquad \left(\because \gamma = \frac{C_p}{C_v} = \frac{7}{5}\right)$$

31.

(b) $\gamma_A = \frac{C_P}{C_v} = \frac{29}{22}$ = 1.32 < 1.4 (diatomic)

and $\gamma_B = \frac{30}{21} = \frac{10}{7}$ = 1.43 > 1.4

Gas A has more than 5-degrees of freedom.

32.

(a) Energy of the gas, E

$$= \frac{f}{2}nRT = \frac{f}{2}PV = \frac{f}{2}\left(3 \times 10^6\right)(2)$$

= f × 3 × 10^6

Considering gas is monoatomic i.e., f = 3

Energy, E = 9 × 10^6 J

33.

(b) Using, $\tau = \dfrac{1}{2n\pi d^2 V_{avg}}$

$\therefore\ t \propto \dfrac{\sqrt{T}}{P}$

$\left[\therefore n = \dfrac{no.of\ molecules}{Volume}\right]$

or, $\dfrac{t_1}{6\times10^{-8}} = \dfrac{\sqrt{500}}{2P} \times \dfrac{P}{\sqrt{300}}$

≈ 4 × 10^{-8} s

34.

(a) $U = \dfrac{f_1}{2} n_1 RT + \dfrac{f_2}{2} n_2 RT$

Considering translational and rotational modes, degrees of freedom $f_1 = 5$ and $f_2 = 3$

$\therefore\ u = \dfrac{5}{2}(3RT) + \dfrac{3}{2} \times 5RT$

U = 15RT

35.

(c) Thermal energy of N molecule

$= N\left(\dfrac{3}{2}kT\right) = \dfrac{N}{N_A}\dfrac{3}{2}RT = \dfrac{3}{2}(nRT) = \dfrac{3}{2}PV$

$= \dfrac{3}{2}P\left(\dfrac{m}{\rho}\right) = \dfrac{3}{2}P\left(\dfrac{2}{8}\right) = \dfrac{3}{2} \times 4 \times 10^4 \times \dfrac{2}{8}$

= 1.5 × 10^4 J

Therefore, order = 10^4 J

36.

(c) Heat transferred,

Q = $nC_v\Delta T$ as gas in closed vessel

To double the rms speed, temperature should be 4 times i.e., T’

= 4T as $v_{rms} = \sqrt{3RT/M}$

$\therefore\ Q = \dfrac{15}{28} \times \dfrac{5 \times R}{2} \times (4T - T)$

$\left[\therefore \dfrac{CP}{CV} = \gamma_{diatomic} = \dfrac{7}{5}\ \&\ C_p - C_v = R\right]$

or, Q = 10000 J = 10 kJ

37.

(c) In an adiabatic process

$TV^{\gamma-1}$ = Constant

or, $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

For monoatomic gas $\gamma = \dfrac{5}{3}$

$(300)V^{2/3} = T_2(2V)^{2/3}$

$T_2 = \dfrac{300}{(2)^{2/3}}$

T_2 = 189 K (final temperature)

Change in internal energy $\Delta U = n\dfrac{f}{2}R\Delta T$

$= 2\left(\dfrac{3}{2}\right)\left(\dfrac{25}{3}\right)(-111)$

= – 2.7 kJ

38.

(b) Using formula,

$$\gamma_{mixture} = \left(\frac{C_p}{C_v}\right)_{mix} = \frac{\dfrac{n_1\gamma_1}{\gamma_1 - 1} + \dfrac{n_2\gamma_2}{\gamma_2 - 1}}{\dfrac{n_1}{\gamma_1 - 1} + \dfrac{n_2}{\gamma_2 - 1}}$$

Putting the value of

$n_1 = 2$, $n_2 = n$, $\left(\dfrac{C_p}{C_v}\right)_{mix} = \dfrac{3}{2}$

$\gamma_1 = \dfrac{5}{3}$, $\gamma_2 = \dfrac{7}{5}$ and solving we get, n = 2

39.

(a) As we know $C_p - C_v = R$

Where C_p and C_v are molar specific heat capacities

or, $C_p - C_v = \dfrac{R}{M}$

For hydrogen $(M = 2)\ C_p - C_v = a = \dfrac{R}{2}$

For nitrogen $(M = 28)\ C_p - C_v = b = \dfrac{R}{28}$

$\therefore\ \dfrac{a}{b} = 14$

or, a = 14b

40.

(d) The ratio of specific heats at constant pressure (C_p) and constant volume (C_v)

$\dfrac{C_p}{C_v} = \gamma = \left(1 + \dfrac{2}{f}\right)$

Where f is degree of freedom

$\dfrac{C_p}{C_v} = \left(1 + \dfrac{2}{5}\right) = \dfrac{7}{5}$

41.

(d) For a polytropic process

$C = C_v + \dfrac{R}{1-n}$

$$\therefore\ C-C_v=\frac{R}{1-n}$$

$$\therefore\ 1-n=\frac{R}{C-C_v}$$

$$\therefore\ 1-\frac{R}{C-C_v}=n$$

$$\therefore\ n=\frac{C-C_v-R}{C-C_v}=\frac{C-C_v-C_p+C_v}{C-C_v}$$

$$\text{n}=\frac{C-C_p}{C-C_v}\quad\left(\because C_p-C_{v=R}\right)$$

42.

(d) Using equipartition of energy, we have

$$\frac{6}{2}KT=mCT$$

$$C=\frac{3\times1.38\times10^{-23}\times6.02\times10^{23}}{27\times10^{-3}}$$

$\therefore$ C = 925 J/kgK

43.

(b)

44.

(b) On giving same amount of heat at constant pressure, there is no change in temperature of monoatomic, diatomic and polyatomic.

$$\left(\Delta Q\right)_P=\mu C_p\Delta T\left(\mu=\frac{No.of\ molecules}{Avogedro's\,no.}\right)$$

or $\quad\Delta T\propto\dfrac{1}{no.of\ molecules}$

45.

Here work done on gas and heat supplied to the gas are zero.

Let T be the final equilibrium temperature of the gas in the vessel.

Total internal energy of gases remains same.

i.e., $u_1+u_2=u_1'+u_2'$

or, $n_1C_v\Delta T_1+n_2C_v\Delta T_2=(n_1+n_2)C_vT$

(0.1) Cv(200) + (0.05)C_v(400) = (0.15)C_vT

$$\therefore\ T=\frac{800}{3}$$

= 266.67 k.

Solutions: Pro Test - I

1.

(b) Given: Temperature $T_1 = 17 + 273 = 290\ K$

Temperature $T_f = 27 + 273 = 300K$

Atmospheric pressure, $P_0 = 1 \times 10^5 Pa$

Volume of room, $V_0 = 30m^3$

Difference in number of molecules, $n_f - n_i = ?$

Using ideal gas equation, $PV = nRT(N_0)$,

$N_0 = Avogadro's\ number$

$\Rightarrow n = \frac{PV}{RT}(N_0)$

$$\therefore n_f - n_i = \frac{P_0V_0}{R}\left(\frac{1}{T_f} - \frac{1}{T_i}\right)N_0$$

$$= \frac{1 \times 10^5 \times 30}{8.314} \times 6.023 \times 10^{23}\left(\frac{1}{300} - \frac{1}{290}\right)$$

$= -2.5 \times 10^{25}$

2.

(d) $P_1M_1 = P_1RT\ and\ P_2M_2 = P_2RT$

$$\therefore \frac{P_1}{P_2} \times \frac{M_1}{M_2} = \frac{P_1}{P_2}$$

$$\frac{4}{3} \times \frac{2}{3} = \frac{P_1}{P_2} \qquad \therefore \frac{P_1}{P_2} = \frac{8}{9}$$

3.

(d) A real gas behaves as an ideal gas at low pressure and high temperature.

4.

(d) Using $V_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow V_{rms} \propto \frac{1}{\sqrt{M}}$

$$\frac{V_{rms}(helium)}{V_{rms}(argon)} = \sqrt{\frac{M_{argon}}{M_{helium}}} = \sqrt{\frac{40}{4}} = \sqrt{10} \approx 3.16$$

5.

(a, b, d)

According to question no. of mole of hydrogen = no. of mole of helium = 1

Total internal energy, u

$$= \frac{f_1}{2}nRT + \frac{f_2}{2}nRT \Rightarrow u = \frac{3}{2}RT + \frac{5}{2}RT = 4RT$$

∴ Average internal energy per mole

$$= \frac{u}{2n} = \frac{4RT}{2} = 2RT$$

We know that $V_{sound} = \sqrt{\frac{\gamma RT}{M}}$

$$\frac{n_1 + n_2}{\gamma_{mix} - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \Rightarrow \frac{2}{\gamma_{mix} - 1}$$

$$= \frac{1}{\frac{5}{3} - 1} + \frac{1}{\frac{7}{5} - 1}$$

$$\frac{2}{\gamma_{mix} - 1} = \frac{3}{2} + \frac{5}{2} = 4 \Rightarrow \gamma_{mix} - 1 = \frac{1}{2} \therefore \gamma_{mix} = \frac{3}{2}$$

$$\frac{(V_s)_{mix}}{(V_s)_{He}} = \sqrt{\frac{\gamma_{mix}}{M_{mix}} \times \frac{M_{He}}{\gamma}}_{He} = \sqrt{\frac{\frac{3}{2} \times 4}{3 \times \frac{5}{3}}} = \sqrt{\frac{6}{5}}$$

$$\left[\because M_{mix} = \frac{1 \times 2 + 1 \times 4}{2} = 3\right]$$

We know that $V_{rms} = \sqrt{\frac{3RT}{M}}$

$$\therefore \frac{(V_{rms})_{He}}{(V_{rms})_{H2}} = \sqrt{\frac{M_{H2}}{M_{H_E}}} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$$

6.

(b, d)

We know for all gases $C_p.C_v = \frac{15}{4}\ and\ C_p + C_v = 4$

For diatomic gas: $C_v = \frac{5}{2R}; C_p = \frac{7}{2}R;\ \gamma = \frac{7}{5}$

$$\therefore C_p.C_v = \frac{35}{4}\ and\ C_p + C_v = 6$$

7.

(b) The total translational kinetic energy of n moles of gas

$= \frac{3}{2}nRT = 1.5PV$

$(\because P_v = nRT)$

Yes, the molecules of a gas collide with each other and the velocities of the molecules change due to collision.

8.

(d) The heat is supplied at constant pressure. i.e., the process is isobaric

$\therefore Q = nC_p\Delta t$

$$= 2\left[\frac{5}{2}R\right] \times \Delta t = 2 \times \frac{5}{2} \times 8.31 \times 5 = 208\,J$$

$\left(\because C_p = \frac{5}{2}R\ for\ mono-atomic\ gas\right)$

9.

(c) $pT = constant\ (given)$

$\therefore \left(\frac{nRT}{V}\right)T_2 = constant\ or\ T^3V^{-1} = constant$

$(\therefore PV = nRT)$

Differentiating the equation, we get

$$\frac{3T^2}{V}.dT - \frac{T^3}{V^2}dV = 0 \; or \; 3dT = \frac{T}{V}.dV \qquad ...(i)$$

From the equation

$dV = V\gamma dT$

γ = coefficient of volume expansion of gas

$$= \frac{dV}{V.dT}$$

from eq. (i) $\gamma = \frac{dV}{V.dT} = \frac{3}{T}$

10.

(900J) Given:$T_i = 100K, V_f = 8V_i$

For and adiabatic process, $TV^{\gamma-1} = constant$

Or, $T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$

$\Rightarrow \frac{T_i}{T_f} = \left(\frac{V_f}{V_i}\right)^{\gamma-1} \qquad \Rightarrow \frac{T_i}{T_f} = \left(\frac{8V_i}{V_i}\right)^{\gamma-1}$

For monoatomic gas $\gamma = \frac{5}{3}$

$$\therefore T_f = \frac{T_i}{(8^{\frac{5}{3}} - 1)} = \frac{T_i}{4}$$

Change in internal energy $\Delta u = nC_v \Delta T$

$= 1 \times \frac{3}{2}R\left(\frac{T_i}{4} - T_i\right) = \frac{3}{2} \times 8\left(-\frac{3}{4}\right) \times 100 =$

$-900J$=900j

11.

(b, c, d)

(a) As we know, Q = mc ΔT

$\Rightarrow \frac{dQ}{dt} = mc\frac{dT}{dT} or, \frac{dQ}{dt} \propto$

$C \; i.e., rate \; of \; heat \; absorption \; \propto$ C.

In the range 0 to 100K from the graph, C increases with temperature but not linearly therefore the rate at which heat is absorbed varies with temperature. But not linearly.

(b) As the value of C is greater in the temperature range 400-500K, the heat absorbed in increasing the temperature from 0-100K is less than the heat required for increasing the temperature from 400-500K.

(c) From the graph the value of C does not change in the temperature range 400-500K, therefore there is no change in the rate of heat absorption in the range.

(d) As the value of C increases from 200-300K, the rate of heart absorption increases in the range 200-300K.

12.

(d) Let T be the final temperature of the gases when equilibrium is achieved.

Heat lost by monatomic gas at constant volume

= Heat gained by diatomic gas at constant pressure

$\therefore nC_{v1}(700 - T) = nC_{p2}(T - 400)$

$\frac{3}{2}R(700 - T) = \frac{7}{2}R(T - 400)$

$\Rightarrow 2100 - 3T = 7T - 2800 \quad \Rightarrow 10T = 4900 \quad \therefore T = 490K$

13.

(d) As the pressure of gases in both compartment is the same

$\therefore nC_{p1}(700 - T) = nC_{p2}(T - 400)$

$\frac{5}{2}R(700 - T) = \frac{7}{2}R(T - 400)$

$\Rightarrow 3500 - 5T = 7T - 2800 \quad \Rightarrow 12T = 6300 \quad \therefore T = 425K$

Applying first law of thermodynamic

$\Delta W_1 + \Delta U_1 = \Delta Q_1$

And $\Delta W_2 + \Delta U_2 = \Delta Q_2$

$\Delta Q_1 + \Delta Q_2 = 0$

Or, $-(\Delta W_1 + \Delta W_2) = \Delta U_1 + \Delta U_2$

$= nC_{v1}(525 - 700) + n_2 C_{v2}(525 - 400)$

$= -2 \times \frac{3TR}{2} \times 175 + 2 \times \frac{5R}{2} \times 125$

$= -525R + 625R = -100R$

Therefore, total work done = -100R

14.

$P_1 = 830 - 30 = 800mm \; Hg \;\; ; P_2?$

$V_1 = V \quad ; \; V_2 = V \; ; \; T_1 = T \;\; ; T_2 = T - 0.01 \; T = 0.99 \; T$

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \therefore P_2 = \frac{P_1T_2}{T_1} = \frac{800 \times 0.099T}{T}$

$= 792mmHg$

$\therefore$ Total pressure in the jar = 792 + 25 = 817 mm Hg

15.

From, PV = nRT

When P,T are same n ∝ V

As volumes are same, i.e., 1c.c of each hydrogen and oxygen, So both samples will have equal number of molecules.

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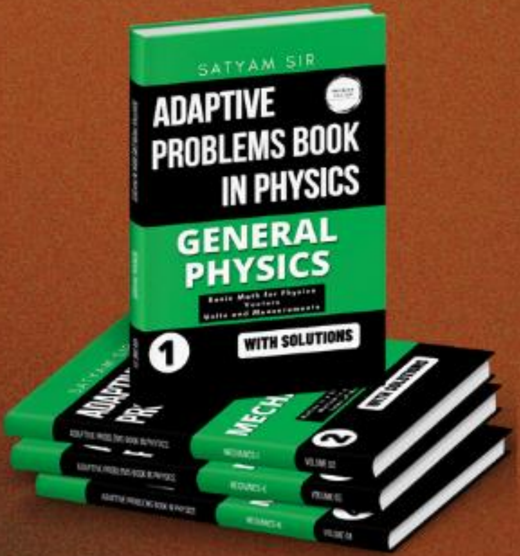
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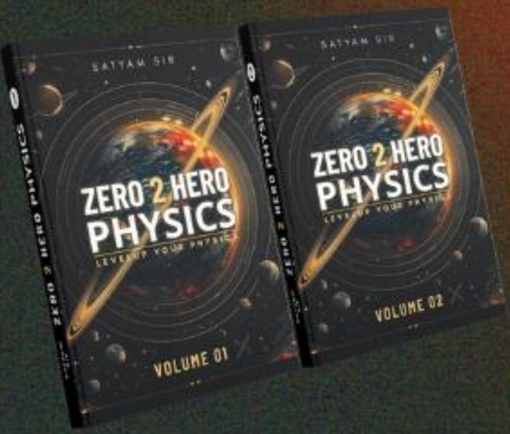
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This concept is not very clear!

I wish, if somebody could monitor my homework

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BOOK *Development* JOURNEY

2016-19

COLLECTIONS OF QUESTIONS & FRAMING

It was mostly from Foriegn author books and Kota Teachers notes. This collections took 3 years total.

2019

IDEA TO MAKE BEST NOTES FOR TEACHER

When I was in FIITJEE, Chennai, after 3 years of work, my teachers self reference note was completed.

2019-20

DEVELOPED EDUCATION APP FOR PHYSICS

I hired MTouch Labs, and made an app for students as well as teachers so that they can refer questions and theory.

2020

PUBLISHED 1ST TWO E-BOOKS ON AMAZON

It was the version 1.0 of the book. The book got decent sales without marketing in next 6 months.

2021-22

IDEA -BOOK TO HAVE MORE SUB-TOPIC QUESTIONS

When version 1.0 was live, I believe something is missing, So I started working on detailed sub-topic oriented style.

2022

TYPING, FORMATING, DIAGRAMS & REVIEWS

Once all subtopic segregations done, I assigned and hired various freelancers to type and format the book.

2023

REVIEWS FROM VARIOUS TEACHERS

I hired more than 15 teachers from all over india to review my book, finding typing mistakes, answer key corrections.

2024

VERSION 3.0 ZERO TO HERO RELEASED

2023 Year, I published Adaptive Problems Book, which was version 2.0 of my content, once all reviews are done. Then I started working on my final book project version 3.0.

Painful Endless Days & Nights, Tremendous Hardwork of Author & His Team

SATYAM SIR

Physics is a subject of complexity and understanding. To make concepts simpler and practice it in a more systematic manner, the Bihar born genius, Satyam Sir has made his deep efforts to publish this book. He has already helped many students to understand in depth subject concepts in the easiest possible manner. He is gifted with rare ability to simplify concepts and motivate the children how to solve complex problems in minutes. Sir has graduated from IIT Kharagpur in Civil Engineering and has been teaching Physics for JEE Mains and Advanced for more than a decade. The main goal of this book is to enhance problem solving ability in students. Sir is having hope that you would enjoy this journey of learning physics.

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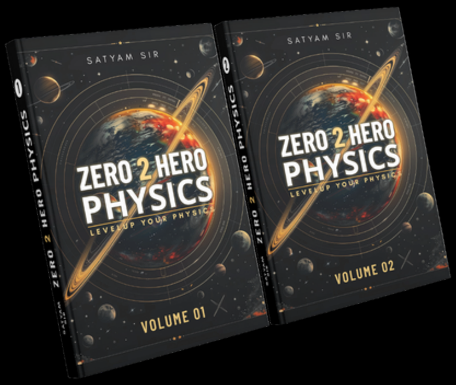

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